Can Cross-Lagged Panel Modeling Be Relied On to Establish Cross-Lagged Effects?

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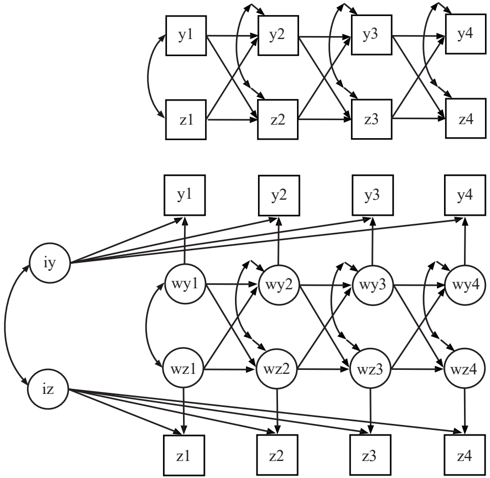
and Noah Hastings for expert assistance.

Outline

- Background, identification, and estimation
- Monte Carlo simulations
- Five examples
- Conclusions

Cross-Lagged Panel Modeling: CLPM and RI-CLPM



- Effects between observed variables vs latent within-person variables
  - Mplus Web Talk No. 4, Part 1 (continuous), Part 2 (categorical)

Focus of Talk

- Key question: Can cross-lagged panel modeling be relied on to establish cross-lagged effects?
  - Alternative models with contemporaneous (lag 0) effects may challenge the conclusions from CLPM and RI-CLPM
  - Are models with reciprocal contemporaneous effects identified?
  - Are models with both cross-lagged and reciprocal contemporaneous effects identified?
- Asparouhov & Muthén (2022). The identification of the reciprocal vector auto-regressive model
  - The answer is yes - under certain conditions
- Do such models work well in practice?
  - What do simulations and real-data examples say?

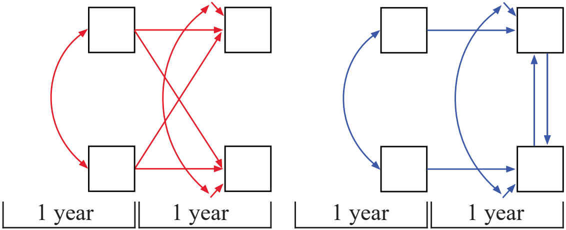
Earlier Research

- Greenberg-Kessler (1982). Equilibrium and identification in linear panel models. Sociological Methods & Research, 435-451
  - Identification by imposing a certain degree of time invariance
  - Disappointing conclusions: ”These results are discouraging”, ”the approach can be used in practice under a very restricted set of circumstances”
- Ormel, Rijsdijk, Sullivan, van Sonderen & Kempen (2002). Temporal and reciprocal relationship between IADL/ADL disability and depressive symptoms in late life. Journal of Gerontology: Psychological Sciences, vol 57B, No. 4, 338-347
  - Model estimated with both cross-lagged and reciprocal effects, as well as random intercepts
  - No proof of identification: ”The full model is identified. Very different starting values gave the same solution”

Panel Data Measurement Instruments

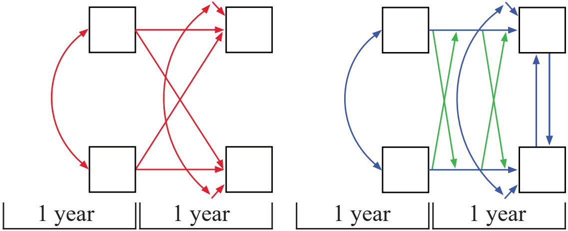
- Two common types of question format:
  - Past status: In the last [time interval], how frequently did you...
  - Current status: How much do you agree with...
- Time intervals for six data sets used in Orth et al. (2021): 2 months, 6 months, 1 year, 2 years
  - Orth et al. (2021). Testing prospective effects in longitudinal research: Comparing seven competing cross-lagged models. Journal of Personality and Social Psychology
- Cross-lagged effects may be less realistic with long time intervals and may call for allowing contemporaneous (lag 0) effects
- Dormann & Griffin (2015). Optimal time lags in panel studies. Psychological Methods, 24, 489-505
  - Deboeck & Preacher (2015). SEM journal

Two Competing Panel Data Models: Cross-Lagged vs Reciprocal Effects



- Cross-lagged effects of events 1-2 years ago may be weak. The reciprocal (lag 0) model on the right may be more realistic with long time intervals and measurements referring to past experiences
- The reciprocal model is identified by the classic econometric rule that each reciprocal DV has its own predictor
- The two models are equivalent and cannot be statistically distinguished: Same number of parameters, same model fit to data: Finding cross-lagged effects does not rule out the reciprocal model and finding reciprocal effects does not rule out the cross-lagged model

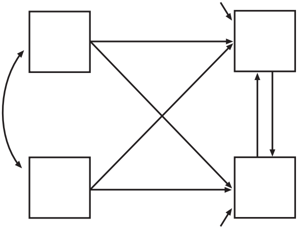
Two Competing Panel Data Models: Cross-Lagged vs Reciprocal Effects, Cont'd



- The reciprocal model on the right in blue can be seen as a summary of several sets of cross-lagged effects (green arrows) and thereby does not disentangle the recursive "causal effects" due to not measuring frequently enough

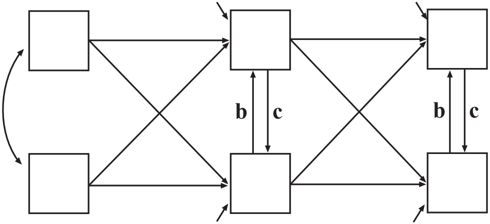
Reciprocal Cross-Lagged Panel Model, T = 2

- Can we include both cross-lagged and reciprocal effects?:



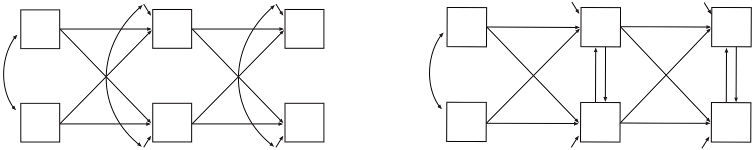
- 6 regression parameters but only 5 sample covariances to identify them (ignoring the time 1 sample covariance)
- The model is not identified - but is identifiable when T>2 !
- Greenberg-Kessler (1982) Soc Meth & Res., Ormel et al. (2002) J of Gerontology, Asparouhov & Muthén (2022)

