Using Mplus To Do Cross-Lagged Modeling of Panel Data Part 2: Categorical Variables

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Outline

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- **Section 2** (12-23): Refresher of Mplus Web Talk 4 Part 1
  - Overview of analysis of continuous outcomes
  - Analysis of data set 1 stress outcome
- **Section 3** (24-33): Brief Introduction to Analysis of Binary Outcomes
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  - Model specification, identification, estimation, and simulations
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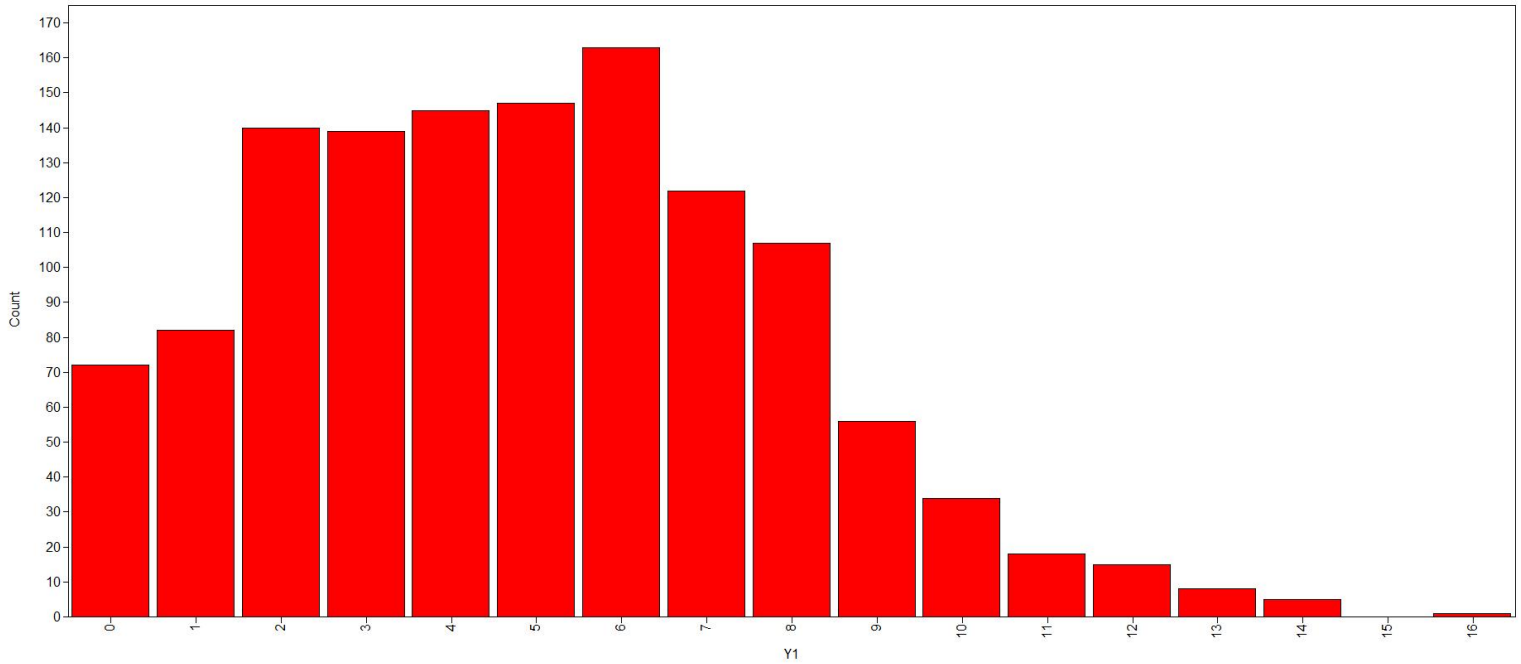
  - Latent Growth Analysis, Longitudinal LCA, Latent Class Growth Analysis, Growth Mixture Modeling
- **Section 14** (152-159): Distal Outcomes

Section 1 Examples from Three Data Sets

Data Set 1: The COMBINE Study of Alcohol Use Disorder

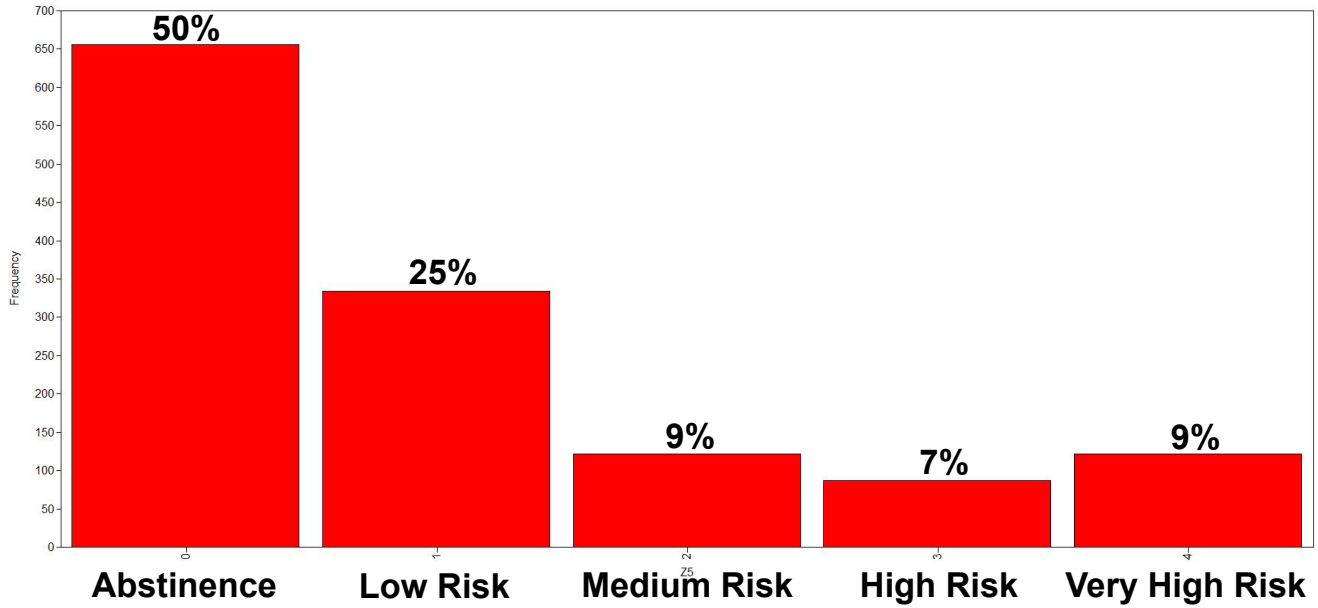
- Data from COMBINE, a 16-week, multisite randomized double-blind clinical trial comparing treatments of alcohol dependence (Anton et al., 2006, JAMA)
  - N = 1,383. Mean age 44
  - Measurement occasions: Baseline, week 1, week 2, week 4, week 6, week 8, week 10, week 12, week 16 and week 52 follow-up
- Stress: Brief version of The Perceived Stress Scale
- Alcohol risk: Abstinence, low risk, medium risk, high risk, very high risk
- Heavy Drinking: Number of heavy drinking days per week
- Covariates:
  - Intervention - 9 groups (medication, placebo, and therapy), gender, race, age, education, marital status, employment
- Stress and alcohol use disorder (AUD). Stress causes drinking (Armeli et al., 2000 in J of Personality and Social Psych)

Distribution of the Stress Variable (Week 1)



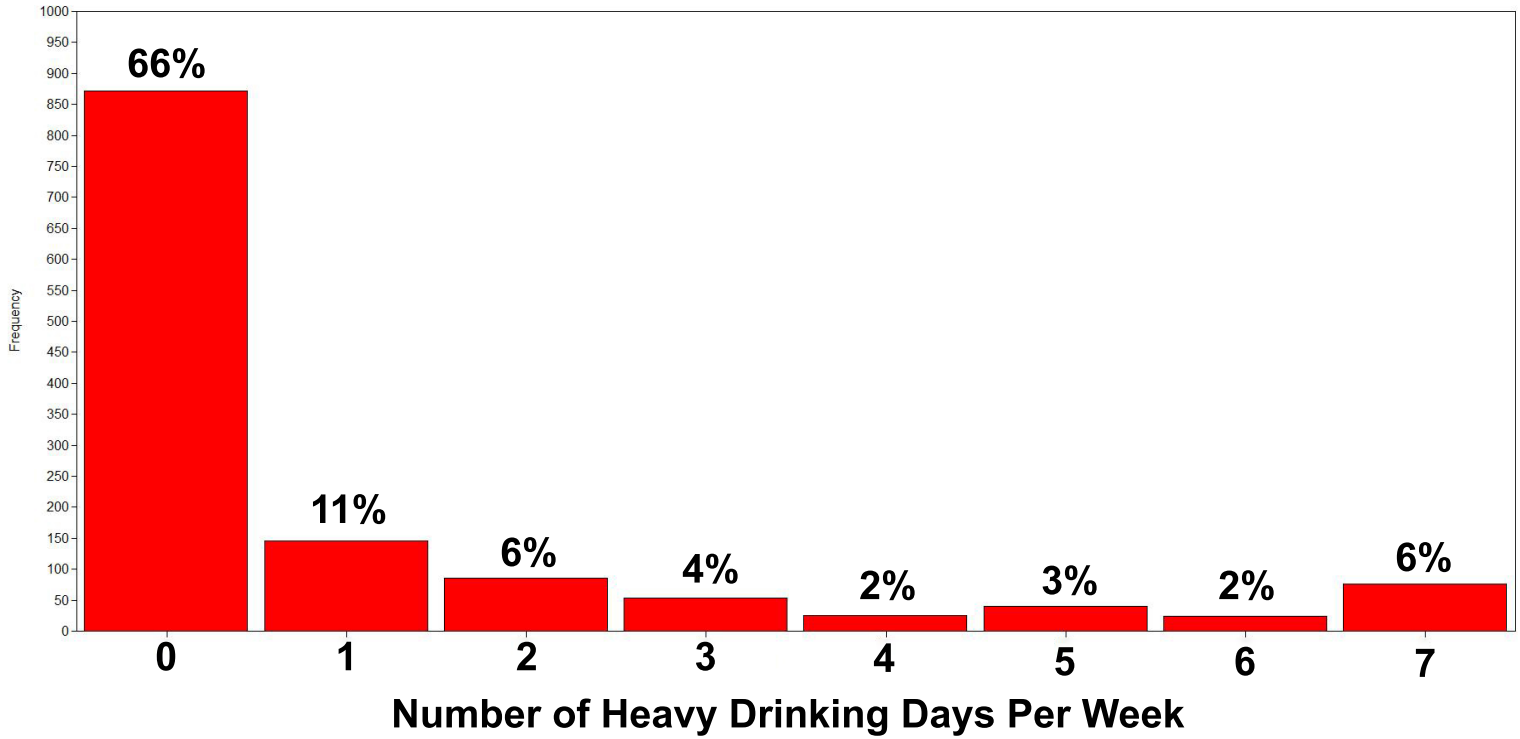
- A 4-item version of The Perceived Stress Scale with scores of 0 to 16 has been used for analyses of the COMBINE data:
  - McHugh et al. (2013). Positive affect and stress reactivity in alcohol-dependent outpatients. J. Studies in Alcohol and Drugs
- Can be treated as a continuous variable using MLR
- Percentage at the lowest score of zero increases with time but does not exceed 15 %

Distribution of the Alcohol Risk Variable



- WHO categories based on grams of pure alcohol per day (separate for males and females)
- Should not be treated as a continuous variable with linear relations due to strong floor effect: Biases in correlations and regressions
- Ordered categorical (ordinal) with 5 categories. Floor effect not a problem
- Binary: Abstinence or not

Distribution of Heavy Drinking Days Per Week

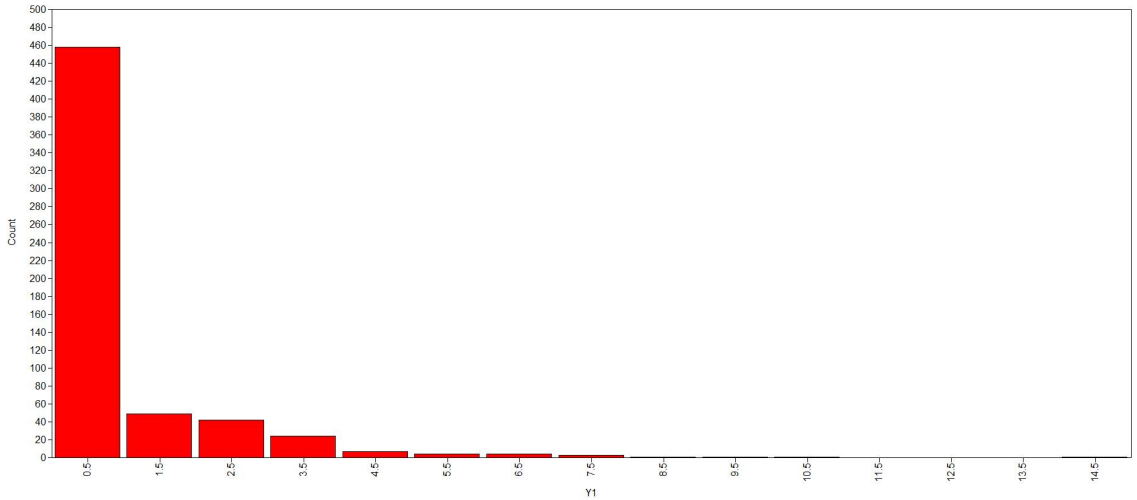


- Not really a count variable, but can be treated as:
  - Ordered categorical (ordinal) variable
  - Dichotomized variable
  - Censored variable
  - Two-part variable

Data Set 2: Suicidal Ideation and Substance Abuse

- Suicidal ideation and substance abuse
- Classic question of what influences what
- Data from a preventive intervention study in Baltimore (Ialongo)
- N = 737
- T = 8: Ages 19-26
- Covariates: Gender, race, lunch (poverty indicator)
- References:
  - Ialongo, Werthamer, Kellam, Brown, & Wang (1999). Proximal impact of two first grade preventive interventions on the early risk behaviors for later substance abuse, depression, and antisocial behavior. American Journal of Community Psychology, 27, 599-641
  - Musci et al. (2016). Suicide & Life Threatening Behavior
  - Thrul et al. (2021). Addiction

Suicidal Ideation and Substance Abuse: Binary Outcomes



- At age 19, 77% are at zero, 8% at 1: Dichotomize into 0 vs higher
  - At least one suicidal ideation and/or behaviors endorsed in the last year (Y)
  - At least one substance abuse or dependence criteria met across all substances assessed in the last year (Z)

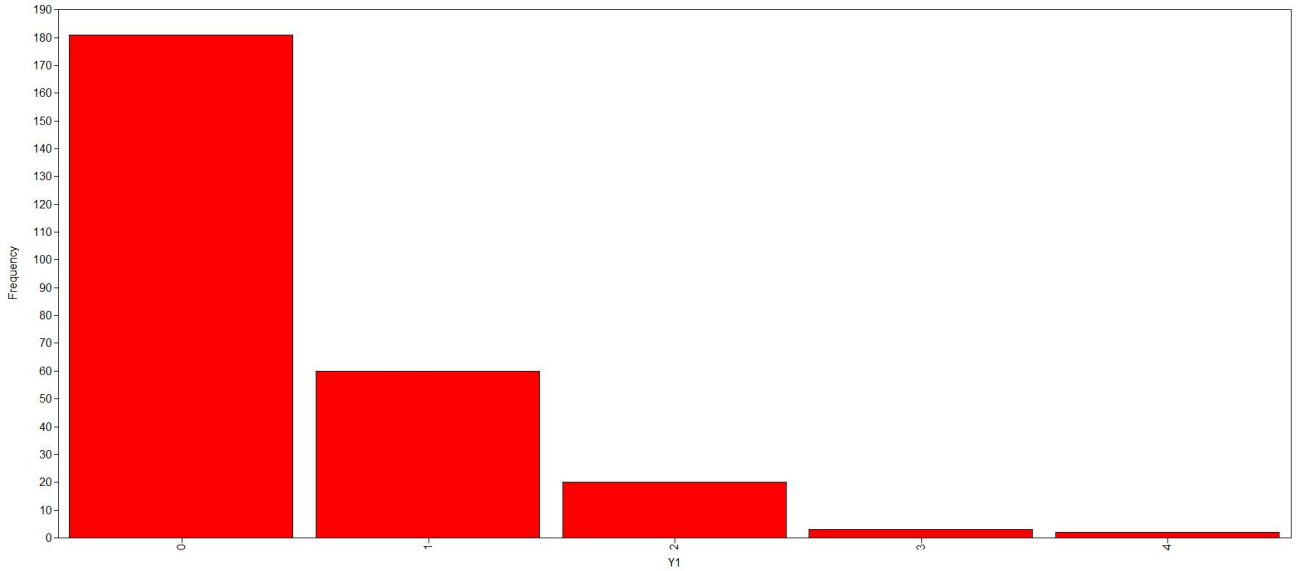
	Age 19	Age 20	Age 21	Age 22	Age 23	Age 24	Age 25	Age 26
Y	23.0%	18.2%	15.2%	19.0%	23.0%	22.3%	23.4%	21.1%
Z	19.9%	19.8%	15.3%	20.4%	18.5%	15.0%	14.3%	13.3%

Data Set 3: Negative Affect

- Data from the older cohort of the Notre Dame Study of Health & Well-being (Bergeman): N = 271, T = 56 (daily measures on consecutive days)
- 10 NA items (5-category scale): afraid, ashamed, guilty, hostile, scared, upset, irritable, jittery, nervous, distressed
- Question format: Today I felt... (1 = Not at all, 2 = A little, 3 = Moderately, 4 = Quite a bit, 5 = Extremely)
- Wang, Hamaker, Bergeman (2012). Investigating inter-individual differences in short-term intra-individual variability. *Psychological Methods*

Negative Affect: Ordered Categorical (Ordinal) Items

- Question format: Today I felt... (1 = Not at all, 2 = A little, 3 = Moderately, 4 = Quite a bit, 5 = Extremely)
- Irritable item: 68% at lowest value - Not at all

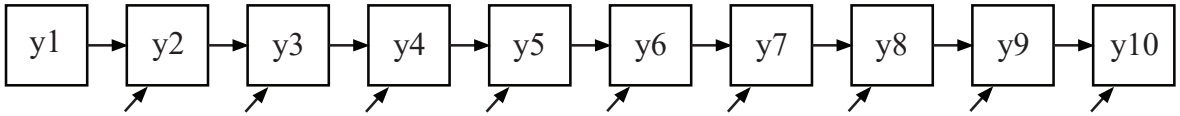


- Not suitable for continuous variable analysis due to strong floor effect
- Can be treated as an ordinal variable

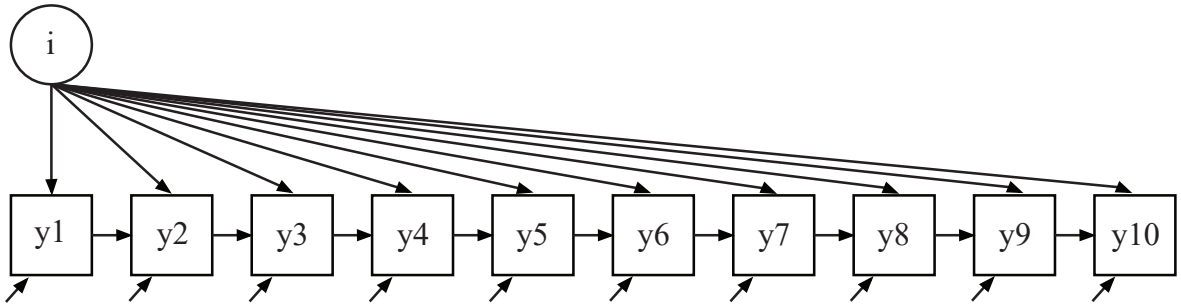
Section 2 Refresher of Mplus Web Talk 4 Part 1 Analysis of Continuous Outcomes

Dynamic Models

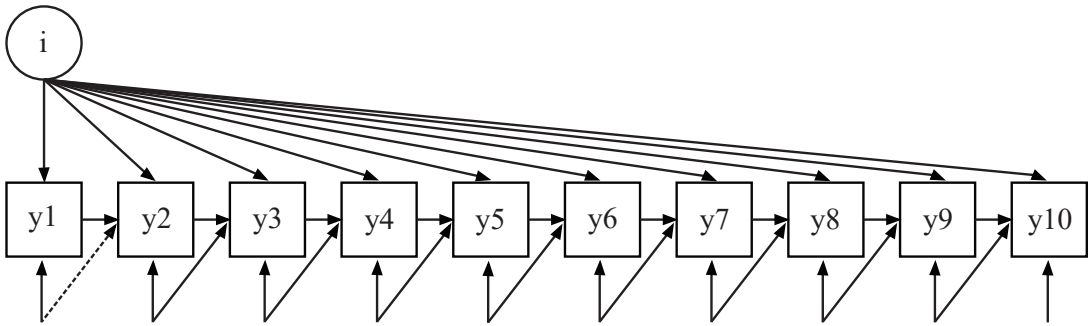
- Auto-Regression of lag 1 (AR1)



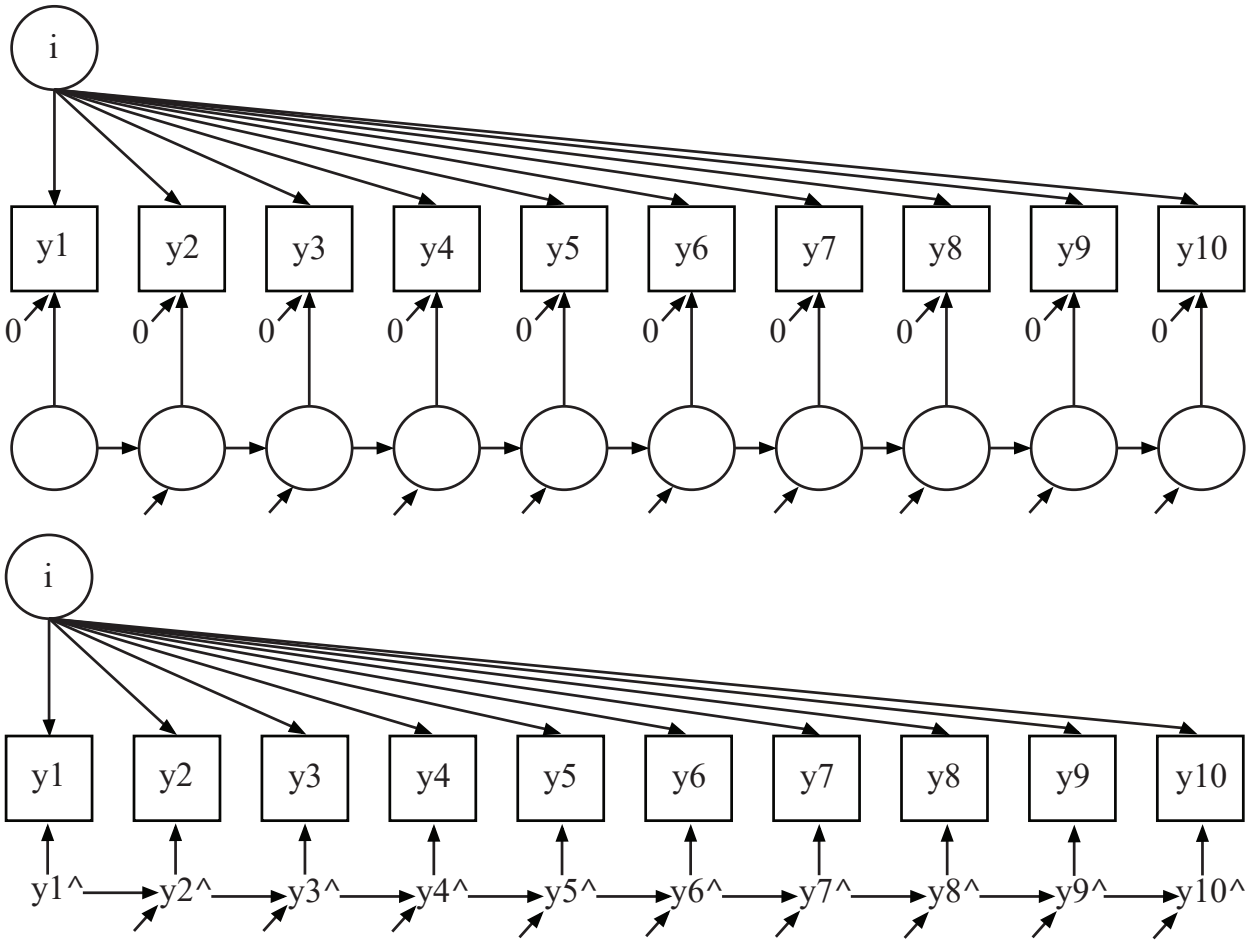
- Dynamic Random Intercept AR1 (D-RI-AR1). Bollen-Brandt (2010)



- Dynamic Random Intercept ARMA (1,1) (D-RI-ARMA11). Zyphur et al. (2020)

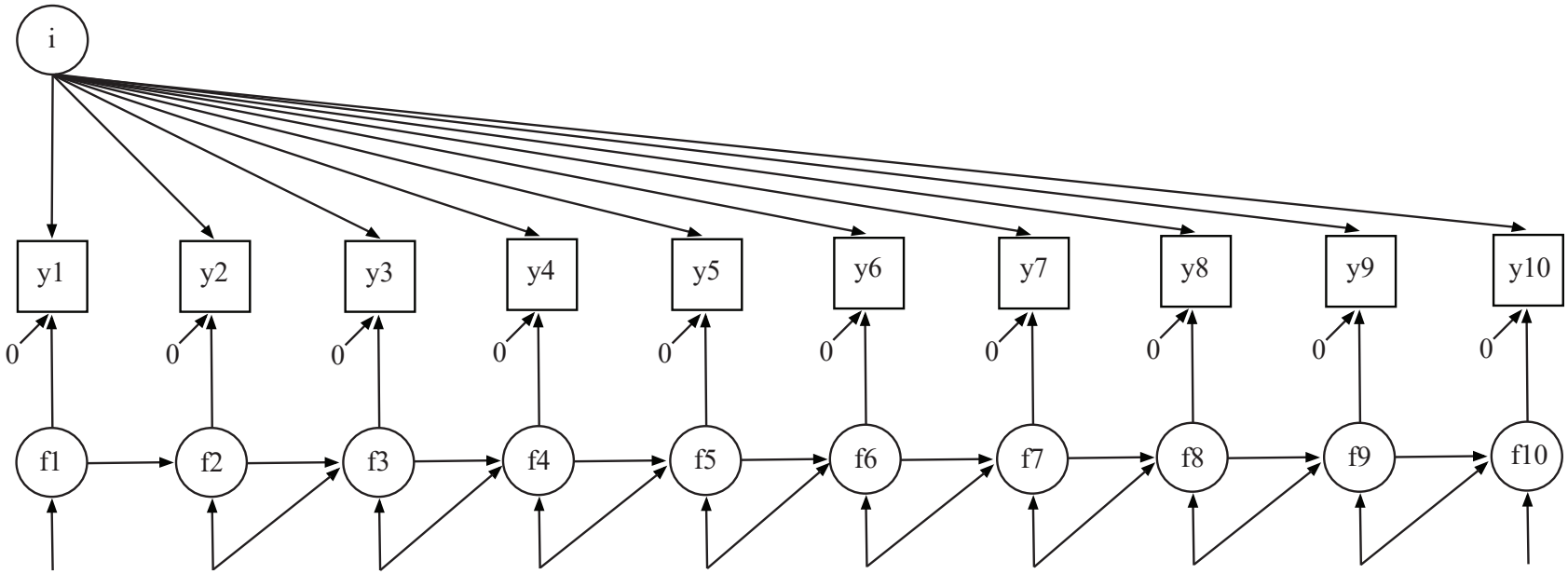


Random Intercept and Auto-Correlated Residuals
RI-AR Modeling Displayed in Two Equivalent Ways