Reciprocal Cross-Lagged Panel Model, T = 3



- The model is identified with equality of the reciprocal effects across time 2 and time 3 (Asparouhov & Muthén, 2022)
- Adding random intercepts does not affect the identification status
- The model is not identified if all parameters are time invariant
  - reciprocal DSEM is not possible
- Special considerations are needed in the analysis

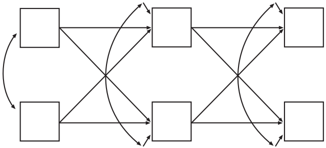
CLPM versus Reciprocal CLPM (T = 3)



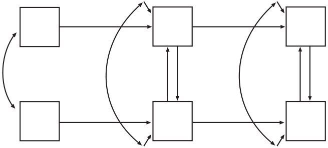
(a) CLPM (b) Reciprocal CLPM

- The residual covariances of model (a) allow time-varying unmeasured common causes to influence the two outcomes while the residual covariances are zero in model (b)
- In line with regular regression, model (a) assumes that the residuals are uncorrelated with the two predictors at the previous time point - if this is not the case, the cross-lagged effects are biased
- If data have been generated by model (b), model (a) residuals are correlated with the predictors because each outcome at time t is influenced by the other variable at time t

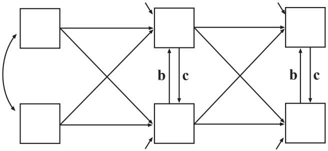
Three Equivalent Models (Random Intercepts or Not)



CLPM



Reciprocal Only



Reciprocal

Key Findings for the Reciprocal Model in Asparouhov & Muthén (2022)

- The model is identified for T$\geq$ 3 with equality of the reciprocal effects across some of the time points
- The model has T-1 more parameters than CLPM/RI-CLPM
- The model is equivalent to CLPM/RI-CLPM under T-1 restrictions such as:
  - T=3: Reciprocal effects are invariant for time 2 and time 3 (2 restrictions)
  - T=5: Reciprocal effects are invariant for time 2 and 3 and for time 4 and 5 (4 restrictions)
  - T=4: No natural choice. Could use reciprocal 2=3, 4 (3 restrictions)
  - Full time invariance for reciprocals may be reasonable, using fewer parameters than CLPM/RI-CLPM
- The cross-lagged effects of the reciprocal model are different from those of CLPM/RI-CLPM

Key Findings for the Reciprocal Model in Asparouhov & Muthén (2022) Cont'd

- The model has 2 solutions - restriction needed to choose the acceptable solution; essential for bootstrapping and Monte Carlo simulations
  - The 2 solutions can be seen as an innocuous type of non-identification in line with 1-factor analysis where factor loadings can be all positive or all negative and give the same fit (so not infinitely many solutions as in typical non-identification)
- R^2 may be negative when the restriction is not fulfilled/applied
- The distribution of the reciprocal estimates is non-normal and non-symmetric CIs may be needed (bootstrapping; Bayes not available yet)
- There is a need for larger sample sizes than for CLPM and RI-CLPM to get sufficient power to detect reciprocal effects
- When cross-lagged effects are not included, there are not 2 solutions, R^2 can still be negative, but non-symmetric CIs are not needed

Restrictions on Reciprocal Effects in Asparouhov & Muthén (2022)

Two types of restrictions on the reciprocal effects r_y and r_z:

- (a): $0 < r_y r_z < 1$
  - Reciprocals are restricted to both be either positive or negative - opposite signs not allowed
  - Avoids dual solutions and negative R-square
  - Suitable for real-data analysis
- (b): $(r_y r_z)^2 < 1$
  - Avoids dual solutions
  - Can be used to study bootstrap distributions before applying (a)
  - Suitable for MonteCarlo studies - restriction (a) always gives power = 1 for reciprocal effects irrespective of sample size

Guidelines for Real-Data Analysis in Asparouhov & Muthén (2022)

1. Estimate the model with the non-duality/R-square constraint (a) $0 < r_y r_z < 1$ and use the STARTS option to ensure getting the best solution
2. If both reciprocal effects are significant, use the model
3. If one or both reciprocals are not significant, eliminate the parameter
4. If one of the reciprocals has a zero estimate, the solution is at the border of the constraint. Re-estimate with non-duality constraint (b) $(r_y r_z)^2 < 1$ to check if a solution is found with different reciprocal signs and better fit

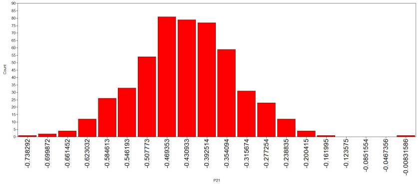
Monte Carlo Simulations

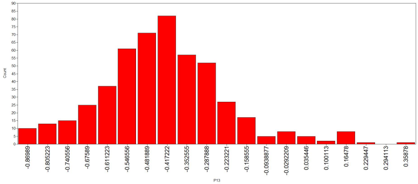
- Data generated under the same model as used in the analysis (H0 model correct)
- Data generated by the reciprocal model and analyzed by regular RI-CLPM - and vice versa (presented later)

Monte Carlo Simulations: H0 Model Correct

- Parameter values based on T = 5 MWI analysis (but normality and no missing data) estimated by a Reciprocal RI-CLPM
- Cross-lagged effects same across time in data generation but time invariance not imposed in analyses
- Reciprocal effects time invariant in both data generation and analyses
- Bootstrap = 500 to capture non-normal estimate distributions and create non-symmetric CIs
- Non-duality reciprocal restriction (b) $(r_y r_z)^2 < 1$ is used
- Number of time points T varied as 3, 4, 5. T = 3 and 4 runs based on first 3 and 4 time points
- Sample size N varied as 500, 750, 1000
- Focus on quality of estimates and SEs as well as power (% Sig Coeff) for both BS and MLR. χ^2 5% reject proportions good

Monte Carlo Distribution of Reciprocal Effect $S_t \rightarrow D_t$ for N = 500: T = 5 and T = 3





- The skewness is only 0.107 for T = 5 and 0.513 for T = 3
  - MLR CIs may be good enough
- How the plot is done:
  - Use the RESULTS option in the MONTECARLO command to save estimates for all replications
  - Do a TYPE=BASIC run on the saved file to plot the distribution

T = 5: Bootstrap Results with Two Added MLR Columns

	Population	ESTIMATES Average	Std. Dev.	S. E. Average	**MLR** S.E. Average	M. S. E.	95% Cover	% Sig Coeff	% Sig **MLR**
T = 5, N = 1000									
S5ˆ ON									
S4ˆ	0.510	0.5121	0.0455	0.0550	0.0491	0.0021	0.976	1.000	1.000
D4ˆ	-0.158	-0.1630	0.0674	0.0835	0.0729	0.0046	0.964	0.670	0.680
D5ˆ	-0.091	-0.0760	0.1680	0.2242	0.1789	0.0284	0.964	0.060	0.120
D2ˆ ON									
D1ˆ	0.214	0.2135	0.0417	0.0469	0.0438	0.0017	0.962	0.992	0.990
S1ˆ	0.059	0.0569	0.0381	0.0394	0.0378	0.0014	0.946	0.294	0.334
S2ˆ	-0.431	-0.4323	0.0966	0.1251	0.1028	0.0093	0.970	0.844	**0.954**
T = 5, N = 750									
S5ˆ ON									
S4ˆ	0.510	0.5114	0.0543	0.0665	0.0612	0.0029	0.972	1.000	0.992
D4ˆ	-0.158	-0.1667	0.0894	0.0994	0.0912	0.0080	0.962	0.504	0.480
D5ˆ	-0.091	-0.0693	0.2267	0.2704	0.2319	0.0518	0.962	0.048	0.112
D2ˆ ON									
D1ˆ	0.214	0.2141	0.0505	0.0560	0.0631	0.0025	0.960	0.966	0.964
S1ˆ	0.059	0.0582	0.0427	0.0472	0.0445	0.0018	0.964	0.228	0.254
S2ˆ	-0.431	-0.4311	0.1265	0.1535	0.1303	0.0160	0.962	0.712	**0.876**
T = 5, N = 500									
S5ˆ ON									
S4ˆ	0.510	0.5115	0.0745	0.0830	0.0782	0.0055	0.970	1.000	0.992
D4ˆ	-0.158	-0.1693	0.1136	0.1211	0.1171	0.0130	0.956	0.368	0.354
D5ˆ	-0.091	-0.0644	0.2925	0.3214	0.2989	0.0861	0.968	0.056	0.130
D2ˆ ON									
D1ˆ	0.214	0.2157	0.0629	0.0699	0.0672	0.0040	0.964	0.898	0.898
S1ˆ	0.059	0.0551	0.0546	0.0610	0.0566	0.0030	0.974	0.118	0.166
S2ˆ	-0.431	-0.4255	0.1639	0.1855	0.1681	0.0269	0.964	0.564	**0.792**

T = 4

	Population	ESTIMATES Average	Std. Dev.	S. E. Average	MLR S.E. Average	M. S. E.	95% Cover	% Sig Coeff	% Sig MLR
T = 4, N = 1000									
S4ˆ ON									
S3ˆ	0.412	0.4113	0.0503	0.0574	0.0538	0.0025	0.974	1.000	0.998
D3ˆ	-0.158	-0.1652	0.0911	0.1004	0.0911	0.0083	0.964	0.451	0.431
D4ˆ	-0.091	-0.0642	0.1969	0.2371	0.2043	0.0394	0.966	0.048	0.100
D2ˆ ON									
D1ˆ	0.214	0.2142	0.0495	0.0515	0.0488	0.0024	0.956	0.988	0.982
S1ˆ	0.059	0.0591	0.0419	0.0434	0.0415	0.0018	0.950	0.248	0.273
S2ˆ	-0.431	-0.4369	0.1073	0.1299	0.1122	0.0115	0.962	0.830	**0.934**
T = 4, N = 750									
S4ˆ ON									
S3ˆ	0.412	0.4103	0.0604	0.0684	0.643	0.0036	0.968	1.000	1.000
D3ˆ	-0.158	-0.1688	0.1064	0.1171	0.1109	0.0116	0.966	0.357	0.301
D4ˆ	-0.091	-0.0577	0.2466	0.2765	0.2540	0.0618	0.978	0.040	0.116
D2ˆ ON									
D1ˆ	0.214	0.2141	0.0578	0.0610	0.0587	0.0033	0.970	0.964	0.956
S1ˆ	0.059	0.0595	0.0481	0.0519	0.0492	0.0023	0.962	0.210	0.240
S2ˆ	-0.431	-0.4348	0.1325	0.1523	0.1362	0.0175	0.972	0.727	**0.866**
T = 4, N = 500									
S4ˆ ON									
S3ˆ	0.412	0.4079	0.0773	0.0876	0.0845	0.0060	0.972	0.988	0.974
D3ˆ	-0.158	-0.1763	0.1464	0.1438	0.1499	0.0217	0.956	0.251	0.196
D4ˆ	-0.091	-0.0427	0.3349	0.3324	0.3491	0.1143	0.956	0.059	0.119
D2ˆ ON									
D1ˆ	0.214	0.2126	0.0729	0.0765	0.0777	0.0053	0.962	0.834	0.826
S1ˆ	0.059	0.0581	0.0561	0.0669	0.0625	0.0031	0.976	0.115	0.145
S2ˆ	-0.431	-0.4329	0.1774	0.1857	0.1911	0.0314	0.960	0.600	**0.739**