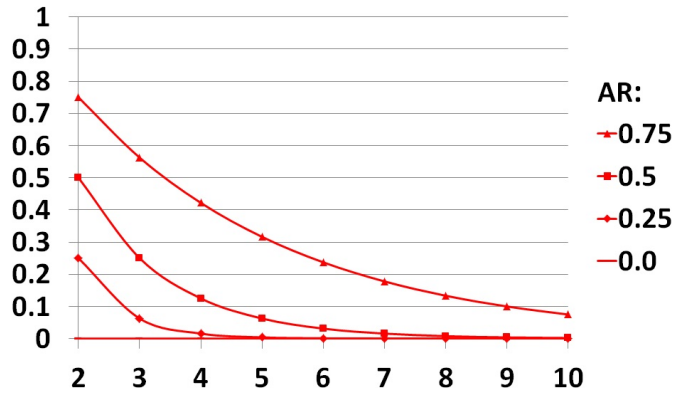
- RI-AR modeling is the univariate part of RI-CLPM
- Time-State-Error (TSE) model allows restricted measurement error
- CLPM does not include the random intercept *i*, so using AR1

RI-ARMA Modeling (Asparouhov-Muthén, 2022; RSEM)

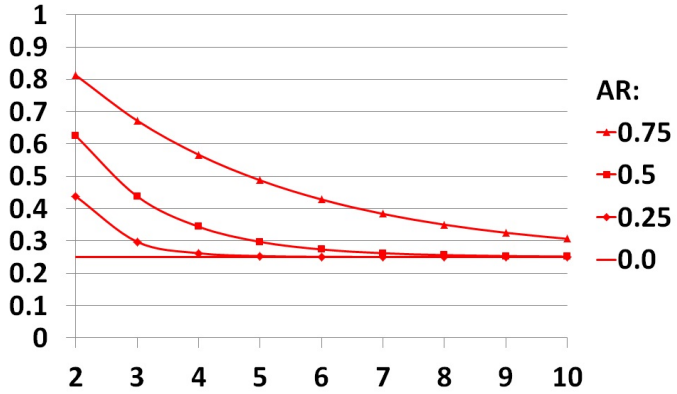


- This model is similar in spirit to RI-AR because of its separation of between- and within-individual variation also referred to as latent centering (centering using the random intercept i), but adds an MA component
- An equivalent measurement error version, RI-MEAR, is available which is more general than TSE but like TSE often presents estimation problems not seen with RI-ARMA

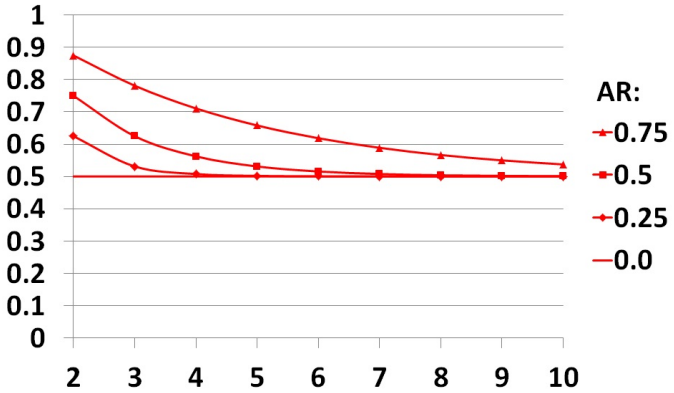
RI and AR1 Impact on Correlations Across Time (T = 10)



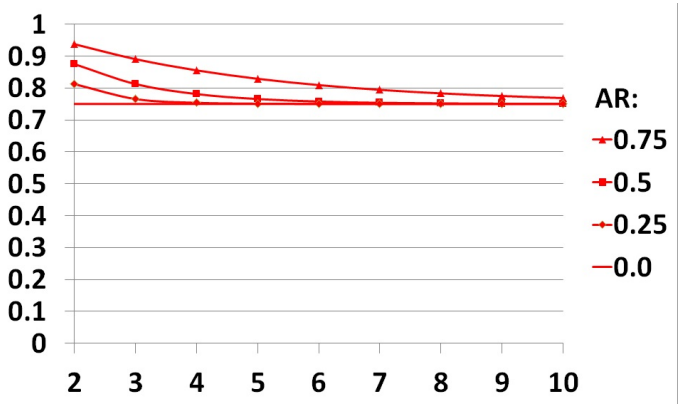
(a) RI variance (R^2) = 0.00



(b) RI variance (R^2) = 0.25



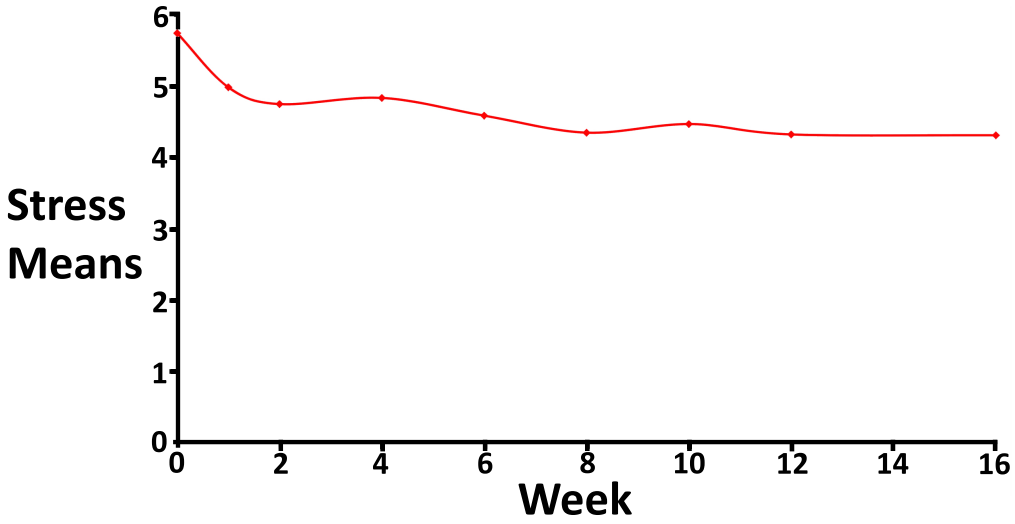
(c) RI variance (R^2) = 0.50

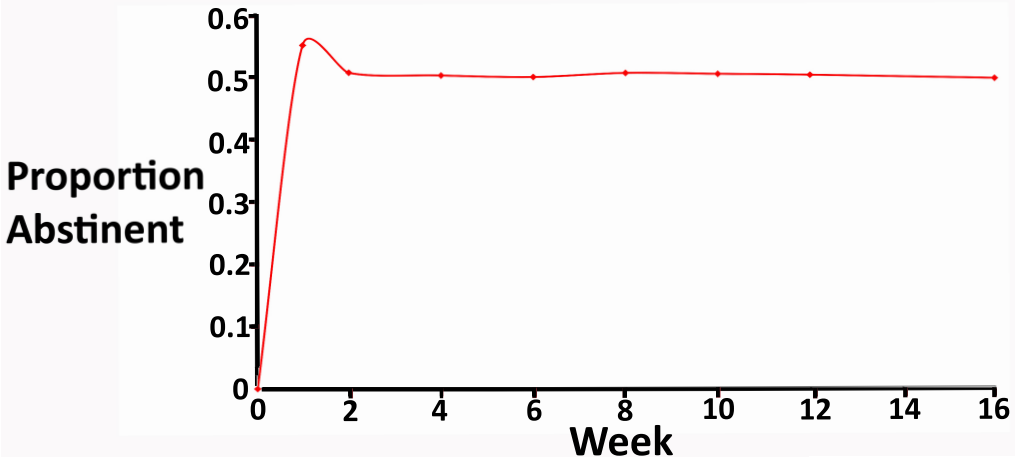


(d) RI variance (R^2) = 0.75

- Correlation $Y_1, Y_t = \psi + \beta^{t-1}(1-\psi)$ where ψ is the random intercept variance, β is the constant auto-regression among the residuals, and Y variances are all 1

COMBINE Stress and Alcohol Risk: Baseline - Week 16





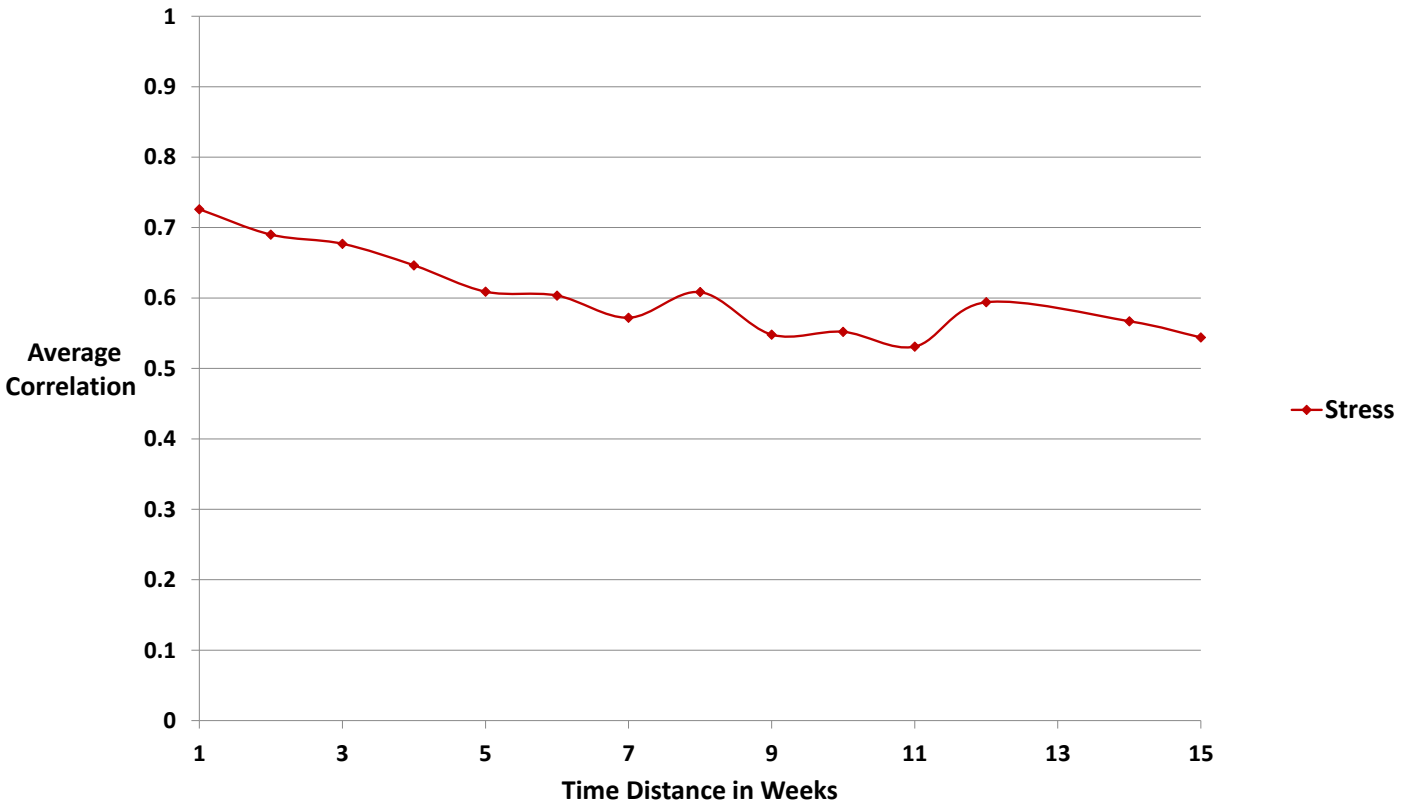
COMBINE Data Stress Outcome: Covariance Coverage for Week 1 - Week 16 (N=1375)

- Week 1 = y1, week 2 = y2, week 4 = y3, week 6 = y4, week 8 = y5, week 10 = y6, week 12 = y7, week 16 = y8

	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8
Y1	0.924							
Y2	0.860	0.915						
Y3	0.823	0.828	0.870					
Y4	0.761	0.767	0.760	0.800				
Y5	0.764	0.771	0.758	0.736	0.811			
Y6	0.673	0.683	0.671	0.666	0.677	0.707		
Y7	0.710	0.719	0.708	0.693	0.704	0.660	0.752	
Y8	0.758	0.763	0.749	0.716	0.742	0.665	0.713	0.817

- 0.817 says that 81.7% have data on Y8 (don't have missing on Y8)
- 0.713 says that 71.3% have data on both Y8 and Y7 (don't have missing on either Y8 or Y7)

Average Correlations for the Stress Outcome as a Function of the Time Distance



- Weeks observed: 1, 2, 4, 6, 8, 10, 12, 16 (T=8)
- Number of correlations at each time distance in weeks:
  - 1 at 1, 5 at 2, 1 at 3, 5 at 4, 1 at 5, 4 at 6, 1 at 7, 3 at 8, 1 at 9, 2 at 10, 1 at 11, 1 at 12, 0 at 13, 1 at 14, 1 at 15 (28 correlations)
- Plot suggests high RI variance and sizeable AR

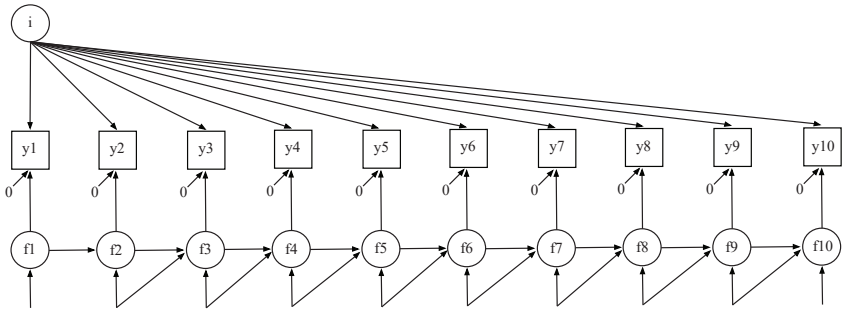
Univariate Analysis of Stress using MLR (N=1375, T=8)

- Outputs for all models are posted on the website of the talk

Model	# par's	LL	BIC	Chi-square	RMSEA	CFI
1. AR1	23	-20409	40984	χ^2(21)=747 (.0000)	0.160 (<.05=.000)	0.818
2. AR2	29	-20062	40334	χ^2(15)=233 (.0000)	0.104 (<.05=.000)	0.945
3. ARMA11	29	-19921	40051	χ^2(15)=18 (.2432)	0.013 (<.05=1.000)	0.999
4. D-RI-AR1	24	-19964	40102	χ^2(20)=86 (.0000)	0.049 (<.05=.520)	0.983
5. D-RI-AR2	30	-19946	40108	χ^2(14)=59 (.0000)	0.048 (<.05=.553)	0.989
6. D-RI-ARMA11*	31	-19918	40060	χ^2(13)=14 (.3787)	0.007 (<.05=1.000)	1.000
7. RI-AR1	24	-19951	40076	χ^2(20)=72 (.0000)	0.044 (<.05=.819)	0.987
8. RI-AR2	30	-19928	40073	χ^2(14)=33 (.0027)	0.032 (<.05=.983)	0.995
9. RI-ARMA	30	-19919	40054	χ^2(14)=15 (.3616)	0.008 (<.05=1.000)	1.000

- * Negative insignificant V(i)

Mplus Input for RI-ARMA



Model:

```
i BY y1-y8@1;

f1 BY y1;
f2 BY y2;
f3 BY y3;
f4 BY y4;
f5 BY y5;
f6 BY y6;
f7 BY y7;
f8 BY y8;
```

```
i WITH f1@0; ! or, MODEL=NOCOV

y1-y8@0;

f2-f8 PON f1-f7; ! AR part
f3-f8 PON f2^-f7^; ! MA part
```

- f's could be replaced by y^, but then we would need y^^ for the moving average part f3-f8 PON f2^-f7^;

Model 9 RI-ARMA Estimates Using MLR

- Substantial random intercept variance = 3.924, S.E. = 0.900
- STDYX estimates:

	Estimate	S.E.	Est./S.E.
I BY			
Y1	0.672	0.073	9.157
Y2	0.636	0.075	8.466
Y3	0.628	0.075	8.336
Y4	0.631	0.072	8.743
Y5	0.626	0.073	8.613
Y6	0.636	0.070	9.076
Y7	0.619	0.068	9.047
Y8	0.637	0.075	8.527

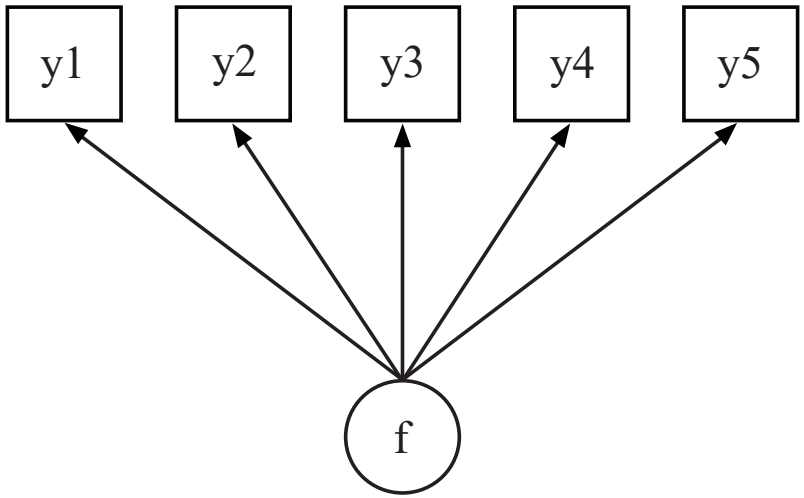
- Corr (Y1, Y8) due to RI = $0.672 \times 0.637 = 0.428$ (total est corr = 0.539)
- R^2 (Y1) = $0.672^2 = 0.452$

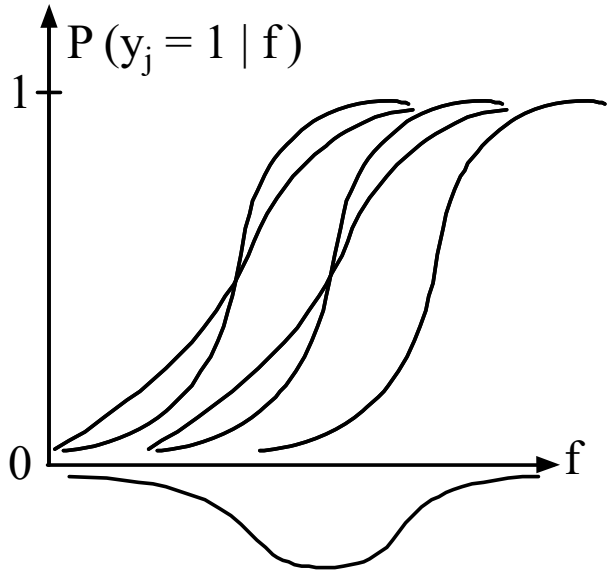
- STDYX estimates:

	Estimate	S.E.	Est./S.E.
F2 ON			
F1	0.522	0.086	6.095
F3 ON			
F2	0.836	0.082	10.192
F2^	-0.345	0.063	-5.485
F4 ON			
F3	0.759	0.096	7.942
F3^	-0.324	0.072	-4.505
F5 ON			
F4	0.809	0.102	7.964
F4^	-0.441	0.074	-5.971
F6 ON			
F5	0.820	0.088	9.265
F5^	-0.432	0.071	-6.099
F7 ON			
F6	1.039	0.074	14.006
F6^	-0.622	0.095	-6.544
F8 ON			
F7	0.848	0.105	8.040
F7^	-0.483	0.081	-5.973

Section 3 Brief Introduction to Analysis of Binary Outcomes

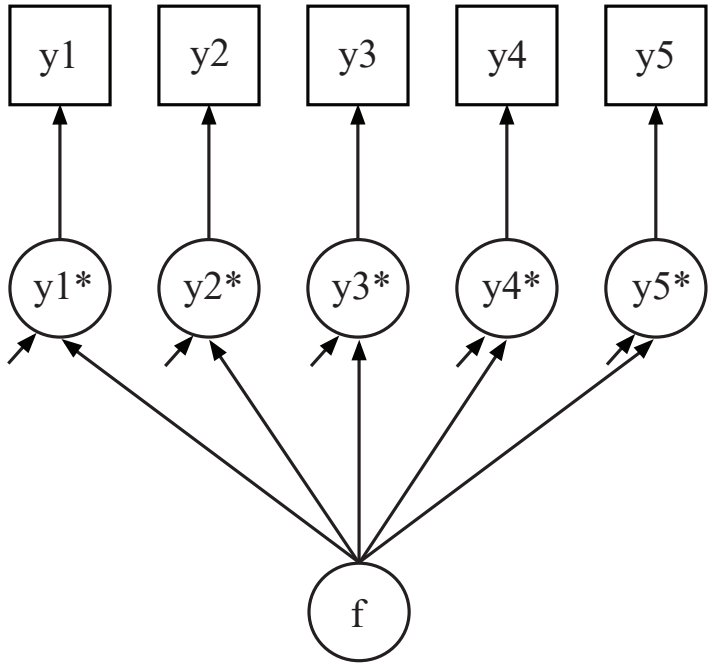
Brief Introduction to Modeling with Binary Outcomes (0/1)





- Three contexts:
  - Item Response Theory (IRT): item difficulty and discrimination
  - Factor analysis: item thresholds and factor loadings
  - Random intercept: factor loadings fixed at 1
- Typical specification:
  - Normally distributed latent variable
  - Logistic or Probit regressions
- See Short Course Topic 2 and WLSMV, ML, and Bayes estimation at `https://www.statmodel.com/download/EstimatorChoices.pdf`

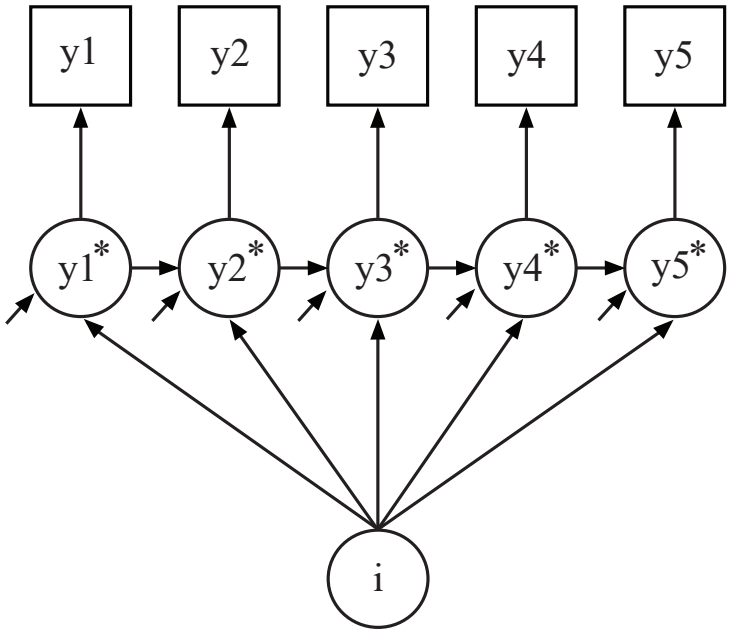
- Equivalent representation with continuous latent response variables Y*:



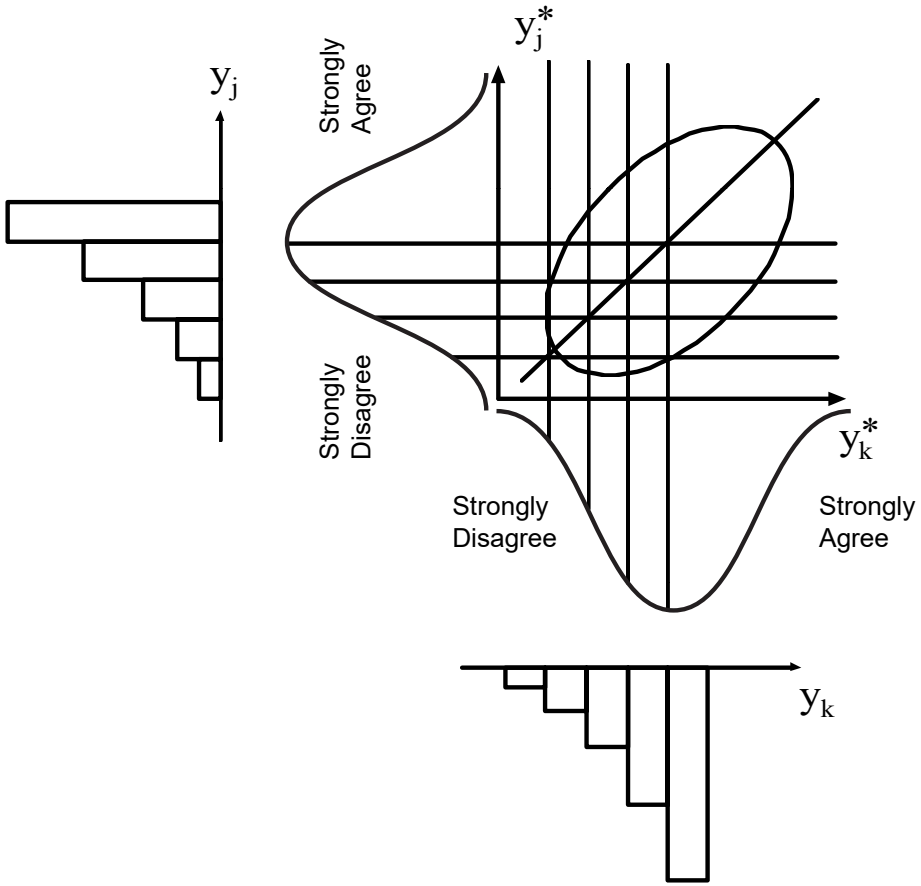
- Y* > threshold results in Y = 1, otherwise Y = 0
- Specifying normally distributed f together with probit regressions is the same as specifying normally distributed latent response variables Y* (N + N = N). Logistic regression does not give Y* normality
- Correlations between Y* variables for binary Y: Tetrachoric correlations

Brief Introduction Continued: Different Interests

- The cross-sectional modeling of IRT and factor analysis focuses on the relationship between f and Y
- The longitudinal modeling of panel data analysis adds a focus on the relationship between Y's at different time points
  - The factor cannot account for all the correlation among the Y*'s
  - Multivariate probit modeling allows linear regressions among normally distributed Y*'s such as in this D-RI-AR1 model:



Brief Introduction Continued: Ordinal Outcomes Polychoric Correlations



- The ordinal case still allows linear regression between Y*'s (as opposed to the nominal case)
- General latent response variable modeling assuming underlying normality for Y* available in Mplus for WLSMV and Bayes

Two Kinds of Model Assessments for Categorical Outcomes

- Fit to correlations among a set of normal, continuous latent response variables Y^* underlying the observed categorical Y's (WLSMV and Bayes)
  - Muthén et al. (1997). Robust inference using weighted least squares and quadratic estimating equations in latent variable modeling with categorical and continuous outcomes. Unpublished technical report. `http://www.statmodel.com/download/Article_075.pdf`
- Fit to the data in the form of response patterns, that is, a frequency table for all variables
  - A model may fit the Y^* correlations but not the frequency table
  - Even a just-identified Y^*s model with all WITH's may not fit the frequency table in some cases
  - Muthén (1993). Goodness of fit with categorical and other non-normal variables. In K.A. Bollen, & J.S. Long (Eds), Testing Structural Equation Models (pp. 205-243). Newbury Park, CA: Sage `http://www.statmodel.com/bmuthen/articles/Article_045.pdf`

Fit to Y* Correlations for Categorical Variables

- Fit to correlations among a set of normal, continuous latent response variables Y^* underlying the observed categorical Y's (WLSMV and Bayes)
  - The Muthén et al. (1997) WLSMV chi-square works well when the number of variables is not large and the sample size is not small: Suitable for cross-lagged panel modeling
  - Bayes PPP idea: Using any fit statistic, compute the fit statistic for the observed data, generate a fit statistic distribution based on generated data from the estimated model, and find the proportion of cases where the latter is larger than the former
    - Bayes PPP for categorical variables: Based on chi-square test of overall model fit for Y^*s
    - Analogous to WLSMV chi-square test of estimated versus sample tetrachorics and polychorics
    - Low power for binary outcomes and less powerful than the WLSMV chi-square test (Asparouhov-Muthén, 2021a)
    - More powerful for polytomous variables

Fit to the Data in Terms of Frequency Tables

- Frequency table test of model fit:
  - With categorical latent class indicators, the model can be tested against data using Pearson and likelihood-ratio chi-square frequency table tests. Summing over the cells of the table:

$$Pearson: \sum_j (o_j - e_j)^2 / e_j$$

$$Likelihood\ ratio: 2 \sum_j o_j\ log(o_j / e_j)$$

  - There are typically too many frequency table cells with many cells having estimated frequencies close to zero, invalidating the tests: Pearson and Likelihood ratio tests disagree
  - Example with 8 binary variables: $2^8 = 256$ possible response patterns, where many patterns are probably not observed (zero cells in the frequency table) - the two tests disagree strongly
- Alternative checks: Fit for univariate and bivariate tables (higher freq's)
- New TECH10 for WLSMV and Bayes in 8.7 and 8.8: Standardized residuals for response patterns, uni- and bi-variate frequency tables, and Bayes PPP for Pearson fit to uni- and bi-variate tables

Example of Frequency Table Fit for Data Set 2 Suicidal Ideation and Substance Abuse (N = 737, T = 8)

- All WITH model to estimate sample correlations: y1-z8 WITH y1-z8;
- For these 16 variables, the most frequent pattern is 0's for both variables at all 8 time points, which is observed for 104 individuals (14%)
- WLSMV and Bayes
- For an introduction to using Bayes estimation in Mplus, see Mplus Short Course Topic 12, Parts 3-4, slides 4-21

```
ANALYSIS:
          ESTIMATOR = WLSMV;
          PARAMETERIZATION = THETA;
          PROCESSORS = 8;
MODEL:
          y1-y8 WITH z1-z8;
OUTPUT:   RESIDUAL TECH10;
PLOT:     TYPE = PLOT3;
```

```
ANALYSIS:
          ESTIMATOR = BAYES;
          BITERATIONS = (1000);
          PROCESSORS = 8;
MODEL:
          y1-y8 WITH z1-z8;
OUTPUT:   TECH8 RESIDUAL TECH10;
PLOT:     TYPE = PLOT3;
```

All WITH Model Fit for WLSMV and Bayes: Chi-Square, PPP, Response Patterns, and Frequency Tables

MODEL FIT INFORMATION

Number of Free Parameters: 136

Chi-Square Test of Model Fit

Value	0.000*
Degrees of Freedom	0
P-Value	0.0000

MOST FREQUENT RESPONSE PATTERNS

Response Pattern	Frequency Observed	Estimated	Standardized Residual (z-score)
1	104.00	98.27	0.65
2	12.00	12.69	-0.20
3	10.00	8.10	0.60
4	7.00	8.48	-0.64
5	7.00	5.44	0.59

Number of Significant Standardized Residuals 0

BIVARIATE PROPORTIONS FOR CATEGORICAL VARIABLES

Overall Number of Significant Standardized Residuals 0

MODEL FIT INFORMATION

Number of Free Parameters 136

Bayesian Posterior Predictive Checking using Chi-Square

95% Confidence Interval for the Difference Between the Observed and the Replicated Chi-Square Values

-49.528 53.388

Posterior Predictive P-Value 0.598

MOST FREQUENT RESPONSE PATTERNS

Response Pattern	Frequency Observed	Estimated	Standardized Residual (z-score)
1	104.00	97.16	0.78
2	12.00	10.62	0.40
3	10.00	8.47	0.48
4	7.00	8.30	-0.54
5	7.00	3.48	1.33

Number of Significant Standardized Residuals 0

BIVARIATE PROPORTIONS FOR CATEGORICAL VARIABLES

Overall Number of Significant Standardized Residuals 0

Section 4 Data Set 2 Binary Outcomes: Suicidal Ideation and Substance Abuse

Coverage for Suicidal Ideation and/or Behaviors (Y) Dichotomized Outcomes, N = 737, T = 8

	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8
Y1	0.808							
Y2	0.727	0.803						
Y3	0.724	0.746	0.823					
Y4	0.705	0.704	0.738	0.832				
Y5	0.700	0.693	0.726	0.754	0.832			
Y6	0.711	0.707	0.735	0.751	0.776	0.844		
Y7	0.701	0.701	0.727	0.739	0.754	0.774	0.848	
Y8	0.700	0.702	0.730	0.730	0.731	0.754	0.780	0.837

- 0.808 says that 80.8% have data on Y1 (don't have missing on Y1)
- 0.727 says that 72.7% have data on both Y1 and Y2 (don't have missing on either Y1 or Y2)
- Similar coverage for Substance abuse

The 20 Most Frequent Response Patterns (* = Missing)

Suicidal Ideation			Substance Abuse		
Pattern	Frequency	Percentage	Pattern	Frequency	Percentage
0 0 0 0 0 0 0 0	155.00	21.0	0 0 0 0 0 0 0 0	201.00	27.3
1 0 0 0 0 0 0 0	25.00	3.4	1 1 0 0 0 0 0 0	18.00	2.4
0 1 0 0 0 0 0 0	14.00	1.9	0 0 0 1 0 0 0 0	15.00	2.0
* 0 0 0 0 0 0 0	12.00	1.6	0 0 1 0 0 0 0 0	14.00	1.9
0 0 0 0 0 0 0 *	11.00	1.5	* 0 0 0 0 0 0 0	12.00	1.6
0 0 0 0 0 0 0 1	11.00	1.5	0 0 0 * 0 0 0 0	10.00	1.4
0 0 0 0 0 0 1 1	10.00	1.4	0 0 0 0 0 0 0 *	9.00	1.2
0 * 0 0 0 0 0 0	9.00	1.2	0 0 0 0 1 0 0 0	8.00	1.1
0 0 0 1 0 0 0 0	9.00	1.2	0 * 0 0 0 0 0 0	7.00	1.0
0 0 0 0 0 0 1 0	8.00	1.1	0 0 0 0 0 0 * 0	7.00	1.0
* * 0 0 0 0 0 0	7.00	1.0	0 0 0 0 0 0 0 1	7.00	1.0
0 0 0 0 0 0 * 0	6.00	0.8	0 0 0 0 0 0 1 0	7.00	1.0
0 0 0 0 * 0 0 0	6.00	0.8	* * 0 0 0 0 0 0	7.00	1.0
1 0 0 0 1 0 0 0	6.00	0.8	1 1 1 1 1 1 1 1	6.00	0.8
0 0 0 0 1 0 0 0	6.00	0.8	1 0 0 0 0 0 0 0	6.00	0.8
0 0 0 * * * * *	5.00	0.7	* * * * * * 0 0	6.00	0.8
0 0 0 * 0 0 0 0	5.00	0.7	0 0 0 0 * 0 0 0	6.00	0.8
0 * * * * * * *	5.00	0.7	0 0 0 0 0 0 * *	6.00	0.8
0 0 0 0 0 0 * *	5.00	0.7	0 * * * * * * *	6.00	0.8
0 0 1 0 0 0 0 0	4.00	0.5	* * * 0 0 0 0 0	6.00	0.8

- Percentage 0's or missing: 43.2% and 53.5%, respectively.

Sample Correlations for Suicidal Behavior and Substance Abuse (Bayes; N = 737, T = 8)

- Tetrachoric correlations using an unrestricted, all WITH model: y1-z8 WITH y1-z8;
- Standard errors ranging from 0.05 to 0.10

Correlations for Suicidal Ideation (Y)

	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8
Y1	1.000							
Y2	0.269	1.000						
Y3	0.422	0.420	1.000					
Y4	0.407	0.348	0.604	1.000				
Y5	0.553	0.357	0.542	0.635	1.000			
Y6	0.400	0.242	0.536	0.560	0.646	1.000		
Y7	0.337	0.197	0.450	0.500	0.649	0.604	1.000	
Y8	0.233	0.267	0.495	0.567	0.540	0.506	0.721	1.000

Sample Correlations Continued

Correlations for Substance Abuse (Z)

	Z1	Z2	Z3	Z4	Z5	Z6	Z7	Z8
Z1	1.000							
Z2	0.931	1.000						
Z3	0.501	0.471	1.000					
Z4	0.491	0.454	0.546	1.000				
Z5	0.507	0.430	0.508	0.652	1.000			
Z6	0.513	0.435	0.476	0.705	0.664	1.000		
Z7	0.424	0.388	0.512	0.659	0.610	0.732	1.000	
Z8	0.430	0.375	0.420	0.466	0.564	0.616	0.646	1.000

Sample Correlations Continued

Correlations for Y and Z: Y $\rightarrow$ Z

	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8
Z1	0.357	0.122	0.123	0.234	0.293	0.253	0.271	0.179
Z2	0.325	0.139	0.072	0.178	0.226	0.235	0.235	0.153
Z3	0.137	0.023	0.242	0.144	0.278	0.282	0.314	0.178
Z4	0.182	0.266	0.355	0.346	0.325	0.432	0.368	0.372
Z5	0.068	0.182	0.097	0.309	0.370	0.370	0.377	0.344
Z6	0.176	0.278	0.334	0.272	0.304	0.490	0.359	0.272
Z7	0.133	0.125	0.279	0.238	0.203	0.494	0.509	0.398
Z8	-0.031	0.057	0.306	0.251	0.177	0.346	0.400	0.403

Sample Correlations Continued

Correlations for Y and Z: Z → Y

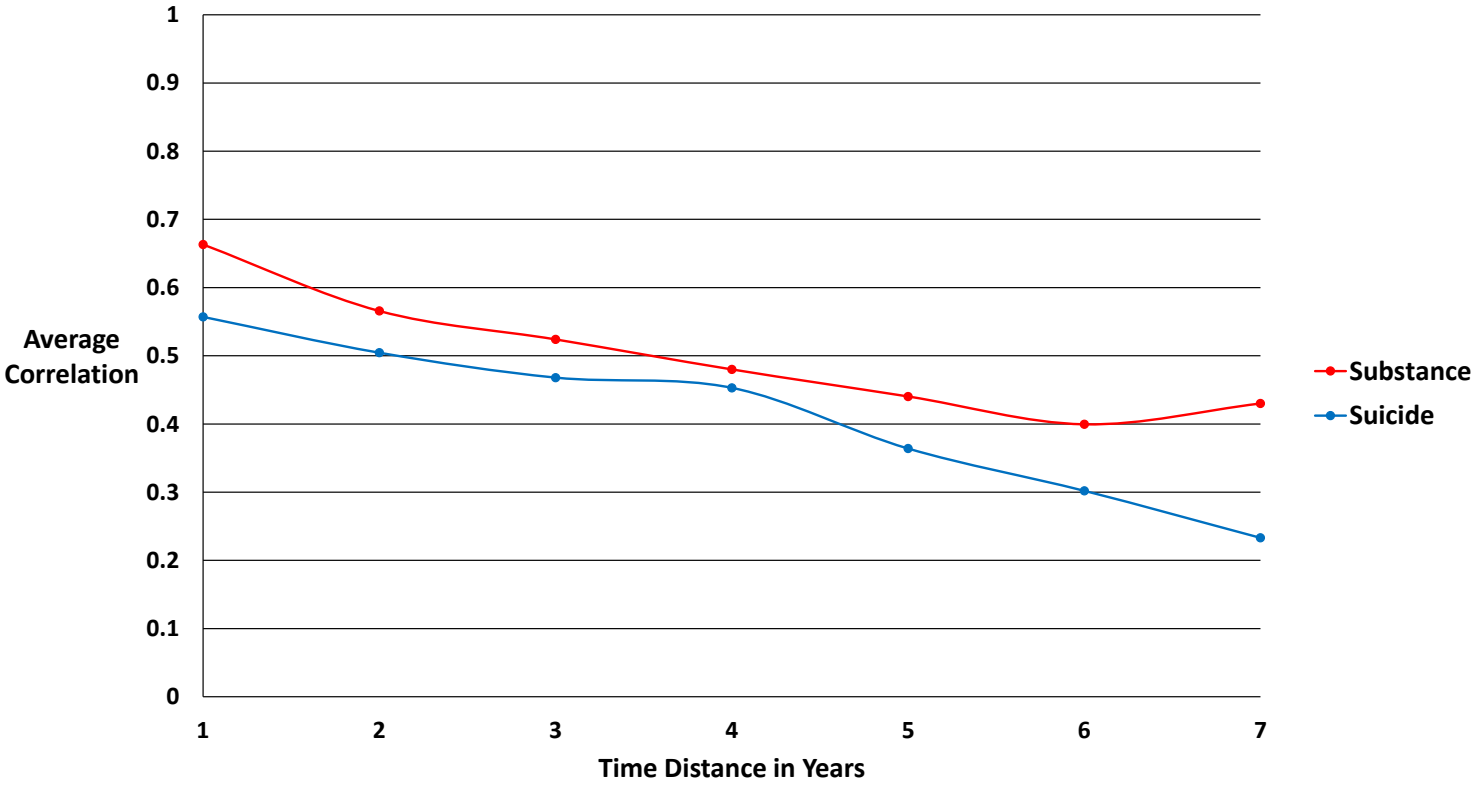
	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8
Z1	0.357	0.122	0.123	0.234	0.293	0.253	0.271	0.179
Z2	0.325	0.139	0.072	0.178	0.226	0.235	0.235	0.153
Z3	0.137	0.023	0.242	0.144	0.278	0.282	0.314	0.178
Z4	0.182	0.266	0.355	0.346	0.325	0.432	0.368	0.372
Z5	0.068	0.182	0.097	0.309	0.370	0.370	0.377	0.344
Z6	0.176	0.278	0.334	0.272	0.304	0.490	0.359	0.272
Z7	0.133	0.125	0.279	0.238	0.203	0.494	0.509	0.398
Z8	-0.031	0.057	0.306	0.251	0.177	0.346	0.400	0.403

CLPM Estimates

Bayes CLPM (AR11)								
	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8
Z1		0.01						
Z2	-0.02		0.17*					
Z3		-0.01		0.03				
Z4			0.26*		0.17*			
Z5				0.06		0.21*		
Z6					0.04		0.05	
Z7						0.12		0.04
Z8							0.05	

- RI-CLPM cross-lagged effects are not expected to be significant unless CLPM estimates are, the exception being cases with large ICC (e.g. 2/3 or 0.75 as in Hamaker et al., 2015)
  - ICC = random intercept variance/(total variance)
  - Total variance = random intercept variance + residual variance
  - With time-invariant parameters, ICC = R-square (Y) due to the random intercept = Corr (Y1, YT) for large T

Average Tetrachoric Correlations Across Time

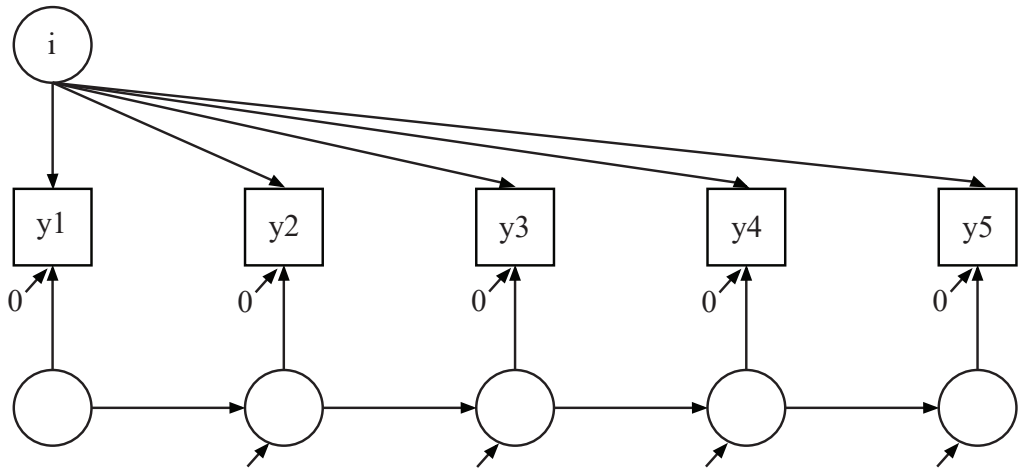


- The plot suggest a lower asymptote larger than zero: Random intercept variance (but much smaller than for the stress outcome in the COMBINE data)
- The plot suggests a higher AR and a higher random intercept variance for Substance abuse than for Suicidal ideation

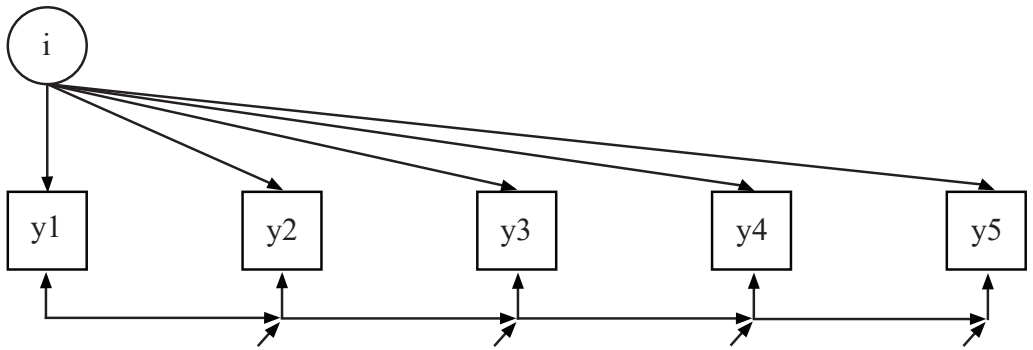
Section 5 Transitioning from Continuous to Categorical Outcomes and from Old to New Mplus Language for RI-CLPM

Random Intercept and Auto-Correlated Residuals RI-AR1 Modeling Displayed in Two Equivalent Ways Continuous Outcomes

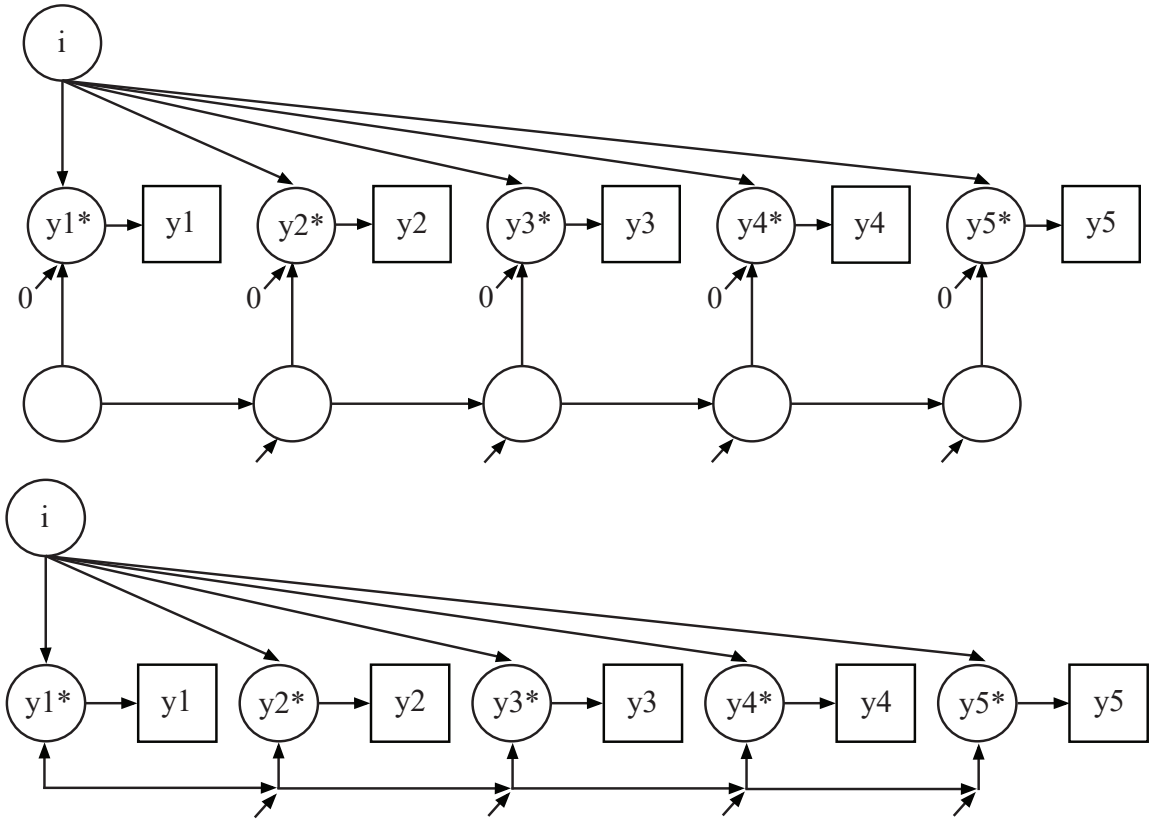
- Old factor specification:



- New residual (hats) specification:



Random Intercept and Auto-Correlated Residuals
RI-AR1 Modeling Displayed in Two Equivalent Ways
Categorical Outcomes



WLSMV Input for Suicide RI-AR1 Model: Naive Version

```
                CATEGORICAL = y1-y8;
ANALYSIS:
                ESTIMATOR = WLSMV;
                PARAMETERIZATION = THETA;
                MODEL = NOCOV;                  OUTPUT:   STANDARDIZED
MODEL:                                                    RESIDUAL
                i BY y1-y8@1;                             TECH1;
                                                          MODINDICES(ALL 0);
                wy1 BY y1;
                wy2 BY y2;                      PLOT:     TYPE = PLOT3;
                wy3 BY y3;
                wy4 BY y4;
                wy5 BY y5;
                wy6 BY y6;
                wy7 BY y7;
                wy8 BY y8;

                y1-y8@0; ! calls for Theta param'n

                wy2-wy8 PON wy1-wy7;
```

Naive WLSMV Problem

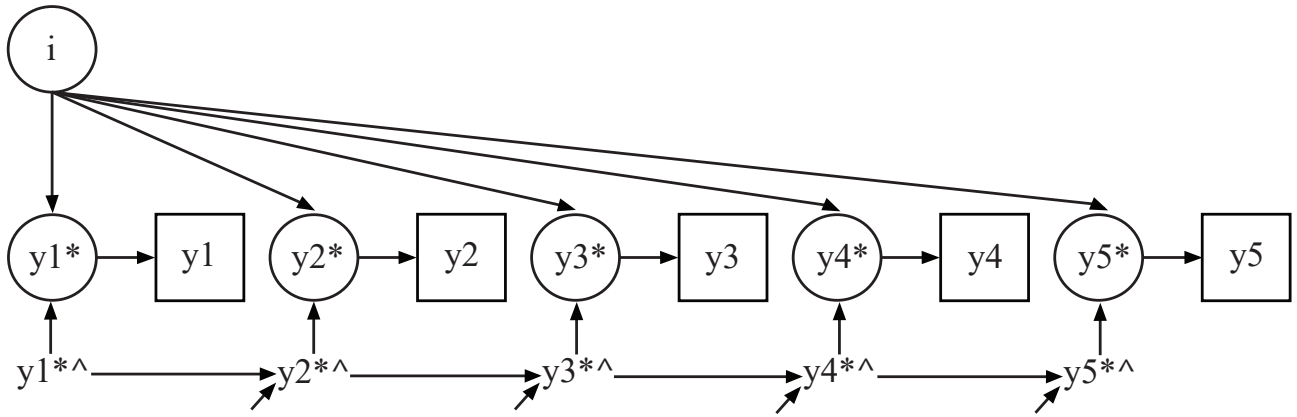
THE STANDARD ERRORS OF THE MODEL PARAMETER ESTIMATES COULD NOT BE COMPUTED. THE MODEL MAY NOT BE IDENTIFIED. CHECK YOUR MODEL. PROBLEM INVOLVING THE FOLLOWING PARAMETER: Parameter 24, WY8
THE CONDITION NUMBER IS -0.157D-16.