T = 3

	ESTIMATES Population	Average	Std. Dev.	S. E. Average	MLR S.E. Average	M. S. E.	95% Cover	% Sig Coeff	% Sig MLR
				T = 3, N = 1000					
S3ˆ ON									
S2ˆ	0.020	0.0195	0.1141	0.1237	0.1183	0.0130	0.956	0.040	0.036
D2ˆ	-0.158	-0.1623	0.0875	0.1020	0.0961	0.0077	0.968	0.396	0.420
D3ˆ	-0.091	-0.0452	0.3015	0.2900	0.2912	0.0928	0.940	0.068	0.080
D2ˆ ON									
D1ˆ	0.214	0.2182	0.0602	0.0663	0.0666	0.0036	0.954	0.944	0.918
S1ˆ	0.059	0.0594	0.0621	0.0598	0.0583	0.0039	0.946	0.180	0.178
S2ˆ	-0.431	-0.4339	0.1337	0.1414	0.1401	0.0179	0.948	0.784	**0.860**
				T = 3, N = 750					
S3ˆ ON									
S2ˆ	0.020	0.0176	0.1310	0.1486	0.1406	0.0171	0.958	0.036	0.034
D2ˆ	-0.158	-0.1587	0.1094	0.1232	0.1183	0.0119	0.964	0.282	0.276
D3ˆ	-0.091	-0.0221	0.3642	0.3221	0.3427	0.1372	0.938	0.068	0.093
D2ˆ ON									
D1ˆ	0.214	0.2189	0.0699	0.0764	0.0771	0.0049	0.958	0.875	0.827
S1ˆ	0.059	0.0631	0.0704	0.0708	0.0687	0.0050	0.948	0.145	0.169
S2ˆ	-0.431	-0.4346	0.1662	0.1597	0.1632	0.0276	0.934	0.710	**0.791**
				T = 3, N = 500					
S3ˆ ON									
S2ˆ	0.020	0.0105	0.1666	0.1965	0.1788	0.0278	0.974	0.022	0.022
D2ˆ	-0.158	-0.1639	0.1338	0.1653	0.1540	0.0179	0.976	0.163	0.187
D3ˆ	-0.091	-0.0054	0.4363	0.3743	0.4169	0.1974	0.936	0.070	0.099
D2ˆ ON									
D1ˆ	0.214	0.2260	0.0946	0.0961	0.0967	0.0091	0.960	0.708	0.648
S1ˆ	0.059	0.0618	0.0862	0.0908	0.0876	0.0074	0.952	0.109	0.113
S2ˆ	-0.431	-0.4311	0.2003	0.1948	0.2021	0.0400	0.940	0.584	**0.682**

Five Examples

- 3 examples from Orth et al. (2021) concerning depression and self-esteem:
  - MWI data: N = 663, T = 5, Interval = 2 months
  - BLS data: N = 404, T = 4, Interval = 1 year
  - NLSY data: N = 8,259, T = 11, Interval = 2 years
- Depression and disability (Ormel et al., 2002):
  N = 753, T = 3, Interval = 1 year
- Academic self-concept and achievement (Nunez-Regueiro et al., 2021; Kenny & McCoach, 2022):
  N = 933, T = 5, Interval = 4 months

Example 1: The MWI Data on Depression and Self-Esteem

- Adult sample, N = 663, T = 5, Coverage = 0.99 - 0.57
- Self-esteem and depression measured two months apart
- Self-esteem: Participants were asked how much they agree with each of the statements included in the scale (no time frame stated, so could include current and past status)
- Depression: Participants were instructed to assess how frequently they had experienced each symptom within the preceding 30 days
- RI-CLPM with time invariant cross-lagged effects points to a small but significant cross-lagged effect of depression on self esteem (Orth et al., 2021, Table 6)
- Orth et al. (2021). Testing prospective effects in longitudinal research: Comparing seven competing cross-lagged models. Journal of Personality and Social Psychology

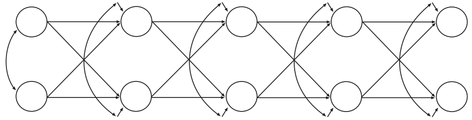
Choice of Estimators for Reciprocal RI-CLPM

- MLR gives good chi-square and SE's but does not provide non-symmetric confidence intervals
- MLR parameter estimates = ML parameter estimates = parameter estimates using bootstrap
- Bootstrap analysis uses ML and gives bootstrap SEs and bootstrap non-symmetric confidence intervals matching a skewed distribution for the reciprocal estimates. The bootstrap SE's are bit inflated and the CIs may be a bit too wide (conservative)
- Conclusion: Do 2 runs, 1 with MLR to get chi-square and 1 with bootstrap to get CIs

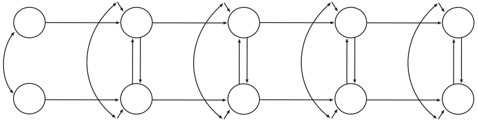
Mplus Input for Reciprocal RI-CLPM with MWI Data

```
TITLE:        Reciprocal RI-CLPM for
              MWI data
DATA:         FILE = mwi.dat;
VARIABLES:    NAMES = id s1-s5
              d1-d5;
              USEVAR = s1-s5 d1-d5;
              MISSING = ALL (-999);

ANALYSIS:     ESTIMATOR = ML;
              ! ML for bootstrap.
              ! Use MLR for chi-2
              BOOTSTRAP = 500;
              STARTS = 20;

MODEL:        ! Random intercepts:
              is BY s1-s5@1;
              id BY d1-d5@1;
              ! ARs:
              s2^-s5^ PON s1^-s4^;
              d2^-d5^ PON d1^-d4^;
              ! Cross-lags:
              s2^-s5^ PON d1^-d4^ ;
              d2^-d5^ PON s1^-s4^ ;
              ! Reciprocals:
              s2^-s5^ PON d2^-d5^ (rsd);
              d2^-d5^ PON s2^-s5^ (rds);
              s1^ WITH d1^;
MODEL
CONSTRAINT:   ! 2 alternatives
              ! (a) R2 pos and non-duality:
              0 <rsd*rds;
              0 <1 - rsd*rds;
              ! (b) Non-duality:
              ! 0 >(rsd*rds)^2 - 1;
OUTPUT:       STDYX RESIDUAL TECH1
              CINTERVAL(BOOTSTRAP);
PLOT:         TYPE = PLOT3;
```

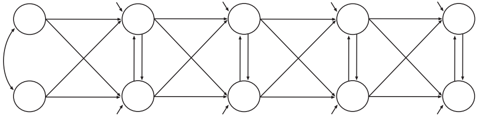
Three Key Models: Within Part of Random Intercept Version



A: RI-CLPM



B: Reciprocal Only



C: Reciprocal

Model Fit for Three Equivalent Random Intercept Cross-Lagged and Reciprocal Models (MLR)

Model	# par's	LogL	BIC	χ^2	Df	P-value	RMSEA	P-value
A. RI-CLPM	44	-1532	3349	34	21	0.0323	0.031	0.958
B. Reciprocal Only	44	-1532	3349	34	21	0.0323	0.031	0.958
C. Reciprocal 4 reciprocals: 2=3, 4=5	44	-1532	3349	34	21	0.0323	0.031	0.958

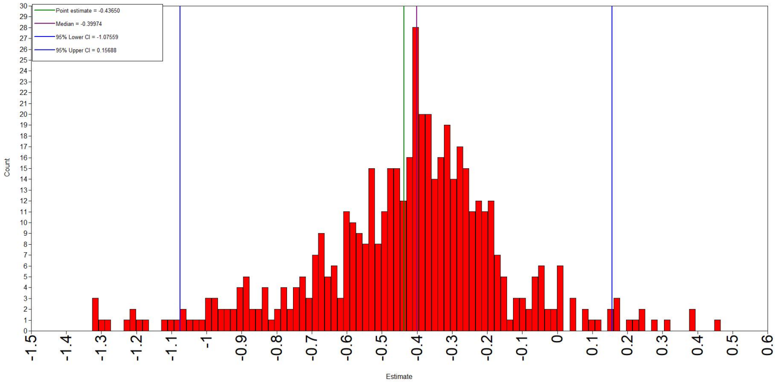
- Model C uses restriction (b) of non-duality and gets 2 negative R-square values: Solution should not be used
- Model C using restriction (a) of non-duality and positive R-square gets a worse logL = -1535 (BIC = 3357): Equivalence with Model A is lost
- Model C with fully time-invariant reciprocals (2 instead of 4 reciprocals estimated) gets the same logL = -1532 in these data with 2 fewer parameters and therefore a better BIC value (see next slide)

Model Fit for Cross-Lagged and Reciprocal Models (MLR)

Model	# par's	LogL	BIC	χ^2	Df	P-value	RMSEA	P-value
1. RI-CLPM	44	-1532	3349	34	21	0.0323	0.031	0.958
2. RI-CLPM Invar. X-lags	38	-1546	3338	60	27	0.0002	0.043	0.763
3. Reciprocal Only	44	-1532	3349	34	21	0.0323	0.031	0.958
4. Reciprocal Only Invar. Recips	38	-1538	3323	45	27	0.0181	0.031	0.975
5. Reciprocal Invar Recips	42	-1532	3337	34	23	0.0637	0.025	0.990
6. Reciprocal Invar. X-lags and Recips	36	-1539	**3312**	45	29	0.0289	0.027	0.992

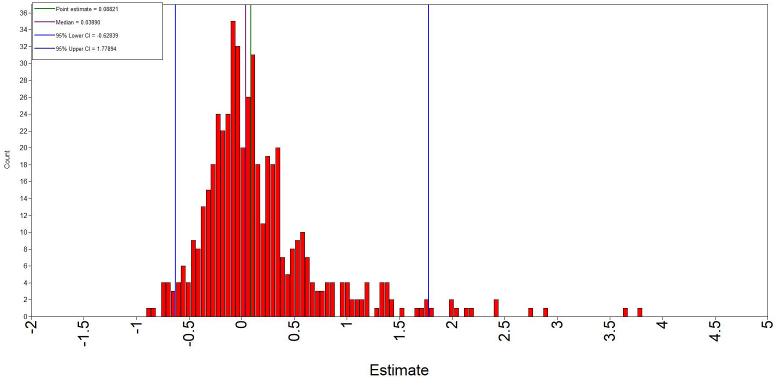
- Model 1 is equivalent to Model 3. Model 1 is also equivalent to a reciprocal RI-CLPM with reciprocals restricted to equality for e.g. times 2=3, 4=5. This means that comparing Model 5 to Model 1 tests full reciprocal invariance 2=3=4=5: Model 5 not rejected (same LogL)

Bootstrap Distribution of Reciprocal Effect St →Dt: Reciprocal RI-CLPM with Time Invariant Cross-Lagged and Reciprocal Effects (Model 6)



- Skewed distribution
- Peak around -0.4 but not quite significant (the run does not apply reciprocal restrictions)

Bootstrap Distribution of Reciprocal Effect Dt →St: Reciprocal RI-CLPM with Time Invariant Cross-Lagged and Reciprocal Effects (Model 6)



- Peak around zero and insignificant - parameter can be fixed at zero

Model Fit Continued (MLR)

Model	# par's	LogL	BIC	χ^2	Df	P-value	RMSEA	P-value
1. RI-CLPM	44	-1532	3349	34	21	0.0323	0.031	0.958
2. RI-CLPM Invar. X-lags	38	-1546	3338	60	27	0.0002	0.043	0.763
3. Reciprocal Only	44	-1532	3349	34	21	0.0323	0.031	0.958
4. Reciprocal Only Invar. Recips	38	-1538	3323	45	27	0.0181	0.031	0.975
5. Reciprocal Invar Recips	42	-1532	3337	34	23	0.0637	0.025	0.990
6. Reciprocal Invar. X-lags and Recips	36	-1539	3312	45	29	0.0289	0.027	0.992
7. Reciprocal Invar. X-lags and Recips $S_t \rightarrow D_t$ only	35	-1539	**3306**	45	30	0.0386	0.027	0.993
8. Reciprocal Invar. X-lags and Recips $D_t \rightarrow S_t$ only	35	-1546	3319	59	30	0.0011	0.038	0.906

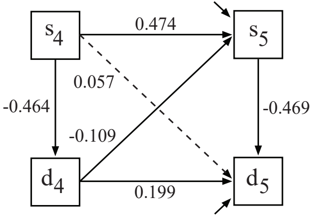
Estimated Effects in Three Key Models for the MWI Data