- Not all within-factor variances are identified with categorical outcomes

Basic Identification Problem with Categorical Outcomes

- For a binary outcome at a given time point, there is only 1 piece of sample information available to estimate P(Y=1) = π, namely the proportion - the variance is not separately identified
- The mean of a binary outcome is $P(Y = 1) = \pi$ and the variance is $\pi\,(1 - \pi)$, that is, the variance is not a separate parameter to be estimated but is a function of the mean
- The problem is that the WY1 - WY8 factor variances were specified to be free as the default but cannot all be identified with categorical outcomes:
  - Indeterminacy: For each variable, the variance can be multiplied by a constant and the corresponding threshold divided by the square root of that constant - the model fit is the same
  - A solution is to add the line WY1-WY8@1;

WLSMV and Bayes RI-AR1 Using New Hats Input Style



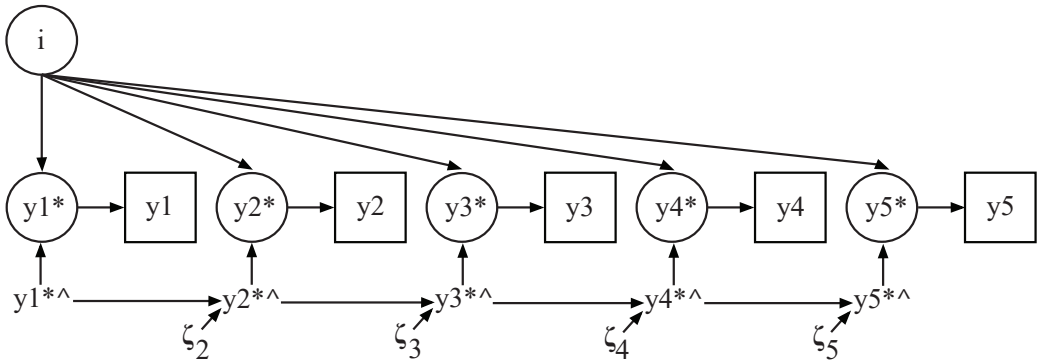

```
ANALYSIS:
          ESTIMATOR = WLSMV;
          PARAMETERIZATION = THETA;
MODEL:
          iy BY y1-y8@1;
          y2^-y8^ PON y1^-y7^;
OUTPUT:   STDYX RESIDUAL TECH10;
PLOT:     TYPE = PLOT3;
```

```
ANALYSIS:
          ESTIMATOR = BAYES;
          BITERATIONS = (2000);
          THIN = 10;
          PROCESSORS = 8;
MODEL:
          iy BY y1-y8@1;
          y2^-y8^ PON y1^-y7^;
OUTPUT:   TECH8 STDYX
          RESIDUAL
          TECH10;
PLOT:     TYPE = PLOT3;
```

- Thresholds are free and y^ residual variances are fixed at 1 as the default as usual in the categorical case

Section 6 Technical Aspects of Modeling with Binary Outcomes: Model Specification, Identification, Estimation, and Simulations

RI-AR1 Probit Model with Hats Specification: Binary Case



- y_t^* continuous latent response variable at time t with threshold τ_t, $y_t^* > \tau_t \rightarrow y_t = 1$, otherwise $y_t = 0$

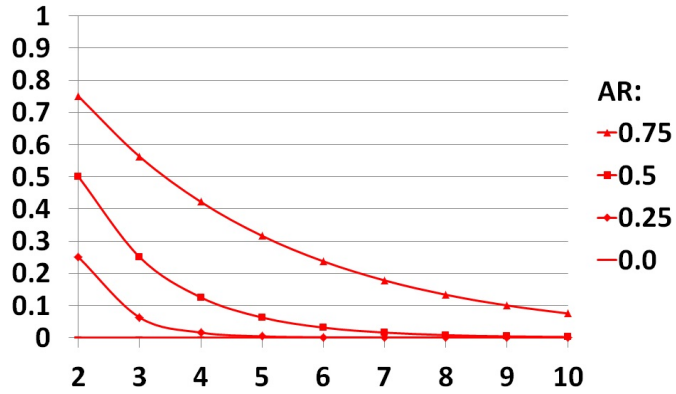
$$y_{it}^* = \alpha_i + \hat{y}_{it}^*, \tag{1}$$

$$\hat{y}_{it}^* = \beta_t \, \hat{y}_{it-1}^* + \zeta_{it}; \; t = 2, \ldots, T \tag{2}$$

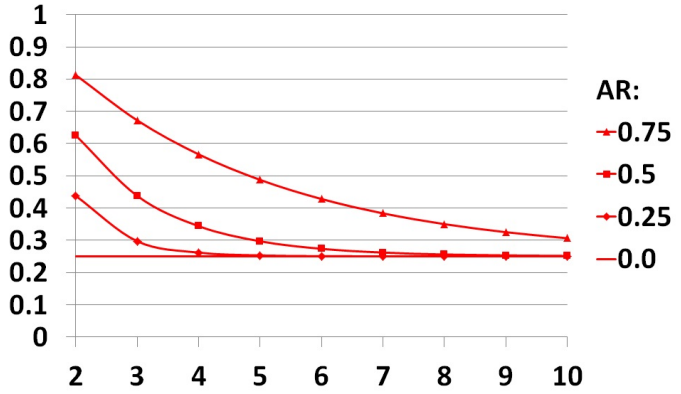
$$\hat{y}_{i1}^* = \zeta_{i1}; \tag{3}$$

- Random intercept α_i and ζ's normally distributed
- A maximum of $T-1$ $V(\zeta_{it})$ variances can be identified
  - Empirical identification issue: Number of identifiable variances depends on the data (correlations across time, # time points)
  - Default of all variances fixed at 1 is often reasonable

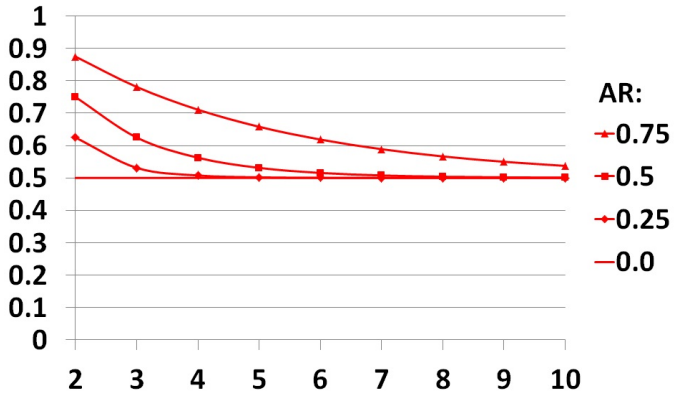
RI and AR1 Impact on Correlations Across Time (T=10)



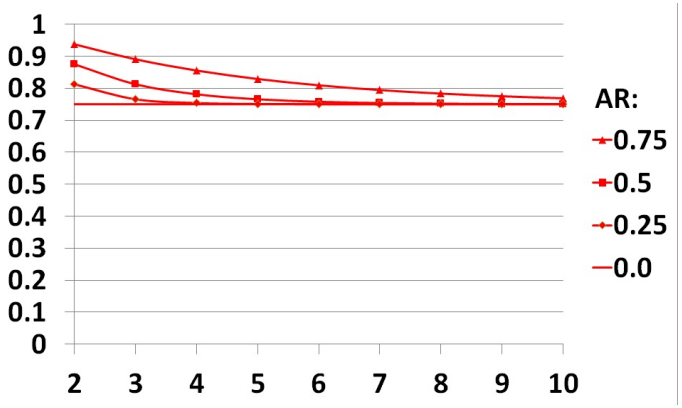
(e) RI variance (R^2) = 0.00



(f) RI variance (R^2) = 0.25



(g) RI variance (R^2) = 0.50

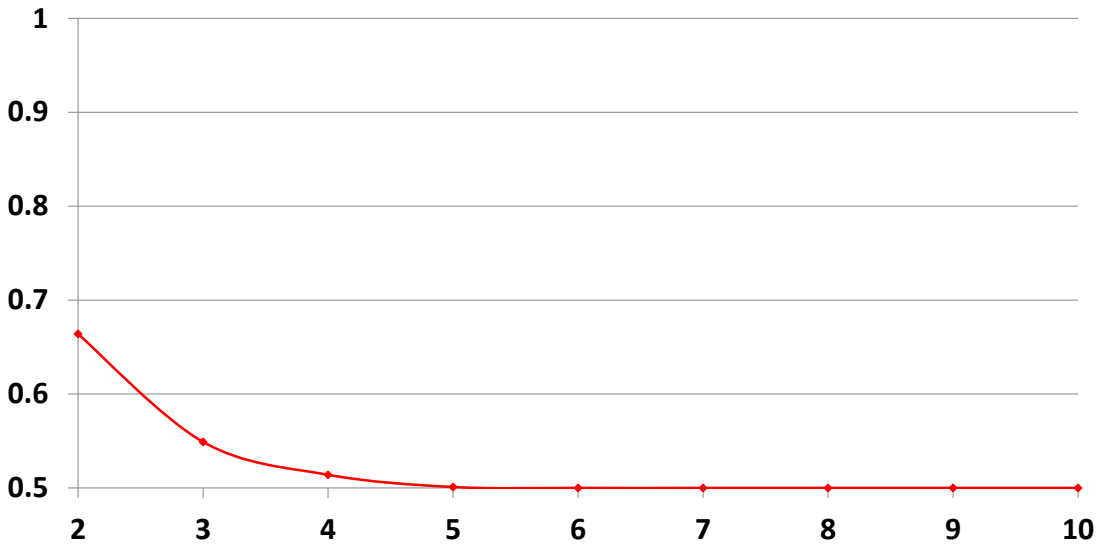


(h) RI variance (R^2) = 0.75

- Correlation $Y_1^*, Y_t^* = \psi + \beta^{t-1}(1-\psi)$ where ψ is the random intercept variance, β is the constant auto-regression among the residuals, and Y* variances are all 1

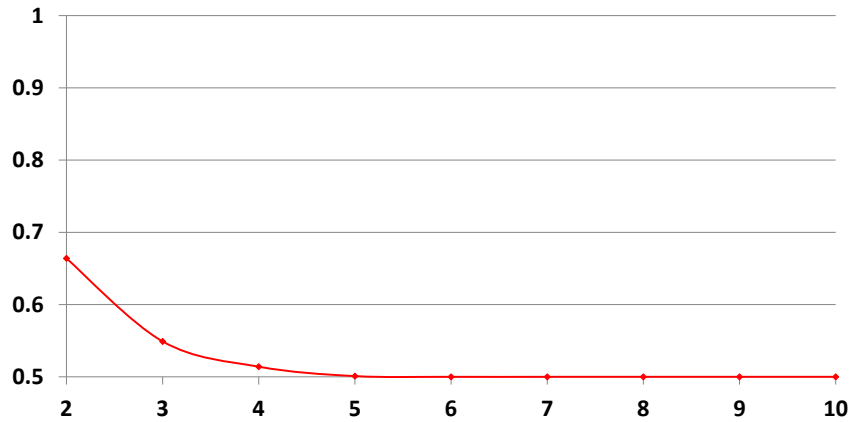
Identification Based on Correlations Across Time

- Allowing time-varying means (probabilities Y=1), the thresholds are identified by the proportions Y=1, while the rest of the model parameters (RI variance, ARs, residual variances) are identified via the correlations
- Simulation example: T=10, RI variance = 1 (R-square of Y*≈0.5); AR's = 0.3, residual variances = 1. Corr (Y_1, Y_t):



- The RI, AR, and residual variance parameter estimates are chosen to fit the curve of the sample correlations at different time distances

Identification Based on Correlations Across Time Cont'd

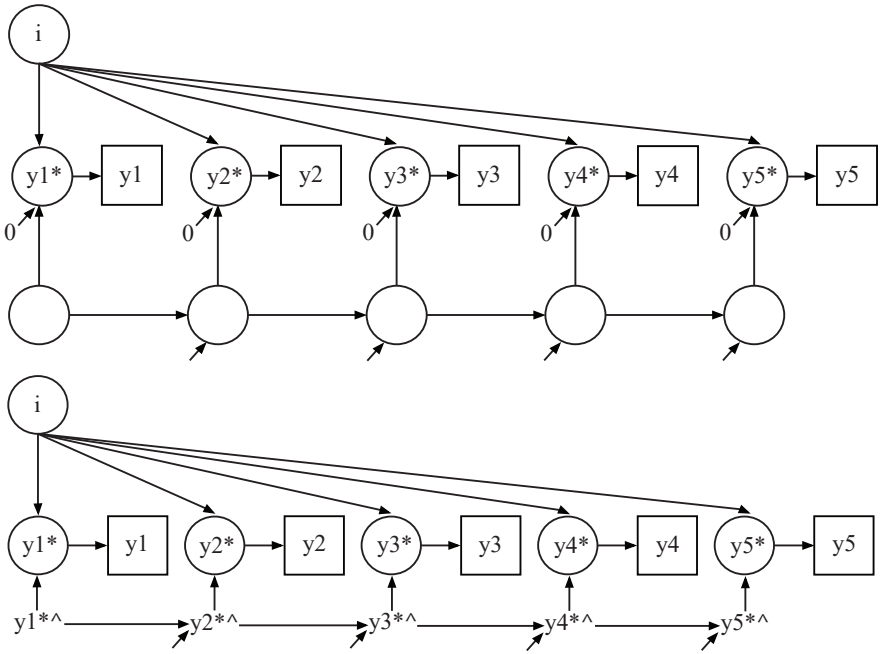


- The large T case:
  - With AR1 models and an autocorrelation of 0.3, the autocorrelation source gives zero contribution at t $\approx$5
  - The correlations over longer time distances are solely due to the random intercept variance
  - This implies that the random intercept variance is identifiable
  - Fixing a residual variance $\theta_1 = 1$ and knowing the random intercept variance ψ, time 1 correlations identify the remaining T-1 θ's: $Corr\,(Y_1, Y_t) = \psi/(\sqrt{\psi+1}\sqrt{\psi+\theta_t})$
  - The T-1 AR's are then identified from among the remaining correlations (T=5 has 10 correlations: 1 identifies ψ, 4 identify 4 θ's, and 4 identify the AR's: 1 df)

More on Identification for a Binary Outcome

- With T time points:
  - There are T sample proportions
  - $T\,(T-1)/2$ sample correlations
- Example of a binary outcome with the RI-AR1 model and fixed residual variances:
  - T parameters for the thresholds (unrestricted means), 1 parameter for RI, $T-1$ parameters for AR1
  - With $T=3$ there are 3+1+2 = 6 parameters = number of proportions and correlations: Just-identified model
  - With $T=4$ there are 4+1+3 = 8 parameters and 10 proportions and correlations: 2 degrees of freedom
  - With $T=8$ there are 8+1+7 = 16 parameters and 36 proportions and correlations: 20 degrees of freedom
- The first variance has different meaning than the other ones because of no regression on previous time point
  - The first variance is identified but often is not significantly different from the other variances which are fixed at 1
  - Depending on the parameter values, some residual variances can also be identified but typically don't change the fit of the model while increasing SEs

Estimation



- ML: Leads to too many dimensions of integration
- WLSMV: Does not handle MAR; good with high coverage; fast
- Bayes: Advantageous due to handling MAR
  - Bayes works poorly with the factor approach, but the residual (hats) approach gives efficient computations - the $\hat{y}^*_{it}$ are not unknowns like factors for which you draw values during the MCMC iterations but are obtained as residuals by subtracting the random intercept from Y* (Asparouhov & Muthén, 2022: RSEM)

Simulations for RI-AR1 with Binary Outcome

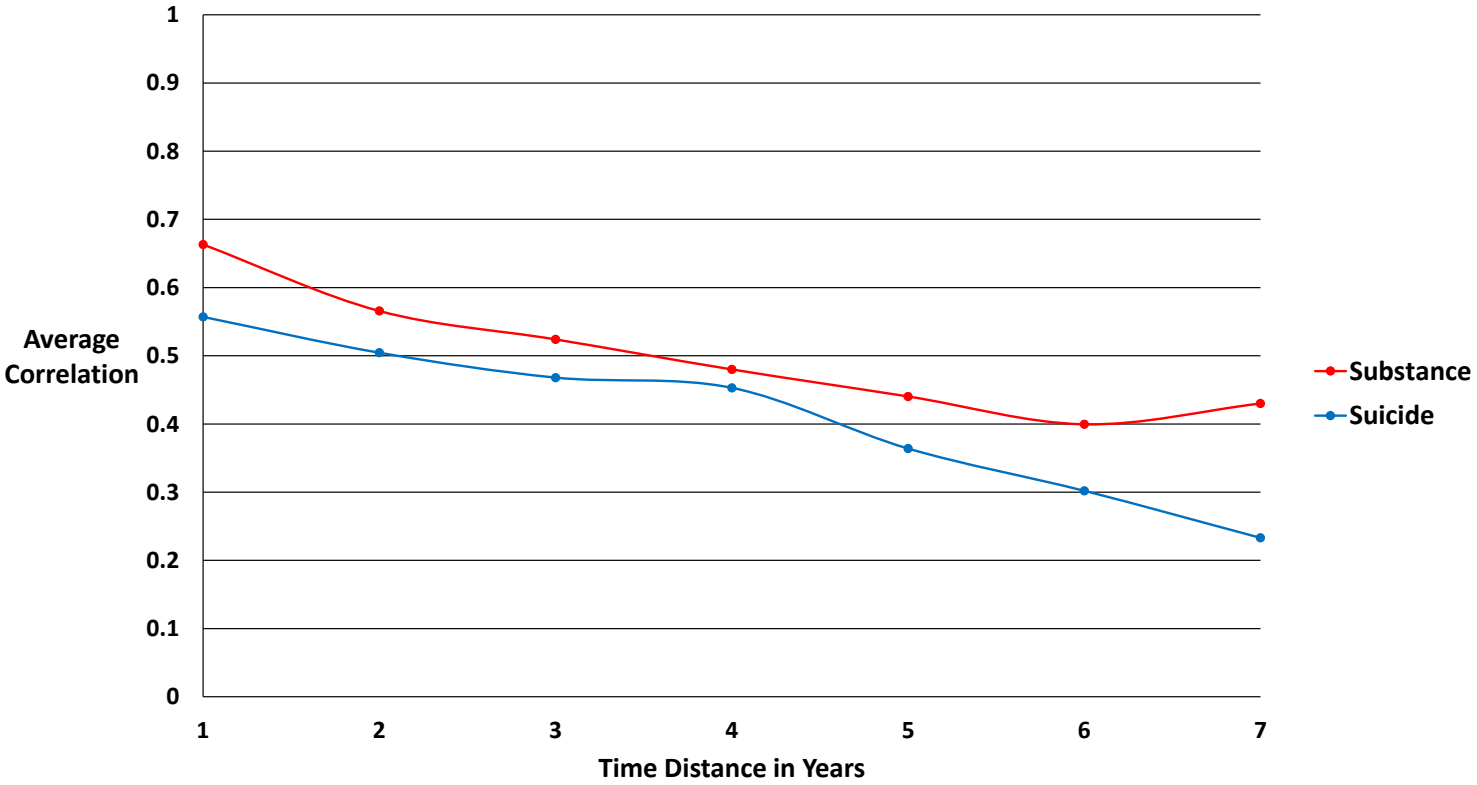
- Time-varying thresholds and ARs
- No Missing Data. WLSMV Estimation (Bayes results similar)
- Parameter values:
  - Autocorrelations = 0.3
  - Random intercept variance = 1, residual variances = 1: R-square due to random intercept = 0.5, R-square for residuals = 0.09
  - P(Y=1) = 0.20 (thresholds = 1.2)
- Fixed variances (default):
  - T=3 gets good results for N=500
  - T=4 gets good results for N=500
  - T=8 gets good results for N=500 (no improvement due to no time-invariant parameters)
- Free 1st variance:
  - T=3 not identified
  - T=4 gets good results for N=2000
  - T=8 gets good results for N=500 (more corr's for 1st var)
- Free T-1 variances (often not a stable model in practice):
  - T=4 is not identified. T=5 gets good results for N=5000
  - T=8 gets good results for N=500

Section 7 Univariate Panel Data Models for Binary Outcomes Analyses of Data Set 2 Suicidal Ideation and Substance Abuse

Panel Data Analysis: Model Notation Introduced in Mplus Web Talk No. 4, Part 1

- Dynamic models (y_t^* regressed on y_{t-1}^*):
  - AR: Auto-regressive, classic model which is dynamic by definition. Used in CLPM
  - ARMA: Auto-regressive, Moving Average, classic model which is dynamic by definition (can only be estimated by WLSMV)
  - D-RI-AR: AR of the classic, dynamic kind but with a random intercept (RI) added
  - D-RI-ARMA: classic ARMA, that is, dynamic but with RI added (can only be estimated by WLSMV)
- RI with dynamic models for residuals (y_t^ regressed on y_{t-1}^):
  - RI-AR: AR is specified for the residual ("within-level", latent-variable centered) part. Used in RI-CLPM
  - RI-ARMA: ARMA is specified for the residual ("within-level", latent-variable centered) part (can only be estimated by WLSMV)

Average Tetrachoric Correlations Across Time



- The plot suggest a lower asymptote larger than zero: Random intercept variance (but much smaller than for the stress outcome in the COMBINE data)
- The plot suggests a higher AR and a higher random intercept variance for Substance abuse than for Suicidal ideation

Univariate Analysis of Suicidal Ideation (N=737, T=8)

- Outputs for all models are posted on the website of the talk
- Frequency of response pattern with 0's throughout = 155
- 4*28 = 112 bivariate cells. 5% = 6
- Good fit, ok fit, marginal fit, poor fit

	Model	# par's	PPP/ χ^2	# Significant Residuals		Comment
				Resp Pattern (obs. freq)	Bivar	
1.	AR1	15	0.133	2 (155, 11)	22	Poor fit
2.	AR2	21	0.378	0	3	OK fit
	AR2*	21	$\chi^2(15)=38$ (.0010)	0	1	OK fit
3.	ARMA11*	21	$\chi^2(15)=17$ (.2972)	0	0	Good fit
4.	D-RI-AR1	16	0.474	0	0	Good fit
	D-RI-AR1*	16	$\chi^2(20)=32$ (.0395)	0	0	Good fit
5.	D-RI-ARMA11*	23	$\chi^2(13)=11$ (.5734)	0	0	Good fit Over-par'd.

Table continues

* denotes WLSMV

Univariate Analysis of Suicidal Ideation Continued

	Model	# par's	PPP/ χ^2	# Significant Residuals		Comment
				Resp Pattern (obs. freq)	Bivar	
6.	RI-AR1	16	0.422	0	1	Good fit
	RI-AR1*	16	$\chi^2(20)=39$ (.0071)	0	0	OK fit
7.	RI-AR2	22	0.466	0	0	Good fit
	RI-AR2*	22	$\chi^2(14)=25$ (.0331)	0	0	Good fit
8.	RI-ARMA11*	22	$\chi^2(14)=10$ (.7423)	Negative V(iy). Over-par'd		
9.	RI-ARMA11* V(iy)=0	21	$\chi^2(15)=17$ (.2972)	0	0	Good fit Same as 3.

* denotes WLSMV

Model 6 RI-AR1 Bayes Estimates for Suicidal Ideation

- Substantial random intercept variance = 0.756, S.E. (SD) = 0.102
- STDYX estimates:

	Estimate	S.E.
I BY		
Y1	0.656	0.025
Y2	0.642	0.026
Y3	0.653	0.026
Y4	0.636	0.032
Y5	0.632	0.032
Y6	0.603	0.038
Y7	0.607	0.034
Y8	0.581	0.035

- Corr (Y1, Y8) due to RI = $0.656 \times 0.581 = 0.381$ (total est corr = 0.383)
- R^2 (Y8) = $0.581^2 = 0.338$

- AR1 estimates (standardized):

	Estimate	S.E.
Y2^ ON		
Y1^	-0.231	0.131
Y3^ ON		
Y2^	0.035	0.117
Y4^ ON		
Y3^	0.304*	0.115
Y5^ ON		
Y4^	0.339*	0.101
Y6^ ON		
Y5^	0.488*	0.099
Y7^ ON		
Y6^	0.473*	0.086
Y8^ ON		
Y7^	0.565*	0.072

Univariate Analysis of Substance Use (N=737, T=8)

- Frequency of response pattern with 0's throughout = 201

	Model	# par's	PPP/ χ^2	# Significant Residuals		Comment
				Resp Pattern (obs. freq)	Bivar	
1.	AR1	15	0.259	1 (201)	14	Poor fit
2.	AR2	21	0.462	0	0	Good fit
	AR2*	21	$\chi^2(15)=26$ (.0339)	1(6)	0	OK fit
3.	ARMA11*	21	$\chi^2(15)=7$ (.9673)	1 (6)	0	OK fit Over-par'd
4.	D-RI-AR1	16	0.458	1 (201)	2	Marginal fit
	D-RI-AR1*	16	$\chi^2(20)=41$ (.0032)	1 (6)	0	Poor fit
5.	D-RI-AR2	22	0.482	1 (201)	1	Marginal fit
	D-RI-AR2*	22	$\chi^2(14)=27$ (.0184)	1 (6)	0	OK fit
6.	D-RI-ARMA*	23	$\chi^2(13)=5$ (.9734)	1 (6)	0	OK fit Over-par'd

Table continues

* denotes WLSMV

Univariate Analysis of Substance Use Continued

	Model	# par's	PPP/ χ^2	# Significant Residuals		Comment
				Resp Pattern (obs. freq)	Bivar	
7.	RI-AR1	16	0.454	1 (201)	3	Marginal fit
	RI-AR1*	16	$\chi^2(20)=61$ (.0000)	1 (6)	2	Poor fit
8.	RI-AR2	22	0.508	1 (201)**	3	Marginal fit
	RI-AR2*	22	$\chi^2(14)=23$ (.0581)	1 (6)	0	OK fit
9.	RI-ARMA*	22	$\chi^2(14)=16$ (.3272)	1 (6)	0	OK fit

* denotes WLSMV
* z-score = 2.01

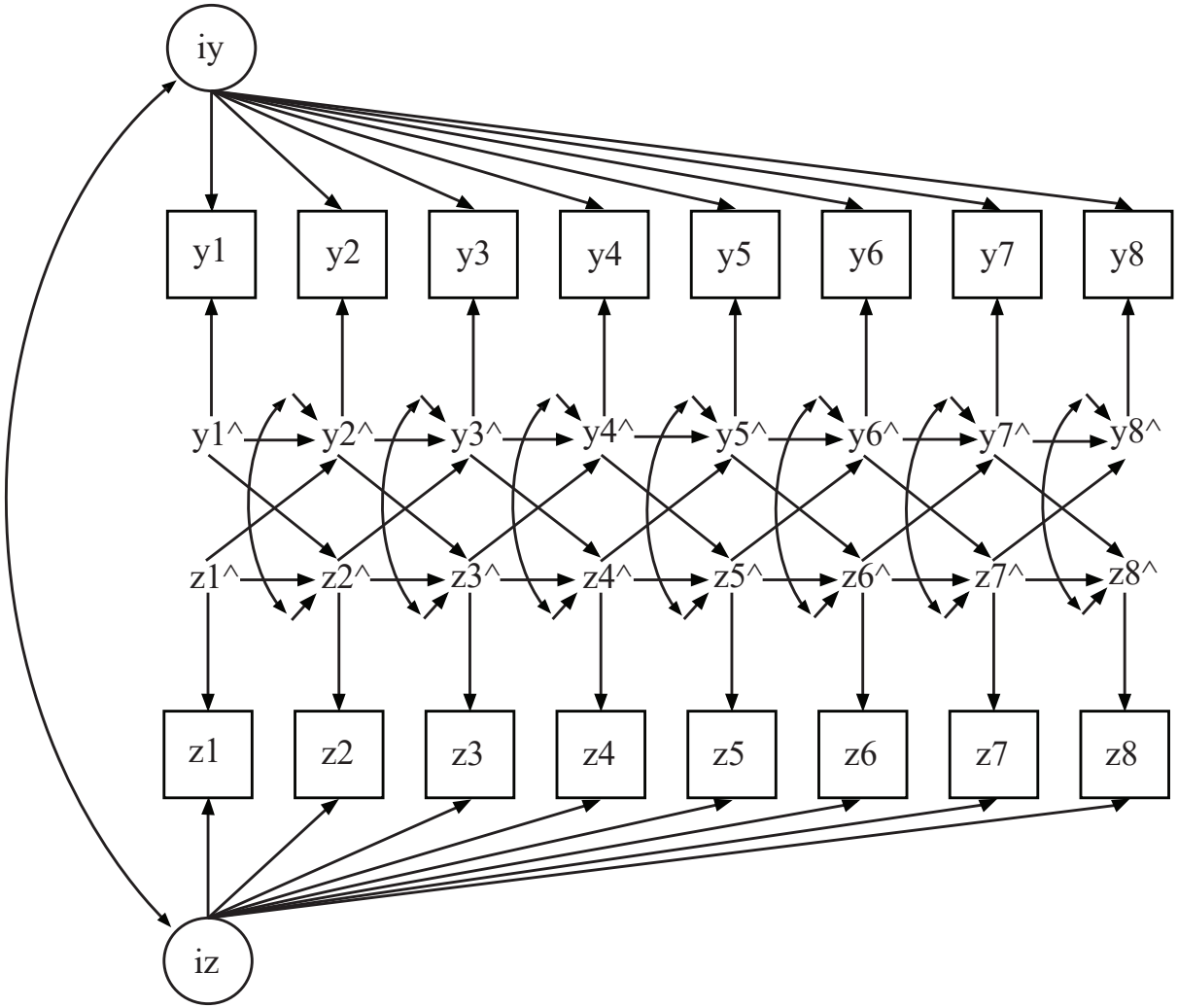
Section 8 Bivariate Binary Cross-Lagged Analyses of Suicidal Ideation and Substance Abuse