- Several models fit the data about the same but have different interpretations

Model	Significant Cross-lags	Significant Reciprocals
2. RI-CLPM Invar. X-lags	$D_{t\text{-}1} \overset{-}{\rightarrow} S_t$	NA
4. Reciprocal Only Invar. Recips	NA	None
7. Reciprocal Invar. X-lags and Recips $S_t \rightarrow D_t$ only	$D_{t\text{-}1} \overset{-}{\rightarrow} S_t$ (sig. also with bootstrap CI)	$S_t \overset{-}{\rightarrow} D_t$ (sig. also with bootstrap CI)

- Model 7 may be preferrable because it is more informative than the others, containing Model 2 as a subset, and does not fit worse. Model 7 has the best BIC (BIC is useful because the 3 models are not nested)
- Standardized $D_{t\text{-}1} \overset{-}{\rightarrow} S_t$ estimates for models 2 and 7 are very close

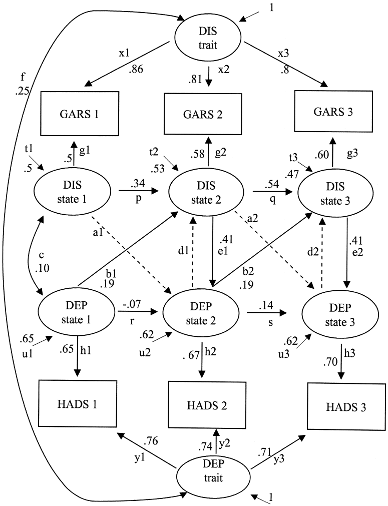
Model 7 Indirect Effects on D5 (Standard'd, Bootstrap CIs)



- **Total s4 →d5**: -0.282, BS CI [-0.421 - 0.148]
- **Total indirect s4 →d5**: -0.339, BS CI [-0.459 -0.206]
- s4 →s5 →d5: -0.223, BS CI [-0.302 -0.128]
- s4 →d4 →d5: -0.092, BS CI [-0.157 -0.015]
- s4 →d4 →s5 →d5: insignificant
- s4 →d5 (direct effect): insignificant
- **Total d4 →d5**: 0.250, BS CI [0.070 0.402]
- **Total indirect d4 →d5**: 0.051, BS CI [0.012 0.103]
- d4 →s5 →d5: 0.051, BS CI [0.012 0.103]
- d4 →d5 (direct effect): 0.199, BS CI [0.028 0.341]

Example 2: Depression and Disability, N=753, T = 3

- Ormel, Rijsdijk, Sullivan, van Sonderen & Kempen (2002). Temporal and reciprocal relationship between IADL/ADL disability and depressive symptoms in late life. Journal of Gerontology: Psychological Sciences, vol 57B, No. 4, 338-347



Example 2 Continued: Depression and Disability, N=753, T = 3, Interval = 1 Year (Ormel et al., 2002)

- Their reciprocal RI-CLPM findings mimic our MWI example:
  - One of the reciprocal effects is found insignificant (small negative effect, whereas the significant effect is positive and larger)
- Comments:
  - The analysis uses time-invariant reciprocal effects (and time-invariant cross-lagged effects), but does not make clear that this model is equivalent to the regular RI-CLPM (with T = 3, there are 2 reciprocal parameters vs 2 residual covariances), that is, reciprocal interaction is just one interpretation of the data
- Raw data no longer available, but Table 2 gives the estimated covariance matrix for the saturated model taking missing data into account (76% have complete data for all 3 time points)
  - This matrix can be used as the sample covariance matrix to give estimates close to what the raw data would give (χ^2 and SE's are not correct)

Example 2: Depression and Disability. Model Fit (ML)

	#par's	LogL	BIC
1. RI-CLPM	20	-9139	18410
2. RI-CLPM Invar X-lags	18	-9139	18397
3. Reciprocals Only	20	-9139	18410
4. Reciprocals Only Invar Recip's	18	-9142	18403
5. Reciprocal Invar Recip's	20	-9139	18410
6. Reciprocals Invar X-lags and Recip's	18	-9139	18397
7. Reciprocals Invar X-lags and Recip's $DIS_t \rightarrow DEP_t$ only	17	-9139	**18391**

- Models 3 and 4 converged only with starting values derived from RI-CLPM but obtained negative R-square values and did not converge with restrictions (a) or (b)
- Model 7 estimates close to those presented in Figure 2 of the Ormel at al. article

Estimated Effects in Three Key Models for the Depression and Disability Data

Model	Significant Cross-lags	Significant Reciprocals
2. RI-CLPM Invar. X-lags	$\mathrm{DIS}_{t\text{-}1} \xrightarrow{+} \mathrm{DEP}_t$ $\mathrm{DEP}_{t\text{-}1} \xrightarrow{+} \mathrm{DIS}_t$	NA
4. Reciprocal Only Invar. Recips	NA	No solution
7. Reciprocal Invar. X-lags and Recips	$\mathrm{DEP}_{t\text{-}1} \xrightarrow{+} \mathrm{DIS}_t$	$\mathrm{DIS}_t \xrightarrow{+} \mathrm{DEP}_t$

Example 3: BLS Data on Depression and Self-Esteem Orth et al. (2021), N = 404, T = 4: Model Fit (MLR)

	#par's	LogL	BIC	χ^2	Df	P-value	RMSEA	P-value
1. RI-CLPM	35	-1579	3368	6	9	0.6910	0.000	0.973
2. RI-CLPM Invar X-lags	31	-1581	3349	10	13	0.6567	0.000	0.984
3. Reciprocals Only	35	-1579	3368	6	9	0.6911	0.000	0.973
4. Reciprocals Only Invar Recip's	31	-1580	3347	9	13	0.7908	0.000	0.994
5. Reciprocal Invar Recip's	34	-1581	3366	12	10	0.3048	0.021	0.871
6. Reciprocals Invar X-lags and Recip's	30	-1582	**3344**	11	14	0.6793	0.000	0.990

- Several models fit the data about the same but have different interpretations

Estimated Effects in Three Key Models for the BLS Data

Model	Significant Cross-lags	Significant Reciprocals
2. RI-CLPM Invar. X-lags	None	NA
4. Reciprocal Only Invar. Recips	NA	None
6. Reciprocal Invar. X-lags and Recips	None *	$D_t \overset{-}{\rightarrow} S_t$ (sig. also with bootstrap CI)

- Model 6 may be preferred over models 2 and 4 because it finds a relationship between the two variables and does not fit worse than alternative models. It has the best BIC
- * Unlike the MWI example, fixing $S_t \rightarrow D_t$ at its Model 6 estimate of zero, does not give a significant cross-lagged effect $D_{t\text{-}1} \rightarrow S_t$

Example 4: NLSY79 Depression and Self-Esteem

- Adolescents and young adults, N = 8,259, T = 11
- Depression and self-esteem measured 2 years apart 1994-2014 (Orth et al., 2021)
- Max 7-8 points for any one person gives low coverage
- Time-invariant modeling using ML (the MLR H1 model cannot be estimated, so no MLR chi-2)

Model Fit for Cross-Lagged and Reciprocal Models (ML)

Model	# par's	LogL	BIC	χ^2	Df	P-value	RMSEA	P-value
1. RI-CLPM Inv Xlags	80	-37461	75644	403	179	0.000	0.012	1.000
2. Reciprocal Only Invar. Recips	80	-37445	75612	371	179	0.000	0.011	1.000
3. Reciprocal Inv Xlags Invar Recips	72	-37465	75580	412	187	0.000	0.012	1.000
4. Reciprocal Inv Xlags Invar Recips $DEP_t \rightarrow SE_t$ only	71	-37466	**75572**	412	188	0.000	0.012	1.000

- Model 4 has the best BIC

Estimated Effects in Three Key Models for the NLSY Data

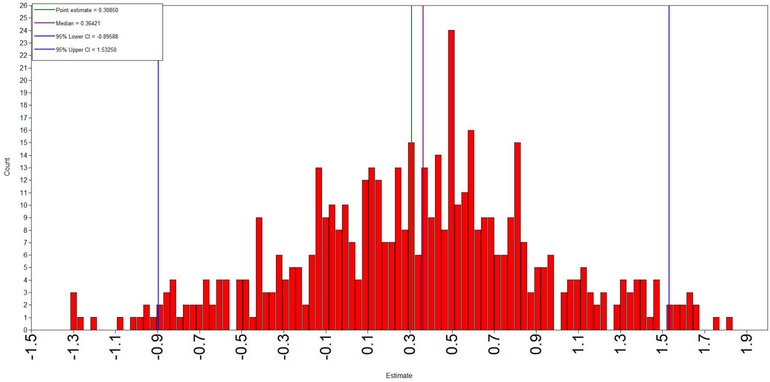
Model	Significant Cross-lags	Significant Reciprocals
1. RI-CLPM Invar. X-lags	$SE_{t\text{-}1} \overset{-}{\longrightarrow} DEP_t$ $DEP_{t\text{-}1} \overset{-}{\longrightarrow} SE_t$	NA
2. Reciprocal Only Invar. Recips	NA	$SE_t \overset{-}{\longrightarrow} DEP_t$ $DEP_t \overset{-}{\longrightarrow} SE_t$
4. Reciprocal Invar. X-lags Invar. Recips $DEP_t \overset{-}{\longrightarrow} SE_t$ only	$SE_{t\text{-}1} \overset{-}{\longrightarrow} DEP_t$	$DEP_t \overset{-}{\longrightarrow} SE_t$

- Model 2 contradicts Model 1 and is partly supported by Model 4

Example 5: Academic Self-Concept and Achievement (GPA)

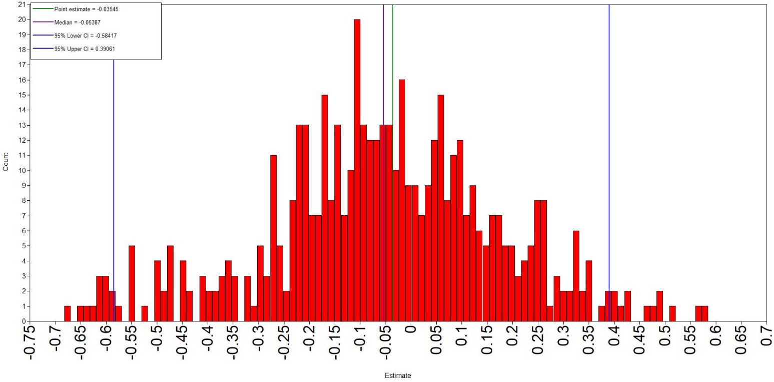
- French high school students, N = 944, T = 5
- Academic self-concept and achievement (GPA) measured 5 times over 6 trimesters during first and second years of high school (no data for the 5th trimester so non-equidistant)
- Nunez-Regueiro et al. (2021) analyzed in Kenny & McCoach (2022)
- Reciprocal RI-CLPM with non-invariant cross-lags and invariant reciprocals (model 4)

Bootstrap Distribution of Reciprocal Effects for Model 4: $\text{GPA}_t \rightarrow$ Academic Self-Concept$_t$



- Peak around +0.4 but insignificant (non-duality restriction only applied)

Bootstrap Distribution of Reciprocal Effects for Model 4: Academic Self-Concept$_t$ $\rightarrow$GPA$_t$



- Peak around zero - parameter can be fixed at zero (non-duality restriction only applied)

Model Fit for Cross-Lagged and Reciprocal Models (MLR)

Model	# par's	LogL	BIC	χ^2	Df	P-value	RMSEA (p-value)
1. RI-CLPM Non-inv Xlags	44	-12876	26054	71	21	0.000	0.051 (.446)
2. RI-CLPM First 3 Xlags Inv	40	-12878	26029	73	25	0.000	0.045 (0.722)
3. Reciprocal Only							
4. Reciprocal Non-inv Xlags Invar Recips	42	-12877	26040	71	23	0.000	0.047 (.608)
5. Reciprocal Non-inv. Xlags Invar. Recips $GPA_t \rightarrow ASC_t$ only	41	-12877	26034	84	24	0.000	0.052 (.382)