The 20 Most Frequent Response Patterns (* = Missing)

Suicidal Ideation + Substance Abuse		
Pattern	Frequency	Percentage
0000000000000000	104	14.1
0000000010000000	12	1.6
0000000001000000	10	1.4
*0000000*0000000	7	1.0
1100000000000000	7	1.0
0000000*0000000*	7	1.0
0010000000000000	6	0.8
0000000000000001	6	0.8
000000000000	6	0.8
0*0000000*000000	5	0.7
0000000000010000	5	0.7
0001000000000000	5	0.7
000000*0000000*0	5	0.7
000*0000000*0000	5	0.7
0000000000001000	5	0.7
0*******0*******	5	0.7
000*****000*****	4	0.5
0000000000000010	4	0.5
0000000000100000	4	0.5
0000000010001000	4	0.5

- Percentage 0's or missing: 28.8%

Bivariate Analysis: RI-CLPM (RI-AR11)



WLSMV Old Input Style for RI-CLPM (RI-AR11)

```
                USEV = y1-y8 z1-z8;
                CATEGORICAL = y1-z8;
ANALYSIS:
                ESTIMATOR = WLSMV;
                PARAMETERIZATION = THETA;
                MODEL = NOCOV;
MODEL:
                iy BY y1-y8@1;
                iz BY z1-z8@1;
                wy1 BY y1; wz1 BY z1;
                wy2 BY y2; wz2 BY z2;
                wy3 BY y3; wz3 BY z3;
                wy4 BY y4; wz4 BY z4;
                wy5 BY y5; wz5 BY z5;
                wy6 BY y6; wz6 BY z6;
                wy7 BY y7; wz7 BY z7;
                wy8 BY y8; wz8 BY z8;

                y1-z8@0;

                wy1-wy8@1;
                wz1-wz8@1;

                wy2-wy8 PON wy1-wy7;
                wz2-wz8 PON wz1-wz7;

                wy2-wy8 PON wz1-wz7;
                wz2-wz8 PON wy1-wy7;

                wy1-wy8 PWITH wz1-wz8;

                iy WITH iz;
```

WLSMV and Bayes New Input Style for RI-CLPM (RI-AR11)

```
ANALYSIS:
          ESTIMATOR = WLSMV;
          PARAMETERIZATION = THETA;
MODEL:
          iy BY y1-y8@1;
          iz BY z1-z8@1;
          y2^-y8^ PON y1^-y7^;
          z2^-z8^ PON z1^-z7^;

          y2^-y8^ PON z1^-z7^;
          z2^-z8^ PON y1^-y7^;

          y1-y8 PWITH z1-z8;
OUTPUT:   STDYX RESIDUAL TECH10;
PLOT:     TYPE = PLOT3;
```

```
ANALYSIS:
          ESTIMATOR = BAYES;
          BITERATIONS = (2000);
          THIN = 10;
          PROCESSORS = 8;
MODEL:
          iy BY y1-y8@1;
          iz BY z1-z8@1;
          y2^-y8^ PON y1^-y7^;
          z2^-z8^ PON z1^-z7^;

          y2^-y8^ PON z1^-z7^;
          z2^-z8^ PON y1^-y7^;

          y1-y8 PWITH z1-z8;
OUTPUT:   TECH8 STDYX RESIDUAL TECH10;
PLOT:     TYPE = PLOT3;
```

Bivariate Analysis Results for Suicidal Ideation and Substance Abuse

- Outputs for all models are posted on the website of the talk
- T=8 gives 16*(16-1)/2 = 120 bivariate tables, 480 cells, 5% = 24

	Model	# par's	PPP χ^2	# Significant Residuals		Comment
				Resp Pattern	Bivar	
1.	AR2	64	0.445	0	3	Good fit
	AR2* (CLPM2)	64	$\chi^2(72)=115$ (.0011)	0	1	Marginal fit
2.	ARMA*	64	$\chi^2(72)=69$ (.5874)	0	0	Good fit Over-par'd
3.	D-RI-AR12	61	0.462	0	1	Good fit
	D-RI-AR12*	61	$\chi^2(75)=102$ (.0197)	0	0	Good fit. Neg. V(iz)
4.	RI-AR12	61	0.450	0	5	Good fit
	RI-AR12* (RI-CLPM12)	61	$\chi^2(75)=98$ (.0372)	0	0	Good fit. Npd
5.	RI-ARMA Simple V(iy)=0	65	$\chi^2(71)=68$ (.5940)	0	0	Good fit Over-par'd

* denotes WLSMV

Testing of Time-Invariance Using Model Test (Wald Chi-2) With Bayes Estimation of the RI-AR12 Model

- Non-invariant model. Invariance testing of cross-lagged effects and covariances

```
ANALYSIS:
        ESTIMATOR= BAYES;
        BITERATIONS = (5000);
        THIN = 10;
        PROCESSORS = 8;
MODEL:
        iy BY y1-y8@1;
        iz BY z1-z8@1;
        y2^-y8^ PON y1^-y7^ (a2-a8);
        z2^-z8^ PON z1^-z7^ (b2-b8);
        z3^-z8^ PON z1^-z6^ (c3-c8);

        y2^-y8^ PON z1^-z7^ (d2-d8);
        z2^-z8^ PON y1^-y7^ (e2-e8);

        y1^-y8^ PWITH z1^-z8^ (f1-f8);
OUTPUT:
        STANDARDIZED RESIDUAL
        TECH8 TECH10 SVALUES;
PLOT:
        TYPE = PLOT3;
```

```
MODEL TEST:
        0 = d3-d2;
        0 = d4-d2;
        0 = d5-d2;
        0 = d6-d2;
        0 = d7-d2;
        0 = d8-d2;
        0 = e3-e2;
        0 = e4-e2;
        0 = e5-e2;
        0 = e6-e2;
        0 = e7-e2;
        0 = e8-e2;
        0 = f2-f1;
        0 = f3-f1;
        0 = f4-f1;
        0 = f5-f1;
        0 = f6-f1;
        0 = f7-f1;
        0 = f8-f1;
```

Results of Time-Invariance Testing with Model Test

Model	WLSMV			Bayes		
	χ^2 test	# Sig Residuals		χ^2/PPP	# Sig Residuals	
Invar	(p-value)	Patterns	Bivar's	(p-value)	Patterns	Bivar's
1. AR2 (CLPM2)						
X-lags	$\chi^2(12)=24$ (.0195)			$\chi^2(12)=14$ (.3318)		
X-lags + Covs	$\chi^2(19)=27$ (.1065)			$\chi^2(19)=18$ (.4970)		
Both invar (45 par's)	$\chi^2(91)=130$ (.0043)	0	2	0.458	0	6
2. D-RI-AR12						
X-lags	$\chi^2(12)=50$ (.0000)			$\chi^2(12)=14$ (.3258)		
X-lags + Covs	$\chi^2(19)=54$ (.0000)			$\chi^2(19)=18$ (.5191)		
Both invar (42 par's)	Poor fit			0.446	0	3
No invar (61 par's)	$\chi^2(75)=102$* (.0192)	0	0	0.462	0	1

* Negative V(iz)

Testing of Time-Invariance Continued

Model	WLSMV			Bayes		
	χ^2 test	# Sig Residuals		χ^2/PPP	# Sig Residuals	
Invar	(p-value)	Patterns	Bivar's	(p-value)	Patterns	Bivar's
3. RI-AR12 (RI-CLPM12)						
X-lags	$\chi^2(12)=28$ (.0056)			$\chi^2(12)=17$ (.1561)		
X-lags + Covs	$\chi^2(19)=31$ (.0375)			$\chi^2(19)=21$ (.3279)		
Both invar (42 par's)	$\chi^2(94)=127$* (.0129)	0	2	0.426	0	9
4. RI-ARMA V(iy)=0						
X-lags	$\chi^2(12)=25$ (.0165)					
X-lags + Covs	$\chi^2(19)=41$ (.0023)					
Both invar (46 par's)	$\chi^2(90)=109$* (.0812)	0	0			

* Negative V(iz)

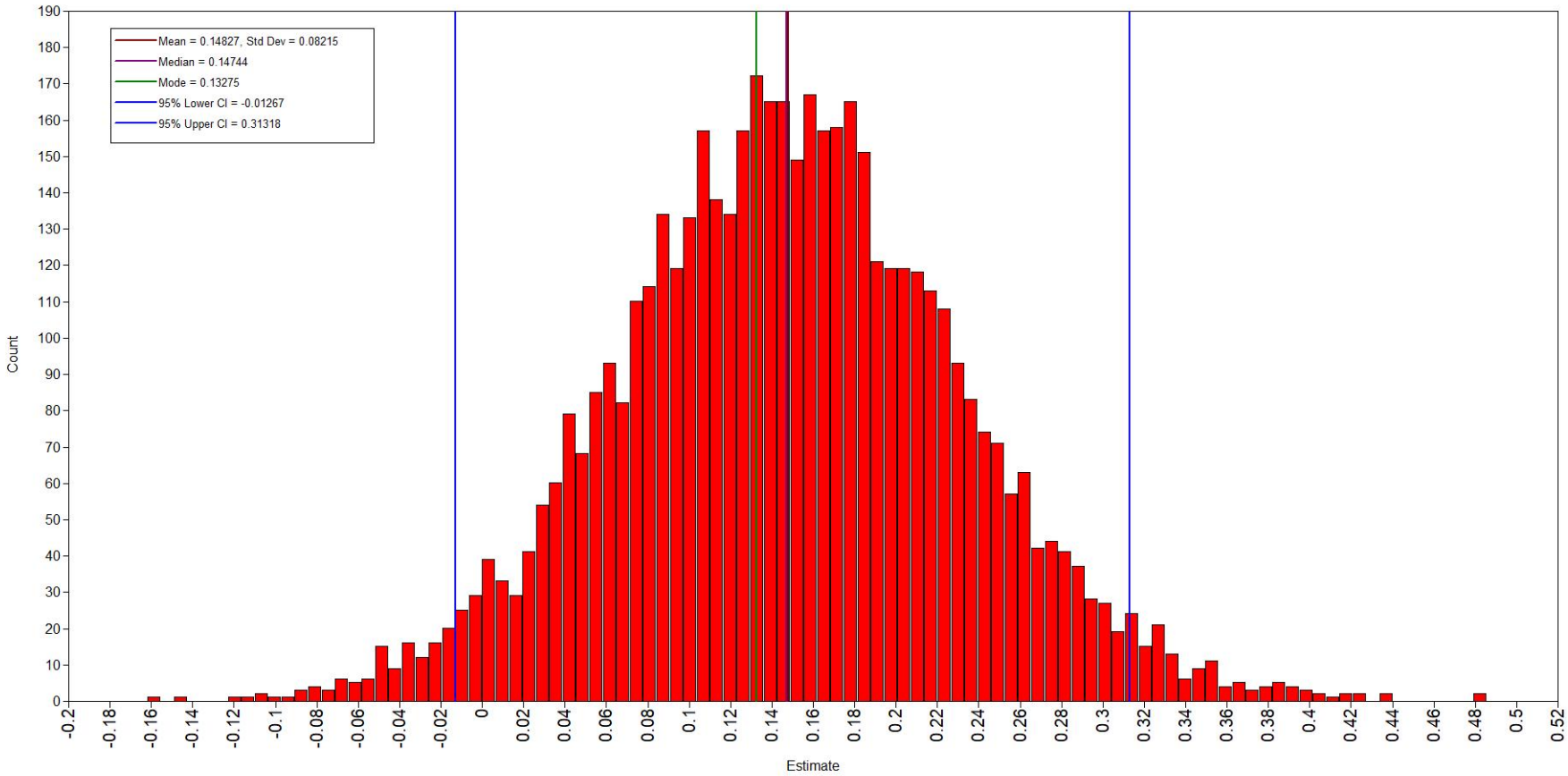
Estimated Cross-Lagged Effects
Y: Suicidal Ideation, Z: Substance Abuse

Model	Cross-Lag	WLSMV # Sig X-Lags			WLSMV BS = 500 # Sig X-Lags			Bayes # Sig X-Lags		
		Pos	Neg	NS	Pos	Neg	NS	Pos	Neg	NS
1. AR2 Invar.	Y ON Z	1	0	0	1	0	0	1	0	0
(CLPM2)	Z ON Y	1	0	0	1	0	0	1	0	0
2. D-RI-AR12	Y ON Z	7	0	0	5	0	2	1	0	6
	Z ON Y	1	0	6	1	0	6	1	0	6
3. D-RI-AR12	Y ON Z							0	0	1
Invar.	Z ON Y							0	0	1
4. RI-AR12	Yˆ ON Zˆ	3	0	4	2	0	5	1	0	6
	Zˆ ON Yˆ	1	0	6	1	0	6	0	0	7
5. RI-AR12	Yˆ ON Zˆ	1	0	0	1	0	0	0	0	1
Invar.	Zˆ ON Yˆ	0	0	1	0	0	1	0	0	1
6. RI-ARMA										
V(iy)=0	fY ON fZ	1	0	0	1	0	0			
Invar.	fZ ON fY	0	0	1	1	0	0			

- Number of successful WLSMV bootstrap draws for the 5 models (not M3): 459, 405, 93, 485, 470 (Asparouhov & Muthén, 2021)

Posterior Distribution of Bayes Estimate of Y7 ON Z6 for D-RI-AR12 Non-Invariant Model

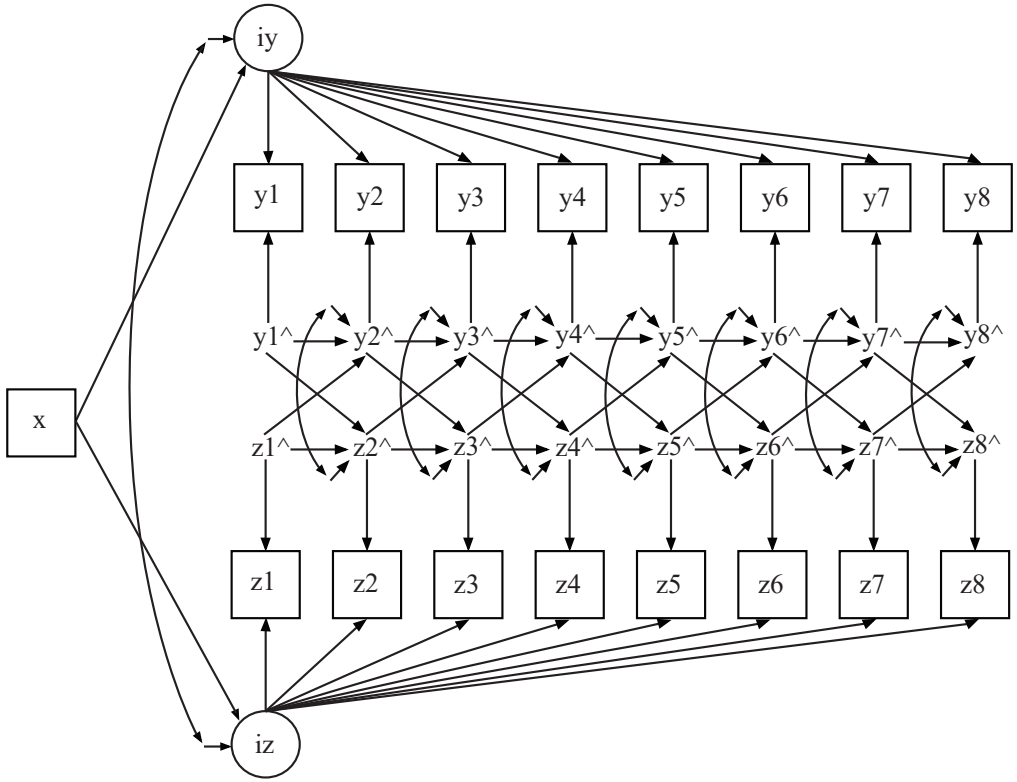
- Bayes posterior distribution has long tails due to uncertainty (small N)
- WLSMV bootstrap distribution has similar shape (obtained by PLOT3)



Results Conclusions

- Using Bayes for AR2 (CLPM2) and bootstrapped WLSMV RI-ARMA with time-invariant cross-lags and residual covariances shows significant but very small cross-lagged effects
- Using Bayes with time-invariant cross-lags and residual covariances, neither D-RI-AR12 nor RI-AR12 show significant cross-lagged effects
  - Lack of power due to small N for low prevalence variables?
  - Lack of power of invariance testing?
  - Is there invariance across part of the time scale?
- Where did the correlations between the two processes go?
  - Intercept correlation and residual correlation

Covariates



- Covariates: Gender, race, lunch (poverty indicator)
- No effect on IY (suicidal ideation)
  - Using only the last 5 time points (T=5), female has a positive effect and Black a negative effect
- Female and Black have a negative effect on IZ (substance abuse)

Section 9 Planning Future Studies: Power Estimation Using Monte Carlo Simulations

Increasing Power

- How can power be increased to reject that cross-lagged effects are zero?
  - Increase N
  - Reformulate variables to increase prevalence
  - Measure more often (e.g. Fall and Spring instead of annually)
    - Useful with time-invariance
  - Use multiple indicators and consider effects for latent constructs
- How large does the sample need to be to make more cross-lagged effects significant?
  - Quadrupling the sample size halves the SE's
  - Monte Carlo simulations

Monte Carlo Simulation Studies

- At which sample size N and number of time points T are the estimates and standard errors trustworthy?
- At which sample size and at which effect size can you detect an effect if it exists? - What is the power? (Last column: % Sig Coeff.)
- How much larger do N and T have to be to detect an effect when the prevalence is lower? How much smaller can N and T be if prevalence is higher?
- How large do N and T have to be for model fit assessment to be trustworthy?
- How large do N, T, and effect sizes have to be for time-invariance testing to be trustworthy?
- How much larger do N and T have to be to distinguish between competing models?
- Muthén & Muthén (2002). How to use a Monte Carlo study to decide on sample size and determine power. Structural Equation Modeling

Bayes Estimates for Non-Invariant D-RI-AR12

	Est	Post SD		2.5%	97.5%		Est/SD	EST/(SD/2)
Y2 ON								
Y1	-0.118	0.104	0.131	-0.319	0.095			
Z1	0.054	0.112	0.322	-0.169	0.273			
Y3 ON								
Y2	0.215	0.110	0.017	0.016	0.445	*		
Z2	0.020	0.035	0.278	-0.047	0.088			
Y4 ON								
Y3	0.305	0.113	0.001	0.093	0.542	*		
Z3	0.058	0.103	0.286	-0.136	0.261			
Y5 ON								
Y4	0.317	0.099	0.000	0.131	0.517	*		
Z4	0.122	0.086	0.072	-0.043	0.301		1.42	2.84
Y6 ON								
Y5	0.219	0.100	0.007	0.043	0.435	*		
Z5	0.233	0.090	0.002	0.064	0.411	*	2.59	5.18
Y7 ON								
Y6	0.209	0.100	0.016	0.016	0.415	*		
Z6	0.147	0.082	0.035	-0.013	0.313		1.79	3.86
Y8 ON								
Y7	0.413	0.122	0.000	0.192	0.667	*		
Z7	0.089	0.086	0.145	-0.077	0.263			

Simulation for Non-Invariant D-RI-AR12, N=737, Bayes

	Pop.	Est. Ave	SD	SE Ave	MSE	Cover	%Sig
Y2 ON							
Y1	-0.118	-0.1235	0.0852	0.0813	0.0073	0.926	0.334
Z1	0.054	0.0598	0.0896	0.0866	0.0081	0.942	0.136
Y3 ON							
Y2	0.215	0.2279	0.0929	0.0920	0.0088	0.956	0.718
Z2	0.020	0.0183	0.0284	0.0297	0.0008	0.952	0.088
Y4 ON							
Y3	0.305	0.3167	0.0954	0.0943	0.0092	0.960	0.960
Z3	0.058	0.0554	0.0853	0.0835	0.0073	0.938	0.102
Y5 ON							
Y4	0.317	0.3254	0.0925	0.0873	0.0086	0.936	0.990
Z4	0.122	0.1225	0.0747	0.0731	0.0056	0.940	**0.434**
Y6 ON							
Y5	0.219	0.2218	0.0819	0.0798	0.0067	0.938	0.826
Z5	0.233	0.2399	0.0779	0.0727	0.0061	0.932	**0.932**
Y7 ON							
Y6	0.209	0.2128	0.0832	0.0839	0.0069	0.946	0.768
Z6	0.147	0.1508	0.0702	0.0683	0.0049	0.922	**0.608**
Y8 ON							
Y7	0.413	0.4243	0.1087	0.1050	0.0119	0.942	0.992
Z7	0.089	0.0893	0.0761	0.0731	0.0058	0.944	0.266

- SE's lower than in real-data run due to real-data model not fitting perfectly

Simulation for Non-Invariant D-RI-AR12, N=3000, Bayes

	Pop.	Est. Ave	SD	SE Ave	MSE	Cover	%Sig
Y2 ON							
Y1	-0.118	-0.1184	0.0398	0.0397	0.0016	0.940	0.832
Z1	0.054	0.0546	0.0423	0.0421	0.0018	0.938	0.246
Y3 ON							
Y2	0.215	0.2153	0.0449	0.0447	0.0020	0.956	0.996
Z2	0.020	0.0204	0.0138	0.0141	0.0002	0.944	0.302
Y4 ON							
Y3	0.305	0.3102	0.0474	0.0457	0.0023	0.938	1.000
Z3	0.058	0.0564	0.0426	0.0410	0.0018	0.946	0.306
Y5 ON							
Y4	0.317	0.3188	0.0414	0.0425	0.0017	0.944	1.000
Z4	0.122	0.1211	0.0349	0.0359	0.0012	0.962	**0.934**
Y6 ON							
Y5	0.219	0.2194	0.0387	0.0390	0.0015	0.960	1.000
Z5	0.233	0.2351	0.0360	0.0357	0.0013	0.954	**1.000***
Y7 ON							
Y6	0.209	0.2114	0.0401	0.0410	0.0016	0.960	1.000
Z6	0.147	0.1477	0.0363	0.0336	0.0013	0.932	**0.996**
Y8 ON							
Y7	0.413	0.4159	0.0525	0.0504	0.0028	0.942	1.000
Z7	0.089	0.0875	0.0366	0.0356	0.0013	0.930	0.684

Simulation for Non-Invariant D-RI-AR12, N=2000, Bayes

	Pop.	Est. Ave	SD	SE Ave	MSE	Cover	%Sig
Y2 ON							
Y1	-0.118	-0.1206	0.0495	0.0487	0.0024	0.950	0.664
Z1	0.054	0.0559	0.0525	0.0518	0.0027	0.954	0.180
Y3 ON							
Y2	0.215	0.2187	0.0568	0.0550	0.0032	0.932	0.986
Z2	0.020	0.0200	0.0180	0.0174	0.0003	0.940	0.222
Y4 ON							
Y3	0.305	0.3103	0.0577	0.0562	0.0033	0.940	1.000
Z3	0.058	0.0564	0.0503	0.0504	0.0025	0.956	0.202
Y5 ON							
Y4	0.317	0.3179	0.0531	0.0521	0.0028	0.952	1.000
Z4	0.122	0.1220	0.0422	0.0440	0.0018	0.952	**0.826**
Y6 ON							
Y5	0.219	0.2187	0.0477	0.0479	0.0023	0.950	0.996
Z5	0.233	0.2389	0.0443	0.0439	0.0020	0.946	**1.000**
Y7 ON							
Y6	0.209	0.2112	0.0484	0.0502	0.0023	0.952	0.992
Z6	0.147	0.1481	0.0417	0.0412	0.0017	0.948	**0.956**
Y8 ON							
Y7	0.413	0.4154	0.0644	0.0620	0.0042	0.934	1.000
Z7	0.089	0.0889	0.0443	0.0438	0.0020	0.946	0.532

Simulation for Non-Invariant RI-AR12, N=737, Bayes

	Pop.	Est. Ave	SD	SE Ave	MSE	Cover	%Sig
Y2^ ON							
Y1^	-0.113	-0.1352	0.1341	0.1311	0.0184	0.934	0.170
Z1^	-0.270	-0.2702	0.1609	0.1572	0.0258	0.946	0.412
Y3^ ON							
Y2^	0.027	0.0380	0.1067	0.1032	0.0115	0.950	0.068
Z2^	-0.056	-0.0725	0.0612	0.0656	0.0040	0.950	0.212
Y4^ ON							
Y3^	0.265	0.2651	0.1066	0.1123	0.0113	0.960	0.666
Z3^	-0.037	-0.0361	0.1028	0.1045	0.0105	0.948	0.070
Y5^ ON							
Y4^	0.242	0.2351	0.1093	0.1045	0.0120	0.940	0.614
Z4^	0.165	0.1686	0.1126	0.1140	0.0127	0.956	0.336
Y6^ ON							
Y5^	0.404	0.3964	0.1154	0.1105	0.0133	0.940	0.934
Z5^	0.314	0.3254	0.1264	0.1219	0.0161	0.948	**0.802***
Y7^ ON							
Y6^	0.402	0.3948	0.1080	0.1071	0.0117	0.936	0.948
Z6^	0.050	0.0589	0.1240	0.1279	0.0154	0.958	0.080
Y8^ ON							
Y7^	0.542	0.5516	0.1444	0.1424	0.0209	0.940	0.958
Z7^	0.116	0.1111	0.1370	0.1355	0.0188	0.936	0.156

Simulation for Non-Invariant RI-AR12, N=3000, Bayes

	Pop.	Est. Ave	SD	SE Ave	MSE	Cover	%Sig
Y2^ ON							
Y1^	-0.113	-0.1139	0.0610	0.0594	0.0037	0.946	0.488
Z1^	-0.270	-0.2748	0.0759	0.0726	0.0058	0.936	**0.962**
Y3^ ON							
Y2^	0.027	0.0285	0.0492	0.0502	0.0024	0.950	0.086
Z2^	-0.056	-0.0565	0.0294	0.0295	0.0009	0.944	0.540
Y4^ ON							
Y3^	0.265	0.2664	0.0526	0.0540	0.0028	0.948	0.998
Z3^	-0.037	-0.0373	0.0502	0.0481	0.0025	0.920	0.134
Y5^ ON							
Y4^	0.242	0.2416	0.0493	0.0503	0.0024	0.960	0.998
Z4^	0.165	0.1652	0.0542	0.0546	0.0029	0.952	**0.854**
Y6^ ON							
Y5^	0.404	0.4041	0.0503	0.0525	0.0025	0.960	1.000
Z5^	0.314	0.3126	0.0598	0.0578	0.0036	0.934	**1.000***
Y7^ ON							
Y6^	0.402	0.4013	0.0497	0.0502	0.0025	0.956	1.000
Z6^	0.050	0.0506	0.0595	0.0600	0.0035	0.956	0.122
Y8^ ON							
Y7^	0.542	0.5450	0.0634	0.0630	0.0040	0.938	1.000
Z7^	0.116	0.1129	0.0593	0.0608	0.0035	0.954	0.458

Power to Reject Time Invariance: Wald Test Simulations Results for CLPM2 (AR2) WLSMV and RI-CLPM Bayes

Real data: Chi-square (19) = 27 (.1065)

Simulated data (N=737):

Degrees of freedom	19
Mean	59.530
Std Dev	13.506
Number of successful computations	472

Proportions

Expected	Observed
0.990	1.000
0.980	1.000
0.950	1.000
0.900	1.000
0.800	1.000
0.700	1.000
0.500	1.000
0.300	1.000
0.200	0.998
0.100	0.994
0.050	**0.994**
0.020	0.977
0.010	0.977

Real data: Chi-square (19) = 21 (.3279)

Simulated data (N=737):

Degrees of freedom	19
Mean	46.463
Std Dev	11.225
Number of successful computations	500

Proportions

Expected	Observed
0.990	1.000
0.980	1.000
0.950	1.000
0.900	1.000
0.800	1.000
0.700	1.000
0.500	0.998
0.300	0.990
0.200	0.988
0.100	0.962
0.050	**0.938**
0.020	0.888
0.010	0.820

Wald Test Simulation Conclusions

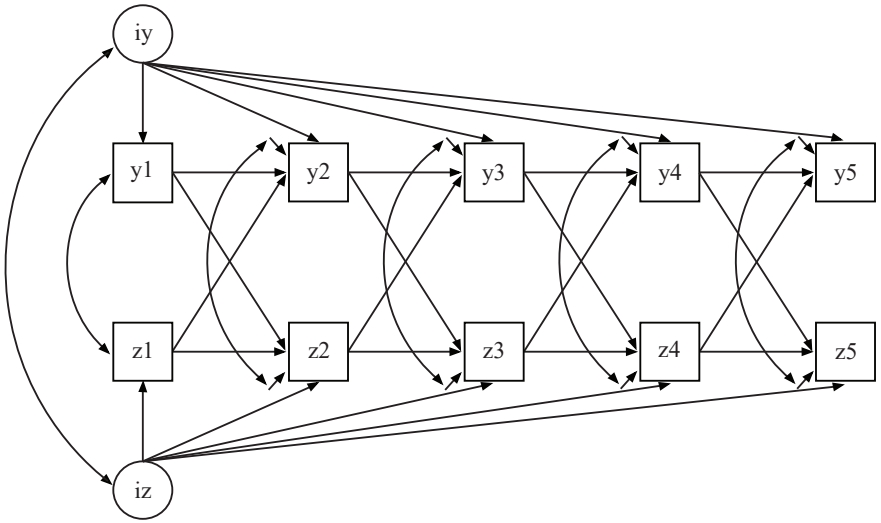
- Is the power to reject time invariance in the real data too low due to sample size and prevalence not being large enough?
- The simulations show that power to reject time invariance is high assuming the model is correct (which it is in the simulations)
- The real-data analysis has larger SEs due to model not being exactly correct. It therefore gets lower power
- The power to reject time invariance may be low in this case

Section 10 Other Analysis Approaches: Measurement Error Observed Y as Predictor Instead of Latent Y* Reciprocal Modeling

Measurement Error

- Measurement error with various restrictions can be added to CLPM and RI-CLPM for categorical outcomes and works ok in simulations, but
  - For the Suicide-Substance data, adding measurement error fits better for some of the models but fails for several other models
  - Time-invariant measurement error does not give time-invariant reliability due to Y* variance changing over time (parameter constraints needed for imposing time-invariant reliability)
- See also MEAR in the Asparouhov & Muthén (2022) RSEM paper
- Difficult in practice even for continuous outcomes - avoid

Observed Y Instead of Latent Y*



- Random intercept, but no separation of between and within:
  - D-RI-AR1 model for observed binary outcomes, not Y*
  - Mplus Analysis option Predictor = Observed using Bayes (WLSMV not available). Bayes PPP not available
  - Predictor = Observed not relevant for RI models with hats variables which are by definition latent
- Related econometric modeling:
  - Individual-specific heterogeneity; individual effects
  - Honoré & Kyriazidou (2019). Panel vector autoregressions with binary data. In Panel Data Econometrics (edited book)

Univariate Results for Observed Y vs Latent Y* (Bayes) Suicidal Ideation and Substance Abuse

Model		# par's	PPP	# Significant Residuals: Resp. patterns (freq.)	# Significant Residuals: Bivar
Suicidal Ideation					
AR1	Observed	15		2 (155,11)	46
	Latent	15	0.133	2 (155,11)	22
AR2	Observed	21		2 (155,11)	12
	Latent	21	0.378	0	3
D-RI-AR1	Observed	16		0	0
	Latent	16	0.474	0	0
Substance Abuse					
AR1	Observed	15		2 (201,6)	49
	Latent	15	0.259	1 (201)	14
AR2	Observed	21		1 (201)	10
	Latent	21	0.462	0	1
D-RI-AR1	Observed	16		0	3
	Latent	16	0.458	1 (201)	2
D-RI-AR2	Observed	22		0	0
	Latent	22	0.482	1 (201)	1

- Most frequent pattern of 0's: 155, 201. Bivar 5%5 = 6

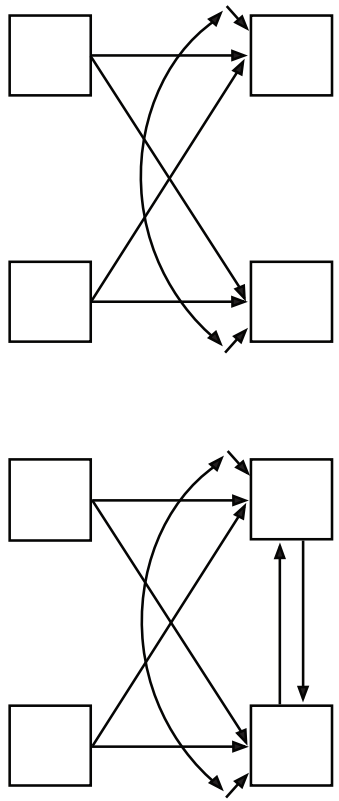
Bivariate Results for Observed Y vs Latent Y* (Bayes) Suicidal Ideation and Substance Abuse

Model		# par's	PPP	# Significant Residuals	
				Resp. patterns (freq.)	Bivar
D-RI-AR11	Observed	55		0	5
	Latent	55	0.486	0	2
D-RI-AR12	Observed	61		0	2
	Latent	61	0.462	0	1

D-RI-AR12 cross-lagged estimates:

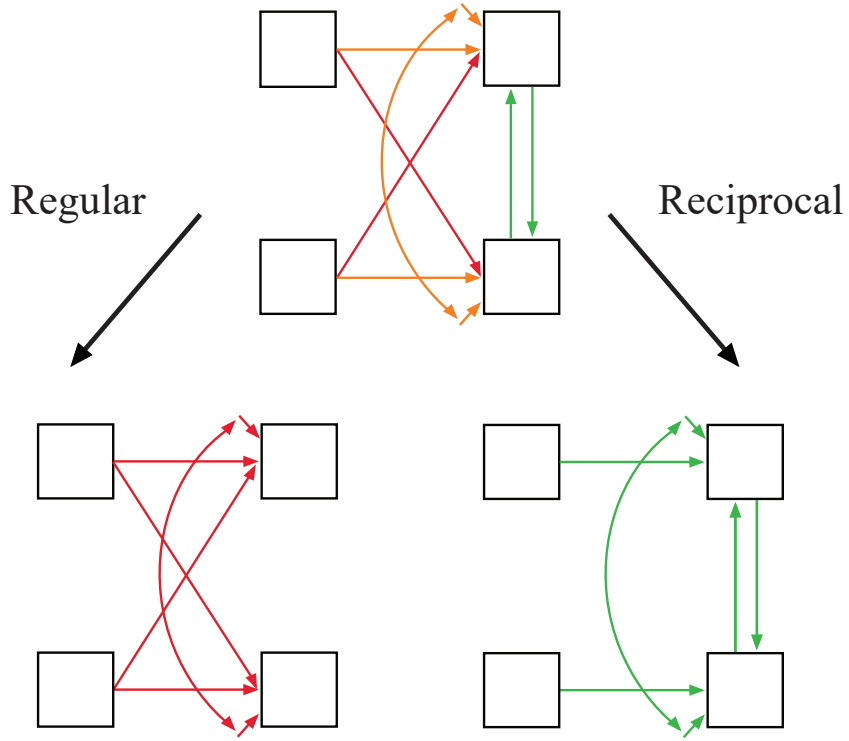
- Observed:
  - Y6 ON Z5 positive and significant
  - Z4 ON Y3 positive and significant
  - Z7 ON Y6 positive and significant
- Latent:
  - Y6 ON Z5 positive and significant
  - Z4 ON Y3 positive and significant

Lagged and Contemporanous Influence for Y*



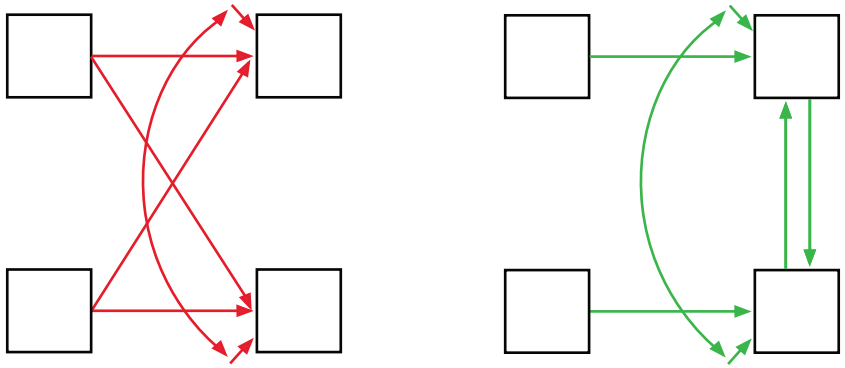
- The bottom model with reciprocal interaction may be more realistic but is not identified without parameter restrictions
- Identification difficulties are described in Greenberg & Kessler (1982)

Lagged versus Contemporaneous Influence for Y*



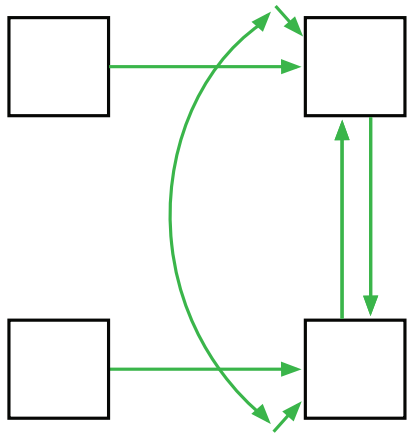
- Cross-lagged effects suitable for variables referring to current status
- Reciprocal effects may be suitable for variables referring to past status (e.g., suicidal ideation and substance abuse during the last year)
  - Cross-lagged effects may be of ignorable magnitude
  - Reciprocal model is identified because each DV has its own predictor (old econometric rule)

Cross-Lagged versus Reciprocal Model for Y*



- The cross-lagged and reciprocal models have the same number of parameters
- For continuous variables, the two models fit the same. But a test of time-invariant cross-lagged versus reciprocal effects gives different results
- For categorical variables, the two models fit differently
- Using the cross-lagged model to analyze data generated by the reciprocal model will show cross-lagged effects: Seeing cross-lagged effects does not rule out the reciprocal model

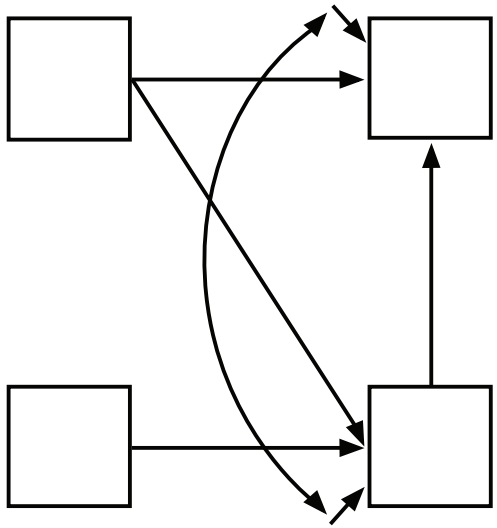
Reciprocal Model Results for Suicide-Substance Data



- Simulated data shows good performance of the model for large samples sizes (N of the order of at least 2,000)
- Reciprocal AR model does not fit the Suicide-Substance data
- Reciprocal RI-AR fits well but does not give believeable results for the reciprocal effects and their residual covariances with some very high correlations among estimates
- The model using Predictor = Observed is not identified (Maddala, 1983, Section 5.7)

Hybrid Model: Lagged and Contemporaneous Effects

- A hybrid model is identified and may be useful if there is strong substantive reason for the specification

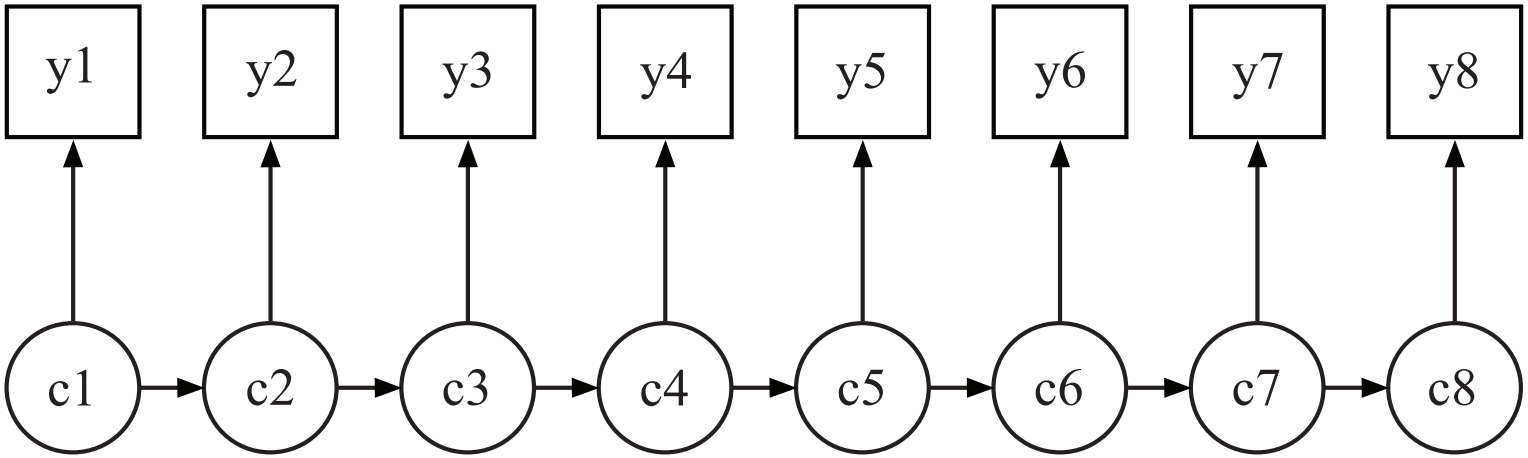


Section 11 Latent Transition Analysis: Mover-Stayer LTA RI-LTA Bivariate LTA

Studying Changes Over Time Using Latent Transition Analysis (LTA)

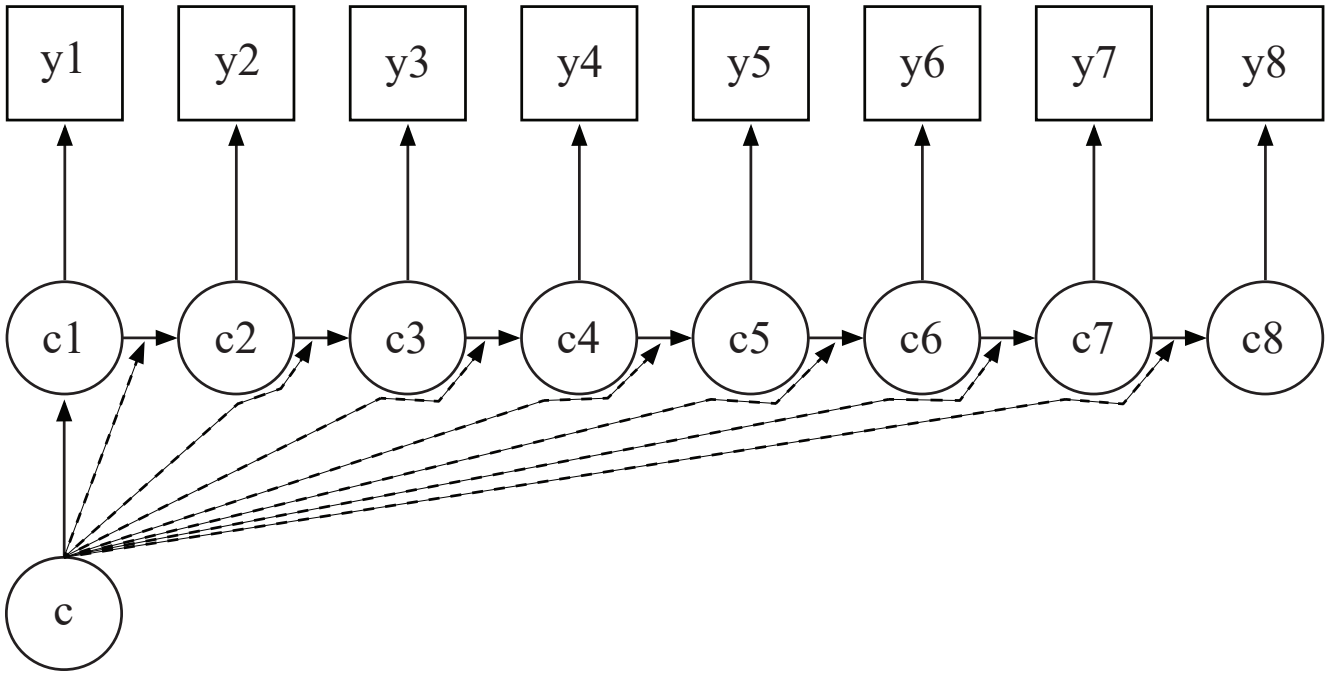
- Cross-lagged panel modeling needs outcomes that show change over time
- LTA focuses on the changes over time
- Mixture modeling
- Maximum-likelihood estimation
- For an introduction to LTA, see Mplus Web Talk No. 2 at `https://www.statmodel.com/Webtalk2.shtml`

LTA (Hidden Markov Model)

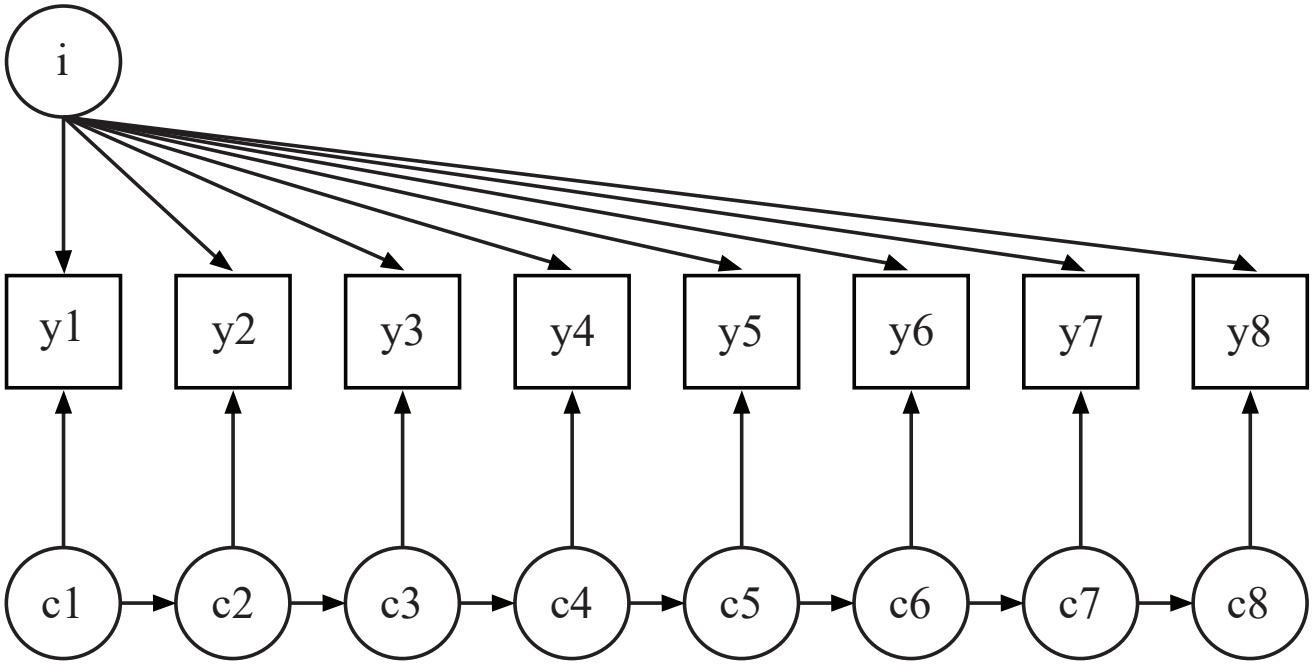


- Transitions over time
- Measurement error
- 3 versions:
  - Regular LTA (2 classes per time point, lag-1, non-stationary, $2^8 = 256$ class combinations)
  - Mover-Stayer (adding an M-S class gives $2^9 = 512$ class combinations)
  - Random intercept (RI-LTA), Muthén & Asparouhov (2022) `https://www.statmodel.com/download/MuthenFINAL.pdf`

Mover-Stayer LTA (Hidden Markov Model)



- An example of Stayers: Individuals who have 0's at all time points



- Also possible: Mover-Stayer RI-LTA

LTA MLR Results for Suicidal Ideation (N=737, T = 8)

- Outputs are posted on the website of the talk
- No χ^2 test of model fit available for LTA (no Y*'s)
  - Comparison of model fit with previous models can be made using the standardized residuals

				# Sig Standardized Residuals		
Model	# par's	LL	BIC	Response Patterns	Bivariate	Comments
1. LTA 2c	17	-2150	4411	2 (155, 6)	6	Poor fit
2. LTA 2c Lag 2	23	-2139	4430	2 (155, 6)	2	Poor fit
3. LTA 2c Mover-Stayer	19	-2138	4401	1 (6)	3	OK fit 36% stayers
4. **RI-LTA 2c**	18	**-2131**	**4380**	1 (6)	3	OK fit
5. RI-LTA Mover-Stayer	20	-2131	4394	1 (6)	3	OK fit 5% stayers

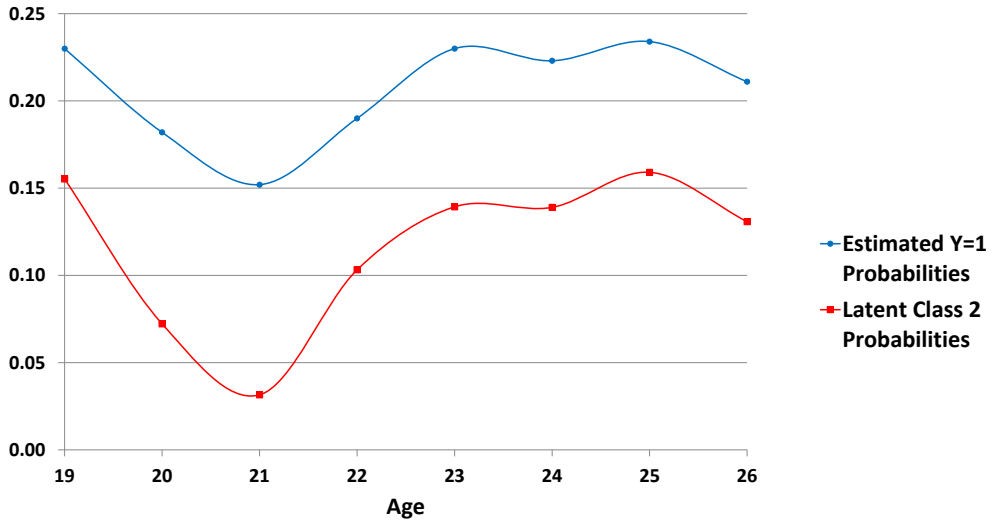
- RI-LTA 2c gives 84% in low class, 16% in high class

RI-LTA Estimates for Suicidal Ideation

Measurement Probabilities

	Latent Class	
	Low	High
y=0	0.87	0.21
y=1	**0.13**	0.79

- Estimated probabilities of y = 1, where P(Y=1) = P(C=High)*P(Y=1 | C=High) + P(C=Low)*P(Y=1 | C=Low)



- Values for top curve obtained from Tech10, bottom curve from Tech15

RI-LTA EstimatedTransition Probabilities For Suicide

C1 → C2

	1	2
1	0.914	0.086
2	1.000	0.000

C2 → C3

	1	2
1	0.966	0.034
2	1.000	0.000

C3 → C4

	1	2
1	0.926	0.074
2	0.008	0.992

C4 → C5

	1	2
1	0.929	0.071
2	0.268	0.732

C5 → C6

	1	2
1	0.968	0.032
2	0.202	0.798

C6 → C7

	1	2
1	0.928	0.072
2	0.299	0.701

C7 → C8

	1	2
1	0.985	0.015
2	0.258	0.742

- First 3 transition tables different from the rest. Invariance for the last 5 time points?