- Model 3 - no solution for the 44-parameter equivalent model version or the invariant reciprocals version: Model 3 not suitable for this data set
- Model 4 - no solution for the 44-parameter equivalent version that applies T-1 = 4 restrictions on the 8 reciprocals (e.g., 2=3, 4=5: 4 par's)

Estimated Effects in Three Key Models for the GPA Data

Model	Significant Cross-lags	Significant Reciprocals
2. RI-CLPM First 3 X-lags inv	$\text{GPA}_{t\text{-}1} \xrightarrow{+} \text{ASC}_t$	NA
3. Reciprocal Only	NA	No solution
5. Reciprocal Non-inv X-lags Invar. Recips GPA $\rightarrow$ASC only	None	$\text{GPA}_t \xrightarrow{+} \text{ASC}_t$ (sig. also with boostrap CI)

- Model 2 and Model 5 disagree

Analysis Summary for the Examples

MWI: N = 663, T =5, Interval = 2 months		
Model	Sig. Cross-lags	Sig. Reciprocals
RI-CLPM	$D_{t-1} \xrightarrow{-} S_t$	NA
Reciprocal Only	NA	None
Reciprocal	$D_{t-1} \xrightarrow{-} S_t$	$S_t \xrightarrow{-} D_t$

Ormel: N = 753, T = 3, Interval = 1 year		
Model	Sig. Cross-lags	Sig. Reciprocals
RI-CLPM	$DIS_{t-1} \xrightarrow{+} DEP_t$ $DEP_{t-1} \xrightarrow{+} DIS_t$	NA
Reciprocal Only	NA	No Solution
Reciprocal	$DEP_{t-1} \xrightarrow{+} DIS_t$	$DIS_t \xrightarrow{+} DEP_t$

Analysis Summary for the Examples, Continued

BLS: N = 404, T = 4, Interval = 1 year

Model	Sig. Cross-lags	Sig. Reciprocals
RI-CLPM	None	NA
Reciprocal Only	NA	None
Reciprocal	None	$D_t \xrightarrow{-} S_t$

GPA: N = 933, T = 5, Interval = 4 months

Model	Sig. Cross-lags	Sig. Reciprocals
RI-CLPM	$GPA_{t-1} \xrightarrow{+} ASC_t$	NA
Reciprocal Only	NA	No solution
Reciprocal	None	$GPA_t \xrightarrow{+} ASC_t$

NLSY: N = 8,259, T = 11, Interval = 2 years

Model	Sig. Cross-lags	Sig. Reciprocals
RI-CLPM	$SE_{t-1} \xrightarrow{-} DEP_t$ $DEP_{t-1} \xrightarrow{-} SE_t$	NA
Reciprocal Only	NA	$SE_t \xrightarrow{-} DEP_t$ $DEP_t \xrightarrow{-} SE_t$
Reciprocal	$SE_{t-1} \xrightarrow{-} DEP_t$	$DEP_t \xrightarrow{-} SE_t$

Monte Carlo Simulations: H0 Model Incorrect, N=500

Generate	Analyze (χ^2 5%)	Data analysis
	MWI	
RI-CLPM	Reciprocal (0.134)	
$D_{t-1} \overset{-}{\rightarrow} S_t$	None	Different
Reciprocal	RI-CLPM (0.342)	
$D_{t-1} \overset{-}{\rightarrow} S_t$	$D_{t-1} \overset{-}{\rightarrow} S_t$ (same)	Same
$S_t \overset{-}{\rightarrow} D_t$		
	BLS	
Reciprocal	RI-CLPM (0.280)	
$D_t \overset{-}{\rightarrow} S_t$	None	Same
	NLSY	
RI-CLPM	Reciprocal Only (0.162)	
$SE_{t-1} \overset{-}{\rightarrow} DEP_t$	None	Different
$DEP_{t-1} \overset{-}{\rightarrow} SE_t$		
Reciprocal Only	RI-CLPM (0.188)	
$SE_t \overset{-}{\rightarrow} DEP_t$	$SE_{t-1} \overset{-}{\rightarrow} DEP_t$ (small)	Similar
$DEP_t \overset{-}{\rightarrow} SE_t$		

Conclusions

- The three model types either fit equally well or about the same - all models have log likelihood values that are not far apart
  - Statistics cannot give a strong indication of which model is best
- What's the answer to the question in the title? It looks like No
  - It is hard to claim that a cross-lagged effect has been found if it might just as well be a contemporaneous effect
    - See for example the GPA and NLSY analyses where the two reciprocal models give different answers than the regular model

GPA: N = 933, T = 5, Interval = 4 months

Model	Sig. Cross-lags	Sig. Reciprocals
RI-CLPM	$GPA_{t-1} \xrightarrow{+} ASC_t$	NA
Reciprocal Only	NA	No solution
Reciprocal	None	$GPA_t \xrightarrow{+} ASC_t$

NLSY: N = 8,259, T = 11, Interval = 2 years

Model	Sig. Cross-lags	Sig. Reciprocals
RI-CLPM	$SE_{t-1} \xrightarrow{-} DEP_t$ $DEP_{t-1} \xrightarrow{-} SE_t$	NA
Reciprocal Only	NA	$SE_t \xrightarrow{-} DEP_t$ $DEP_t \xrightarrow{-} SE_t$
Reciprocal	$SE_{t-1} \xrightarrow{-} DEP_t$	$DEP_t \xrightarrow{-} SE_t$

Conclusions Continued

- With similarly fitting models, it is tempting to choose RI-CLPM because cross-lagged effects offer a more interesting “causal” interpretation given the time lag between cause and effect
- This may be wishful thinking because RI-CLPM assumes zero lag0 effects - don’t overemphasize lag1 effects over lag0 effects
- Recommendation: Report results of all 3 model types
    - Reciprocal and Reciprocal Only models are useful complements to RI-CLPM, enriching the understanding of the data
- The reciprocal model may facilitate the search for a parsimonious model by its natural time invariance restrictions
    - Regular RI-CLPM typically has worse BIC in the five examples
- Is the situation different with Intensive Longitudinal Data?
    - Depending on the subject matter and type of measurements, intensive longitudinal data with more frequent measurements may have less of a need to include contemporaneous effects