LTA MLR Results for Substance Abuse (T = 8)

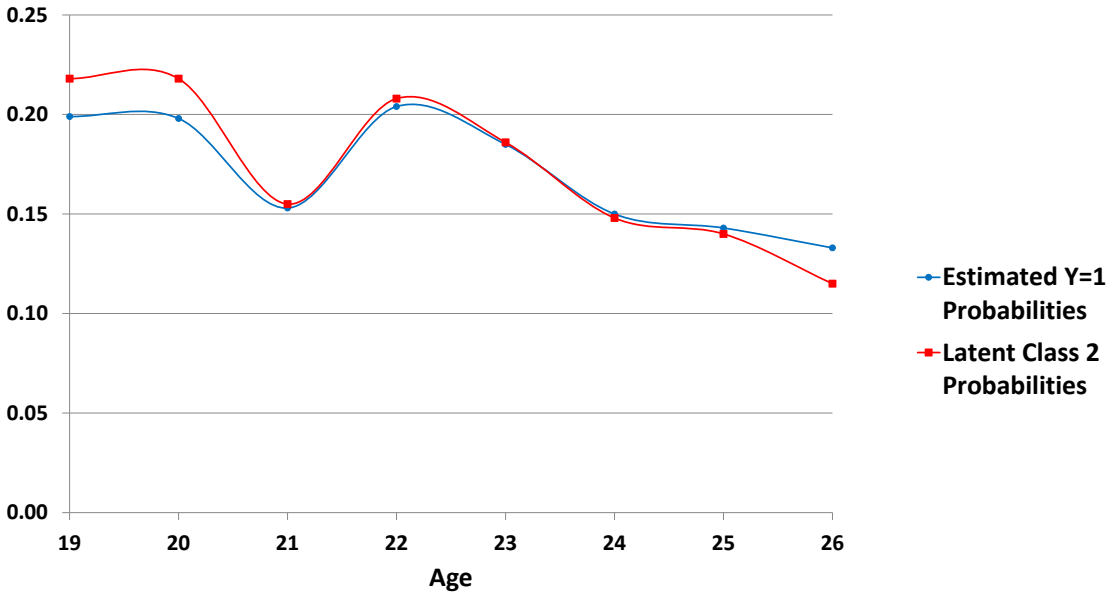
Model	# par's	LL	BIC	# Sig Standardized Residuals		Comments
				Response Patterns	Bivariate	
1. LTA 2c	17	-1804	3721	2 (201, 6)	10	Poor fit
2. LTA 2c Lag 2	23	-1785	3722	2 (201, 6)	1	Poor fit
3. **LTA 2c Mover-Stayer**	19	-1784	3695	0	0	Good fit 58% stayers
4. RI-LTA 2c	18	-1783	3685	0	0	Good fit Poor meas.
5. RI-LTA Mover-Stayer	20	-1783	3699	0	0	Good fit 5% stayers

LTA Mover-Stayer Estimates for Substance Abuse

Measurement Probabilities

	Latent Class	
	Low	High
y=0	0.98	0.12
y=1	0.02	0.88

- Estimated probabilities of y = 1, where P(Y=1) = P(C=High)*P(Y=1 | C=High) + P(C=Low)*P(Y=1 | C=Low)



LTA Mover-Stayer Estimated Transition Probabilties For Substance Abuse Movers Obtained from TECH15

C1 → C2

	1	2
1	1.00	0.00
2	0.00	1.00

C2 → C3

	1	2
1	0.72	0.28
2	0.71	0.29

C3 → C4

	1	2
1	0.62	0.38
2	0.53	0.47

C4 → C5

	1	2
1	0.78	0.22
2	0.45	0.55

C5 → C6

	1	2
1	0.87	0.14
2	0.50	0.50

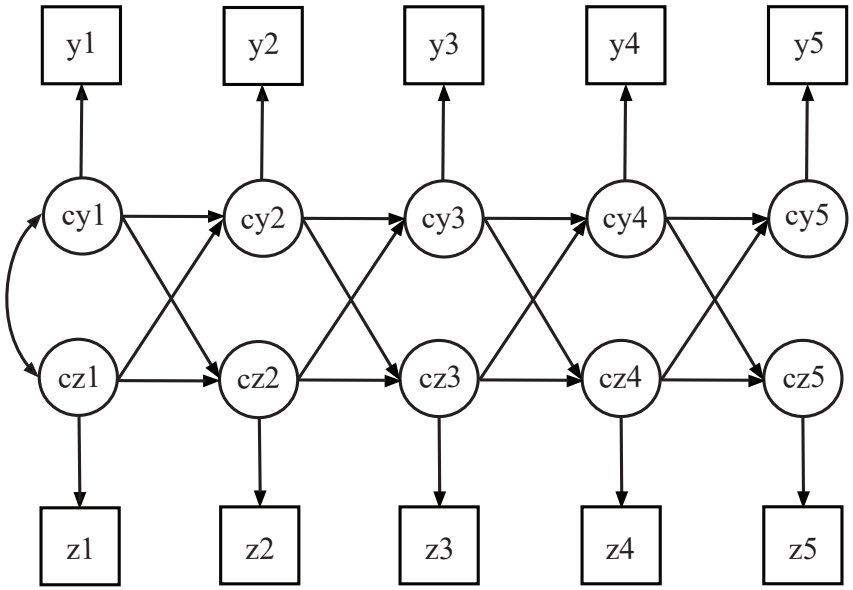
C6 → C7

	1	2
1	0.88	0.12
2	0.39	0.61

C7 → C8

	1	2
1	0.90	0.10
2	0.54	0.46

Bivariate LTA



- Related bivariate LTA:
  - ALTA (Associate Latent Transition Analysis): Flaherty (2008), Witkiewitz & Villarroel (2009), Bray et, al (2010)
- Mover-Stayer bivariate LTA with variations of the M-S latent class variable:
  - For each outcome, one for both, for only one outcome
- Bivariate RI-LTA

Bivariate LTA: Computational Considerations

- Bivariate LTA is time and memory demanding for high T even with only 2 classes due to proliferation of latent class combinations:
  - Regular LTA with T = 7 results in $2^{14} = 16,384$ combination classes which is slow but doable on a good computer
  - Mover-Stayer LTA adds a latent class variable for each outcome which for T = 7 results in $2 \times 2 \times 2^{14} = 2^{16} = 65,536$
  - If 16,384 is the max, $2 \times 2 \times 2^{x} = 16,384$ gives x = (ln 16,384 - ln 4)/ln 2 = 12, that is T = 12/2 = 6
  - The T = 8 analyses show more stable development after the first 3 occasions, suggesting analysis of the last 5 (T = 5), which results in $2 \times 2 \times 2^{10} = 4,096$

Univariate LTA MLR Results for T = 5 Imposing Stationarity (Time-Invariant Transitions)

Model	# par's	LL	BIC	# Sig Standardized Residuals: Response Patterns	# Sig Standardized Residuals: Bivariate	Comments
Suicidal Ideation						
1. LTA 2c	11	-1389	2851	2 (7, 7)	0	OK fit
2. **LTA 2c Mover-Stayer**	13	-1383	2852	1 (7)	0	Good fit 36% stayers
3. RI-LTA 2c	12	-1383	2844	1 (7)	0	Good fit Poor Meas.
Substance Abuse						
1. LTA 2c	11	-1142	2356	1(6)	0	Good fit
2. **LTA 2c Mover-Stayer**	13	-1142	2370	0	0	Good fit 58% stayers
3. RI-LTA	12	-1140	2360	0	0	Good fit Poor meas.

Bivariate LTA MLR Results for T = 5 Imposing Stationarity (Time-Invariant Transitions)

- Outputs are posted on the website of the talk
- Most frequent pattern of all 0's is observed for 193 individuals

				# Sig Standardized Residuals		
Model	# par's	LL	BIC	Response Patterns	Bivariate	Comments
1. LTA 2c	13	-2497	**5080**	5 (193, 7, 7, 6, 6)	1	Marginal fit
2. LTA 2c Mover-Stayer for both	18	-2488	5094	3 (193, 7, 6)	3	Marginal fit 66 % stayers (Y) 45 % stayers (Z)

Cross-Lagged Effects for LTA Using Model 1 in Logit and OR Terms

- CY ON CZ = 2.172 (SE = 0.651). OR = exp (2.172) = 8.77
- Translating logit CI to OR CI by exponentiation: [2.45 31.14]
  - Significant effect
- CZ ON CY = 0.471 (SE = 0.528). OR = exp(0.471) = 1.602
- Translating logit CI to OR CI by exponentiation: [0.57 4.51]
  - Non-significant effect
- Conclusion: Substance abuse influences suicidal ideation, not the other way around
- The model is analogous to bivariate AR1 (CLPM) with time-invariant cross-lagged effects - but adding measurement error

- The Mover-Stayer Model 2 results also point to significant CY ON CZ and insignificant CZ ON CY (among Movers)

Further Detail: Model Test of Probability Differences

- Results in Probability terms obtained by Model Constraint:
  1. P(CYt=1 | CYt-1=1, CZt-1=1) = 0.980
  2. P(CYt=1 | CYt-1=1, CZt-1=2) = 0.851
  3. P(CYt=1 | CYt-1=2, CZt-1=1) = 0.090
  4. P(CYt=1 | CYt-1=2, CZt-1=2) = 0.011
- 1 - 2: Effect of CZ on CY for the low CY class (CYt-1=1)
  - The probability of staying in the low CY class is decreased from 0.980 to 0.851 by previously being in the high compared to the low CZ class
- 3 - 4: Effect of CZ on CY for the high CY class (CYt-1=2)
  - The probability of transitioning from the high to the low CY class is decreased from 0.090 to 0.011 by previously being in the high compared to the low CZ class
- Testing both the 1 -2 and the 3 - 4 differences using Model Test: Chi-2 (2) = 7.2 (.0280)
- Testing first difference: Chi-2 (1) = 6.3 (.0124)
- Testing second difference: Chi-2 (1) = 2.8 (.0937)

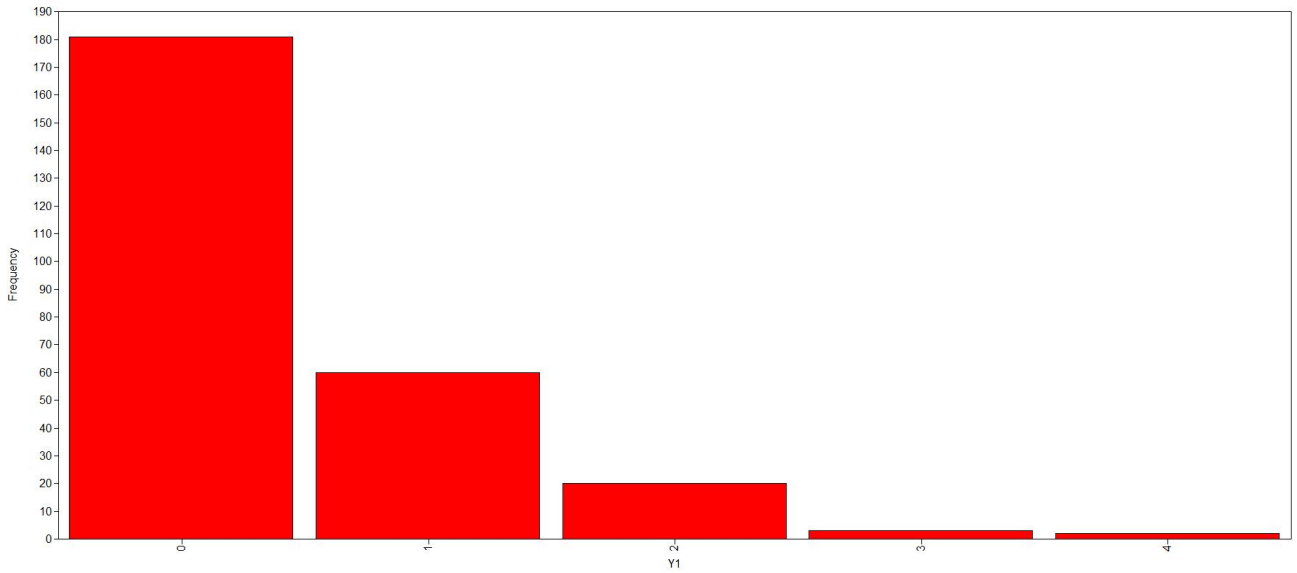
Other LTA Models

- LTA also possible for ordinal variables (and nominal variables)
  - 5-category alcohol risk outcome for T = 5 analyzed with up to 5 classes with regular LTA and up to 4 classes with Mover-Stayer LTA and RI-LTA
- How do you bring the two-part ordinal parts into the LTA?
- How do you combine an LTA for alcohol risk with a model for the continuous stress variable to capture cross-lagged effects?

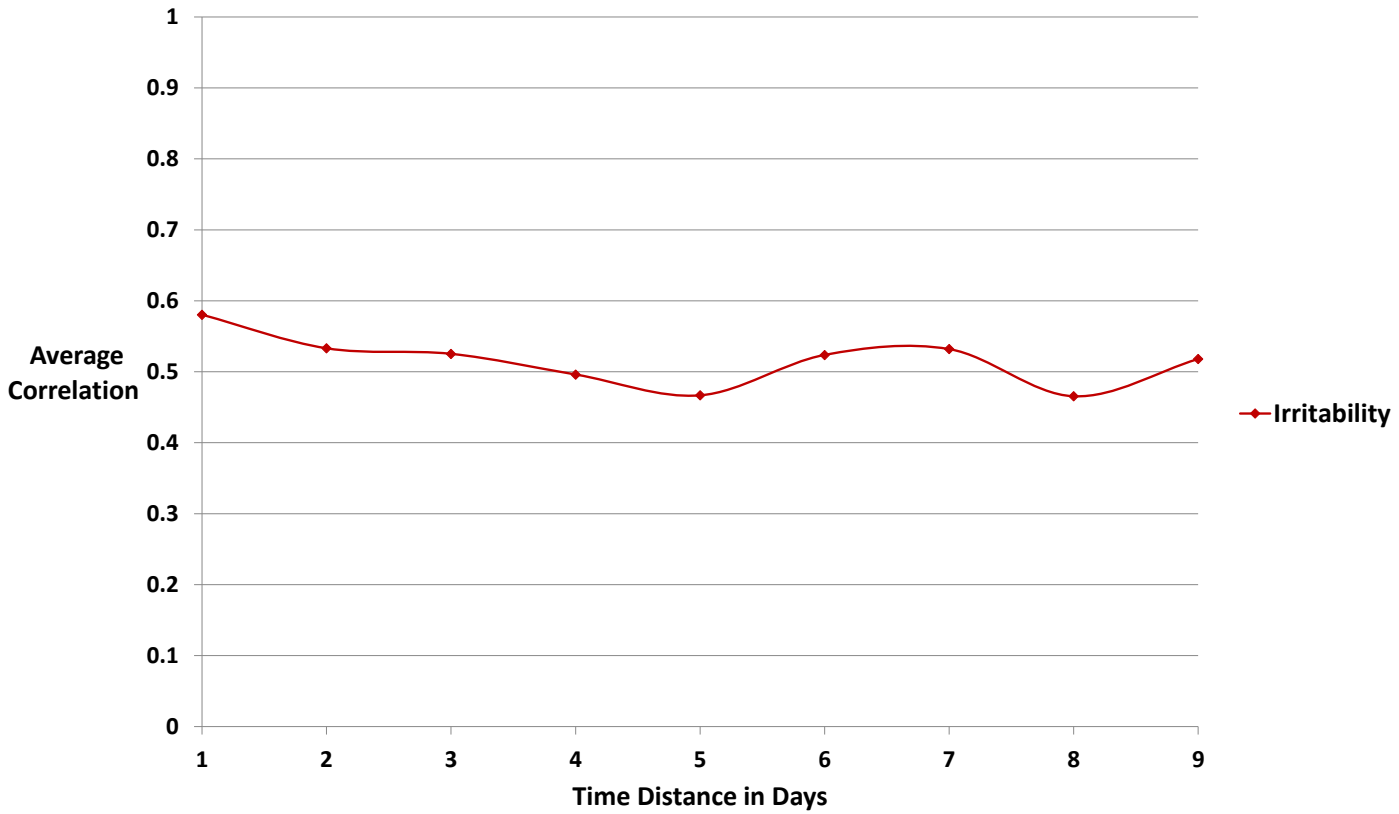
Section 12 Ordered Categorical (Ordinal) Outcomes Two-Part Ordinal Model

Ordered Categorical (Ordinal) Outcomes

- Data set 3: 5-category negative affect item Irritability
  - Question format: Today I felt... (1 = Not at all, 2 = A little, 3 = Moderately, 4 = Quite a bit, 5 = Extremely)
- Analysis of days 1-10
- N = 271



Average Correlations as a Function of the Time Distance for 5-Category Irritability Item (N=271, T=10)



- The plot suggests a low AR and a high random intercept variance

Univariate Analysis of 5-Categ. Irritability (N=271, T=10)

- Number of subjects with 0's throughout = 52 (19%). This is the most frequent response pattern
- Total number of bivariate cells: 5*5*10(10-1)/2 = 1125. 5% = 56
- The unrestricted multivariate probit model has 10*4 thresholds and 10(10-1)/2 = 45 correlations: 85 parameters

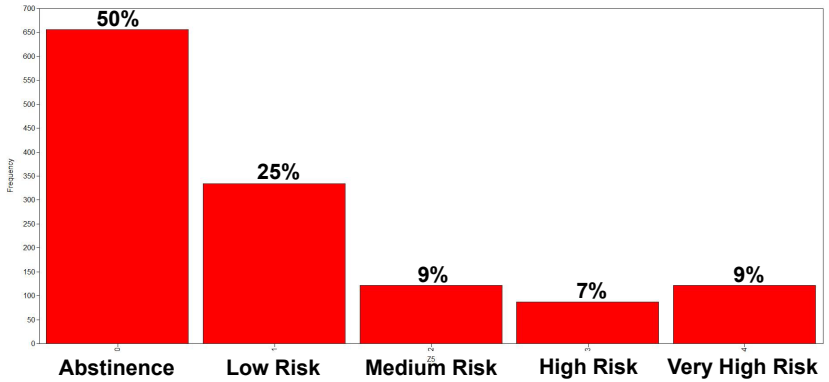
	Model	# par's	PPP/ χ^2	# Significant Residuals		Comment
				Resp Pattern (obs. freq)	Bivar	
1.	Unrestr.	85	0.498	3 (2,1,1)	35	Good fit
2.	AR1	49	0.002	4 (52, 2, 1, 1)	121	Poor fit
3.	RI-AR1	50	0.382	3 (2, 1, 1)	32	Good fit
	RI-AR1*	50	$\chi^2(35)=64$ (.0018)	3 (2, 1, 1)	49	Marginal fit
4.	RI-ARMA11*	58	$\chi^2(27)=35$ (.1295)	3 (2, 1, 1)	47	Good fit

* denotes WLSMV

- Models 3 and 4 both have sizeable random intercept variance - the random intercept R-square for RI-ARMA Y*'s is about 0.5
- Only 3 of 9 ARs are significant, agreeing with correlation plot

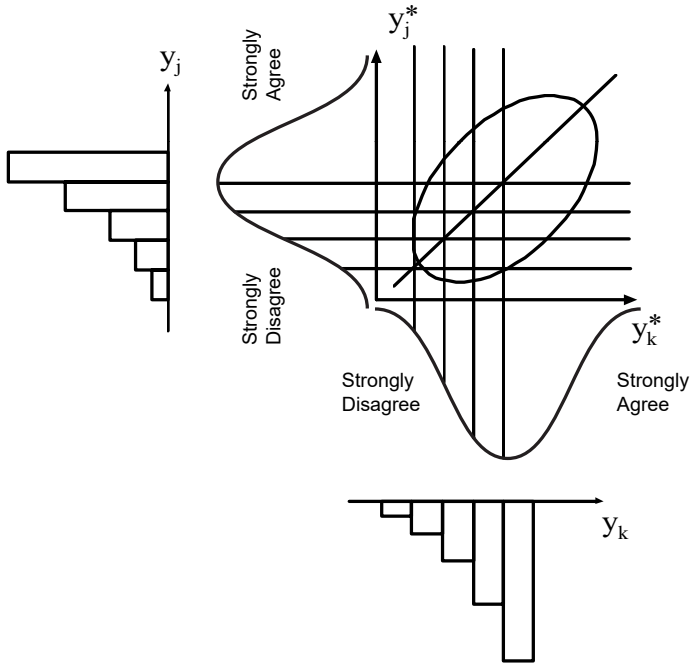
Data Set 1: Analysis of 5-Category Alcohol Risk and 8-Category Heavy Drinking (N=1375, T=8)

- 5-category alcohol risk:



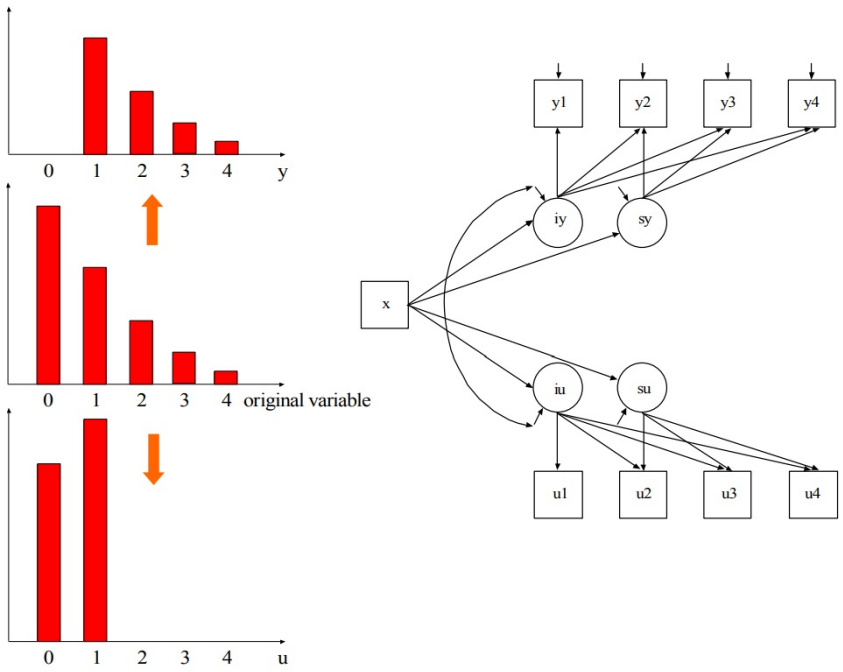
- Most frequent pattern is 0's throughout with frequency 312 (23 %)
- Total number of bivariate cells = 25*8(8-1)/2 = 700. 5% = 35
- The unrestricted multivariate probit model has 8*4 thresholds and 8(8-1)/2 = 28 correlations: 60 parameters
  - Number of significant response pattern residuals: 5 for patterns with observed frequencies 312 (estimated as 267), 46, 12, 6, 6
  - Number of significant bivariate cells = 273 which is 39% of all bivariate cells
- Poor fit: A new model is needed

Bivariate Models for Ordered Categorical (Ordinal) Variables



1. The probit model for 2 variables in the figure has C1-1+C2-1+1 (thresholds+1 polychoric corr) = C1+C2-1 parameters where C is the number of categories: 9 for C=5
2. The unrestricted multinomial model for 2 variables has C1*C2-1 parameters: 24 for C=5. This is the model tested against in the TECH10 bivariate tests
3. Intermediate models are possible. E.g. a **two-part ordinal model** for 2 variables has C1+C2 parameters: 10 for C=5 (1 more than probit)

Two-Part Ordinal Model



- Two-part growth modeling with continuous outcomes: Olsen & Schafer (2001), JASA; two-part GMM - Muthén (2001)
- Two-part regression analysis with an ordinal outcome: Muthén, Muthén & Asparouhov (2016). RMA book, page 292
- Two-part ordinal multivariate probit model: Version 8.8 TECH10
  - With two C-category ordinal variables there are C1-1+C2-1 thresholds and two correlations: a correlation between the binary parts as well as the ordinal parts (C1-1+C2-1+2 = C1+C2)

Mplus Input for Univariate (Single Process, T=8) Unrestricted Two-Part Ordinal Model Using Bayes

```
DATA TWOPART:
    USEVARIABLES = u1-u8 p1-p8;
    CATEGORICAL = u1-u8 p1-p8;

    NAMES = z1-z8; ! from Names list
    BINARY = u1-u8;
    CONTINUOUS = p1-p8;
    CUTPOINT = 0;
    TRANSFORM = NONE;
ANALYSIS:
    ESTIMATOR = BAYES;
    BITERATIONS = (10000);
    THIN = 10;
    PROCESSORS = 8;

MODEL:
    u1-u8 WITH u1-u8;
    p1-p8 WITH p1-p8;
    ! Approach that keeps ut and pt
    ! uncorrelated in line with
    ! Asparouhov-Muthén (2022)
    ! Residual Structural Equation
    ! Models, Section 3.5:
    p1^-p8^ ON u1^-u8^;
    p1^-p8^ PON u1^-u8^@0;
OUTPUT:
    STANDARDIZED RESIDUAL
    TECH8 TECH10;
PLOT:
    TYPE = PLOT3;
```

- Correlations among 16 variables (binary and ordinal parts), where 8 are fixed at zero (u-p correlations are not well identified since p is not observed when u=0). 2T*(2T-1)/2 - T: 112
- For each variable, there is one threshold for the binary part of and C-2 thresholds for the ordinal part. T + T*(C-2): 32

Mplus Input for Univariate (Single Process) RI-AR2 Two-Part Ordinal Model Using Bayes

```
                    USEVARIABLES = u1-u8 p1-p8;
                    CATEGORICAL = u1-u8 p1-p8;
DATA TWOPART:
                    NAMES = z1-z8;
                    BINARY = u1-u8;
                    CONTINUOUS = p1-p8;
                    CUTPOINT = 0;
                    TRANSFORM = NONE;
ANALYSIS:
                    ESTIMATOR = BAYES;
                    BITERATIONS = (10000);
                    THIN = 10;
                    PROCESSORS = 8;
MODEL:
                    ib BY u1-u8@1;
                    ip BY p1-p8@1;
                    u2^-u8^ PON u1^-u7^;
                    u3^-u8^ PON u1^-u6^;
                    p2^-p8^ PON p1^-p7^;
                    p3^-p8^ PON p1^-p6^;
                    ! u1^-u5^ PWITH p1^-p5^;
OUTPUT:
                    STANDARDIZED RESIDUAL
                    TECH8 TECH10;
PLOT:
                    TYPE = PLOT3;
```

- The two parts are correlated only via the correlation between the two random intercepts. The residual correlations are not well identified since p is not observed when u=0

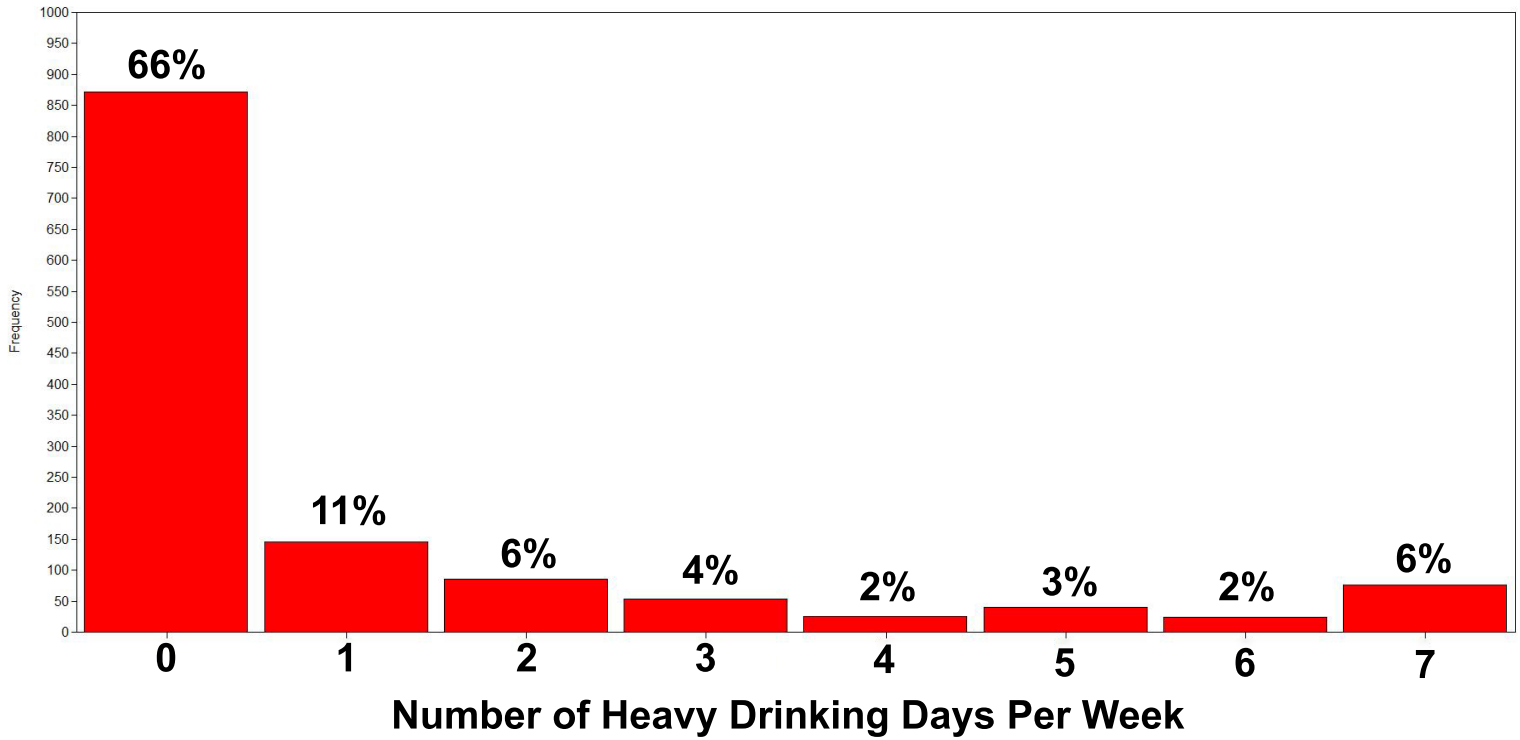
Analysis of 5-Category Alcohol Risk Using Regular and Two-Part Ordinal Models (N=1375, T=8)

	Model	# par's	PPP/ χ^2	# Significant Residuals Resp Pattern (obs. freq)	Bivar	Comment
			Regular ordinal probit			
1.	Unrestricted	60	0.498	5 (312)	273 (39%)	Poor fit
2.	AR2	45	0.151	5 (312)	277	Poor fit
3.	RI-AR2	46	0.135	5 (312)	274	Poor fit
4.	RI-ARMA11*	46	χ^2 (14)=39	5 (312)	279	Poor fit
			Two-part ordinal probit			
5.	Unrestricted	144	0.472	1 (12)	29 (4%)	Good fit
6.	AR2	58	0.145	2 (46, 13)	106 (15%)	Poor fit
7.	RI-AR2	61	0.228	1 (12)	52 (7%)	OK fit
8.	RI-ARMA11†	NA				

* denotes WLSMV. † No WLSMV for two-part

- Model 7 (RI-AR2) has sizeable variances for the two random intercepts - the random intercept R-squares range from 0.3 to 0.6 with slightly higher values for the binary part
- Regular ordinal probit Model 3 (RI-AR2) has ignorable random intercept variance (est. less than s.e.) and R-squares about 0.04

Analysis of 8-Category Heavy Drinking Using Two-Part Ordinal Models (N=1375, T=8)



- Most frequent pattern is 0's throughout with frequency 503 (37 %)
- Total number of bivariate cells = 64*8(8-1)/2 = 1792. 5% = 90

Analysis of 8-Category Heavy Drinking Using Regular and Two-Part Ordinal Models (N=1375, T=8)

	Model	# par's	PPP/ χ^2	# Significant Residuals Resp Pattern (obs. freq)	Bivar	Comment
			Regular ordinal probit			
1.	Unrestricted	84	0.492	3 (38, 8, 7)	211 (12%)	Poor fit
2.	AR2	69	0.233	3 (503 7, 5)	228	Poor fit
3.	RI-AR2	70	0.230	3 (503, 7, 5)	229	Poor fit
4.	RI-ARMA11*	70	χ^2 (14)=30	4 (7,7,6,5)	225 (13%)	Poor fit
			Two-part ordinal probit			
5.	Unrestricted	168	0.454	1 (7)	45 (3%)	Good fit
6.	AR2	82	0.053	6 (503)	124 (7%)	Poor fit
7.	RI-AR2	85	0.165	3 (8, 7, 5)	70 (4%)	OK fit
8.	RI-ARMA11†	NA				

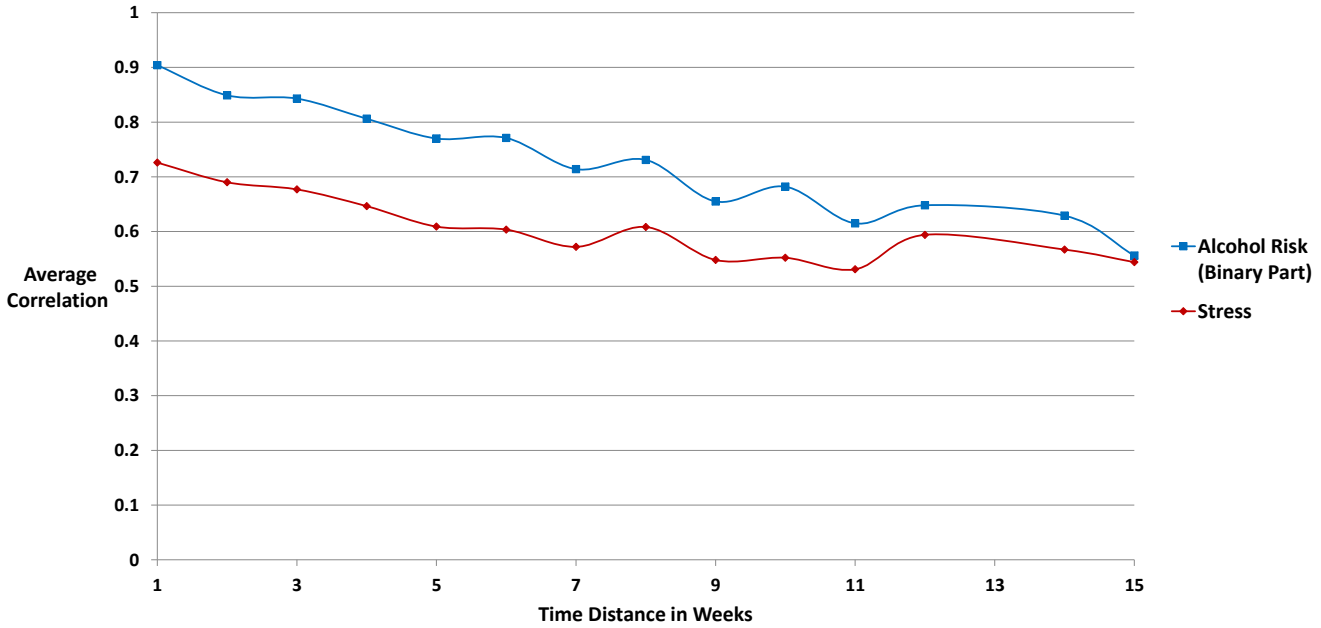
* denotes WLSMV. † No WLSMV for two-part

- Model 7 (RI-AR2) has sizeable variances for the two random intercepts - the random intercept R-squares range from 0.4 to 0.6 with somewhat higher values for the binary part
- Regular ordinal probit Model 3 (RI-AR2) has ignorable random intercept variance (est. less than s.e.) and R-squares about 0.04

Bivariate Analysis of Stress and Alcohol Risk Week 1 - Week 16 (N=1375, T=8)

- Returning to the question in the introduction: Does stress influence alcohol risk or the other way around?
- Stress causes drinking (Armeli et al., 2000 in J of Personality and Social Psych)
- The alcohol treatment setting may produce a different picture

Bivariate Analysis of Stress and Alcohol Risk
Week 1 - Week 16 (N=1375, T=8)
Average Correlations as a Function of the Time Distance



- Two-part ordinal RI-AR2 model chosen for 5-category alcohol risk
- RI-AR2 model chosen for stress
  - RI-ARMA model for stress fits better but cannot be combined with two-part ordinal modeling because RI-ARMA requires WLSMV which does not handle two-part (not MAR)

Mplus Input for Bivariate Analysis of Stress and Alcohol Risk Using RI-AR2 and Two-Part Ordinal

```
                    USEVARIABLES = y1-y8 u1-u8 p1-p8;
                    CATEGORICAL = u1-u8 p1-p8;
DATA TWOPART:
                    NAMES = z1-z8;
                    BINARY = u1-u8;
                    CONTINUOUS = p1-p8;
                    CUTPOINT = 0;
                    TRANSFORM = NONE;
ANALYSIS:
                    ESTIMATOR = BAYES;
                    BITERATIONS = (10000);
                    THIN = 10;
                    PROCESSORS = 8;
MODEL:
                    iy BY y1-y8@1;
                    ib BY u1-u8@1;
                    ip BY p1-p8@1;
```

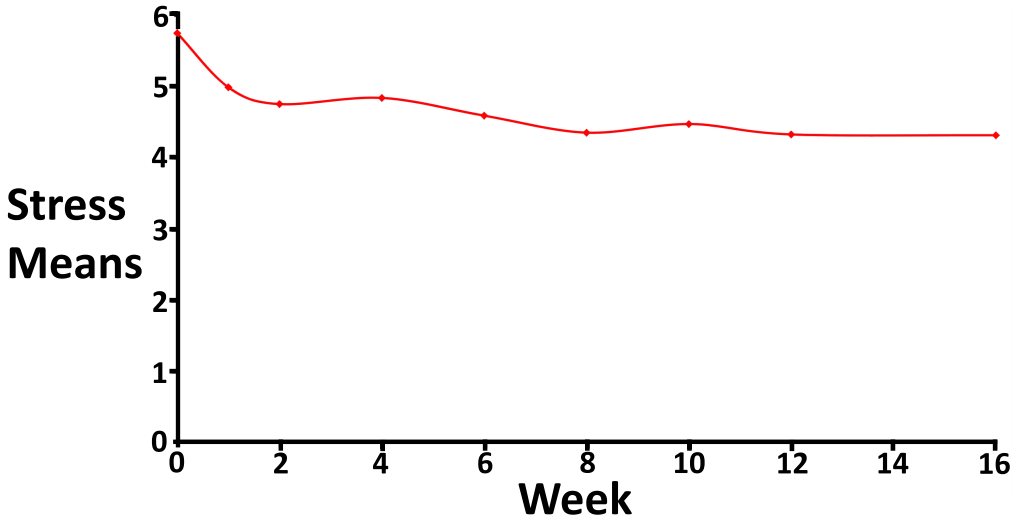
```
! univariates:
y2^-y8^ PON y1^-y7^;
y3^-y8^ PON y1^-y6^;
u2^-u8^ PON u1^-u7^;
u3^-u8^ PON u1^-u6^;
p2^-p8^ PON p1^-p7^;
p3^-p8^ PON p1^-p6^;
! bivariates:
y2^-y8^ PON u1^-u7^;
y2^-y8^ PON p1^-p7^;
u2^-u8^ PON y1^-y7^;
p2^-p8^ PON y1^-y7^;

OUTPUT:

STANDARDIZED  RESIDUAL
TECH8 TECH10;

PLOT:

TYPE = PLOT3;
```

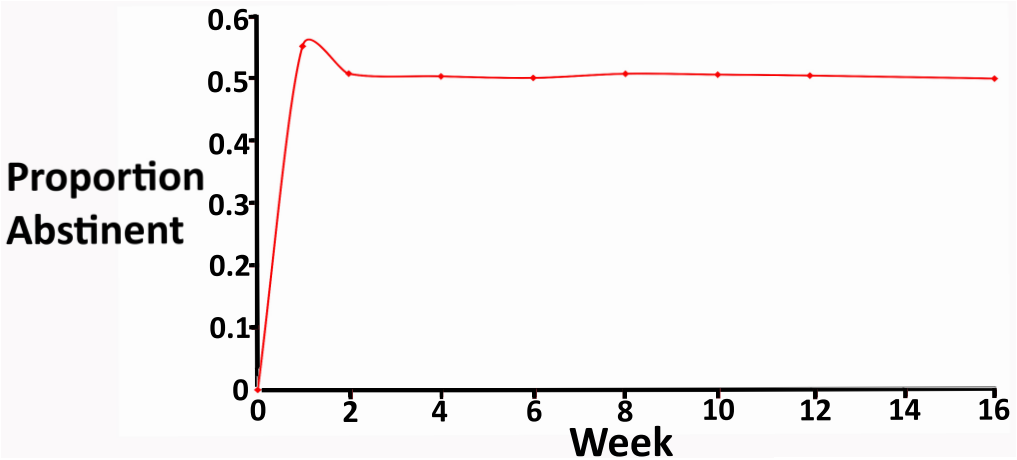
Results for Bivariate Analysis of Stress and Alcohol Risk Using RI-AR2 and Two-Part Ordinal

- Stress = Y, Alcohol risk = Z
- #par.'s = 121, PPP = 0.054, # signif. response pattern residuals = 1 (freq. = 12), #significant bivariate residuals = 55
- # significant cross-lagged effects:
  - Y ON binary part of Z: 4 out of 7, stdyx = 0.12 - 0.17
  - Y ON ordinal part of Z: 7 out of 7, stdyx = 0.16 - 0.21
  - Binary part of Z ON Y: 0
  - Ordinal part of Z ON Y: 0
- Increased alcohol risk has a significant effect on increase of stress, not the other way around
- Abstinence or not has less of an effect on stress than higher degree of alcohol risk (non-abstinence)

Section 13 Allowing for a Trend: Latent Growth Analysis Longitudinal LCA Latent Class Growth Analysis Growth Mixture Modeling

COMBINE Stress and Alcohol Risk: Baseline - Week 16





- Can a left-out random effect capturing a trend distort RI or AR? Would cross-lagged effects be affected?

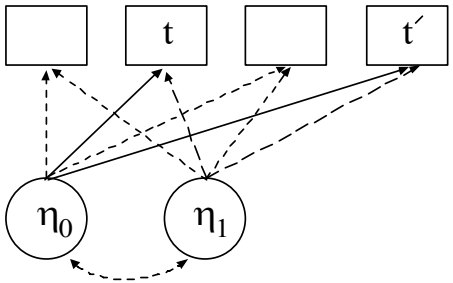
Allowing for a Trend: Latent Growth Analysis (Cont's Y)

- Adding linear and quadratic random effects to the random intercept also contributes to correlations across time:

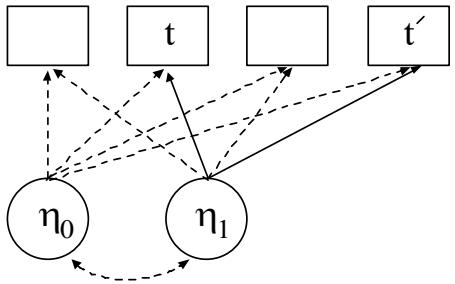
$$y_{ti} = \eta_{0i} + \eta_{1i}\, x_t + \varepsilon_{ti},$$
$$x_t = 0, 1, \ldots, T-1.$$

$$Cov(y_{ti}, y_{t'i}) = V(\eta_{0i}) + V(\eta_{1i})\, x_t\, x_{t'} + Cov(\eta_{0i}, \eta_{1i})\,(x_t + x_{t'}) + Cov(\varepsilon_{ti}, \varepsilon_{t'i}).$$

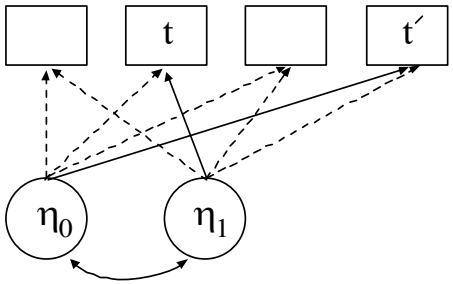
$V(\eta_{0t})$:



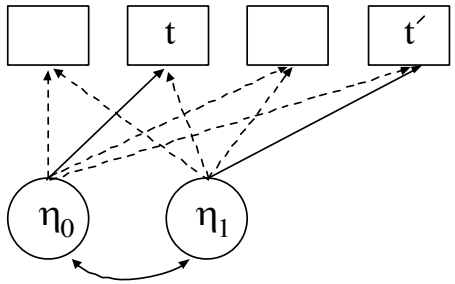
$V(\eta_{1i})\, x_t\, x_{t'}$:



$Cov(\eta_{0i}, \eta_{1i})\, x_t$:



$Cov(\eta_{0i}, \eta_{1i})\, x_{t'}$:



Analysis of Stress: Comparing Model Fit for RI-AR2 vs Linear Growth AR2 Using MLR Week 1 - Week 16 (N=1375, T=8)

Model	# par's	LL	BIC	Chi-square	RMSEA	CFI
RI-AR2	30	-19928	40073	$\chi^2(14)=33$ (.0027)	0.032 (<.05=.983)	0.995
Linear Growth with AR2	26	-19935	40057	$\chi^2(18)=46$ (.0003)	0.034 (<.05=.987)	0.993

- Input for linear growth AR2 model:

Model:

```
! Weeks: 1, 2, 4, 6, 8, 10, 12, 16
i s | y1@0 y2@.1 y3@.3 y4@.5 y5@.7 y6@.9 y7@.1.1 y8@.1.5;
! Auto-regression among residuals - new in version 8.7:
y2^-y8^ PON y1^-y7^;
y3^-y8^ PON y1^-y6^;
```

Analysis of Stress: Comparing MLR Estimates for RI-AR2 vs Linear Growth AR2

Y2^ ON			
Y1^	0.413	0.045	9.210
Y3^ ON			
Y2^	0.307	0.046	6.683
Y1^	0.137	0.041	3.345
Y4^ ON			
Y3^	0.232	0.050	4.674
Y2^	0.128	0.046	2.795
Y5^ ON			
Y4^	0.179	0.050	3.563
Y3^	0.068	0.052	1.296
Y6^ ON			
Y5^	0.215	0.062	3.484
Y4^	0.105	0.048	2.171
Y7^ ON			
Y6^	0.326	0.049	6.596
Y5^	0.148	0.055	2.703
Y8^ ON			
Y7^	0.199	0.046	4.348
Y6^	0.099	0.050	1.984
Variances			
I	5.442	0.276	19.705

Y2^ ON			
Y1^	0.373	0.081	4.618
Y3^ ON			
Y2^	0.324	0.050	6.472
Y1^	0.156	0.054	2.906
Y4^ ON			
Y3^	0.261	0.047	5.529
Y2^	0.152	0.050	3.047
Y5^ ON			
Y4^	0.205	0.042	4.930
Y3^	0.081	0.045	1.812
Y6^ ON			
Y5^	0.130	0.052	2.481
Y4^	0.088	0.040	2.219
Y7^ ON			
Y6^	0.153	0.075	2.034
Y5^	0.084	0.045	1.843
Y8^ ON			
Y7^	-0.183	0.151	-1.211
Y6^	-0.136	0.105	-1.291
S WITH			
I	-0.158	0.361	-0.438
Means			
I	4.896	0.077	63.820
S	-0.505	0.052	-9.626
Variances			
I	5.340	0.503	10.610
S	1.427	0.468	3.048

Covariance Contributions in the Linear Growth AR2 Model for Stress

- $Cov(y_s, y_t) = V(i) + V(s)(x_s x_t) + Cov(i,s)(x_s + x_t) + Cov(\hat{y}_s, \hat{y}_t)$
- Estimated $Cov(y_s, y_t)$ is obtained from Residual or TECH4
- Estimated $Cov(\hat{y}_s, \hat{y}_t)$ is obtained from TECH4
- Using y_4 and y_5 as an example:

$$
\begin{aligned}
Cov &= 5.340 + 1.427 * 0.5 * 0.7 - 0.158 * (0.5 + 0.7) + 1.038 \\
&= 5.340 + 0.499 - 0.1896 + 1.038 \\
Corr &= \underbrace{0.511}_{i} + \underbrace{0.050 - 0.019}_{s} + \underbrace{0.103}_{AR}
\end{aligned}
$$

- This shows that the contribution from the random slope is relatively small

Analysis of Abstinence: Comparing Categorical Model Fit for RI-AR2 vs Linear Growth AR2 Using Bayes Week 1 - Week 16 (N=1375, T=8)

	Model	# par's	PPP/	# Significant Residuals		Comment
				Resp Pattern (obs. freq)	Bivar	
1.	Unrestricted	36	0.520	1 (38; Z=1.98)*	0	Good fit
2.	AR2	21	0.474	0	0	Good fit
3.	RI-AR2	22	0.472	0	0	Good fit
4.	Linear growth AR2	18	0.436	0	1	Good fit

* 3rd most frequent pattern

- Model 3: Estimated V(i) smaller than its SE (posterior SD), suggesting that the trait is not substantial so that Model 2 is sufficient
- Model 4: Estimated V(i) = 0.256 (.096), V(s) = 0.190 (.076), correlation between i and s = -0.409, mean of s = 0.109 (.043) which means that the probability of non-abstinence increases over time

Mplus Input for Linear Growth AR2 for Abstinence

```
                    USEVARIABLES = z1-z8;
                    CATEGORICAL = z1-z8;
DEFINE:
                    CUTPOINT z1-z8 (0.5);
ANALYSIS:
                    ESTIMATOR = BAYES;
                    BITERATIONS = (2000);
                    THIN = 100;
                    PROCESSORS = 8;
MODEL:
                    i s | z1@0 z2@.1 z3@.3 z4@.5
                    z5@.7 z6@.9 z7@1.1 z8@1.5;
                    z2^-z8^ PON z1^-z7^;
                    z3^-z8^ PON z1^-z6^;
                    ! Fix residual variances to avoid
                    ! growth model Theta param'n
                    ! default of last T-1 free
                    z1-z8@1;
OUTPUT:
                    STANDARDIZED RESIDUAL
                    TECH8 TECH10;
PLOT:
                    TYPE = PLOT3;
```

Bivariate Analysis of Stress and Abstinence: Comparing Model Fit for RI-AR2 vs Linear Growth AR2 for Both Processes, Week 1 - Week 16 (N=1375, T=8) Bayes and WLSMV Estimation

	Model	# par's	PPP/ χ^2	# Significant Residuals		Comment
				Resp Pattern (obs. freq)	Bivar	
1.	RI-AR2	75	0.299	0	0	Good fit
	RI-AR2*	75	$\chi^2(69)=83$ (.1218)	0	0	Good fit
2.	Linear Growth AR2	66	0.144	0	1	Good fit
	Linear Growth AR2*	66	$\chi^2(78)=107$ (.0151)	0	0	Good fit but npd

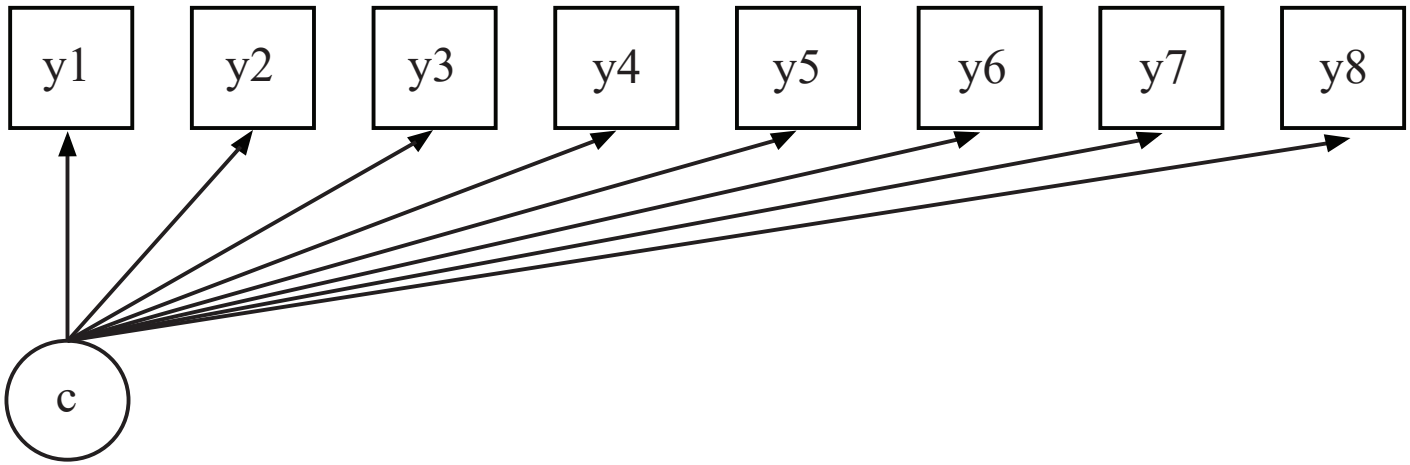
* denotes WLSMV

- To achieve convergence with Bayes, the linear growth model fixes the variance of the slope of abstinence at 0 (actually 0.1) and corresponding covariances. The WLSMV run gives a npd solution

Bivariate Analysis of Stress and Abstinence: Comparing Cross-Lagged Estimates for RI-AR2 vs Linear Growth AR2

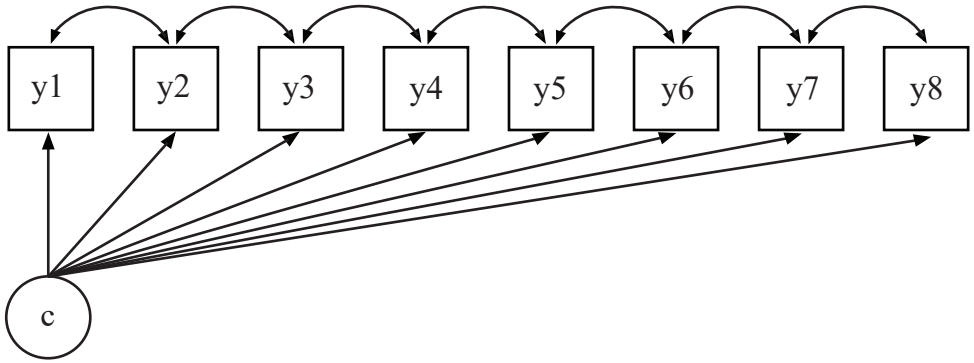
- Agreement in terms of # significant cross-lagged effects for the two models:
  - Y ON Z: 7 out of 7
  - Z ON Y: 0/1 out of 7
- Non-abstinence has a significant effect on increase of Stress, not the other way around
- Including a trend using linear growth is not essential in these data from the point of view of cross-lagged effects

Allowing for a Trend Using Longitudinal LCA: Growth Modeling with Flexible Curve Shapes



- LCA typically used for cross-sectional analysis but here used for longitudinal analysis
- The latent class variable serves as a non-parametric representation of a combination of growth factors: i, s, q
- ML and Bayes can be used
  - With Bayes, label switching is a threat (CHAIN=1 helpful)
- ML can add residual covariances (RESCOV parameterization)
  - Bayes can add AR but convergence problems due to mixture are possible

Residual Associations



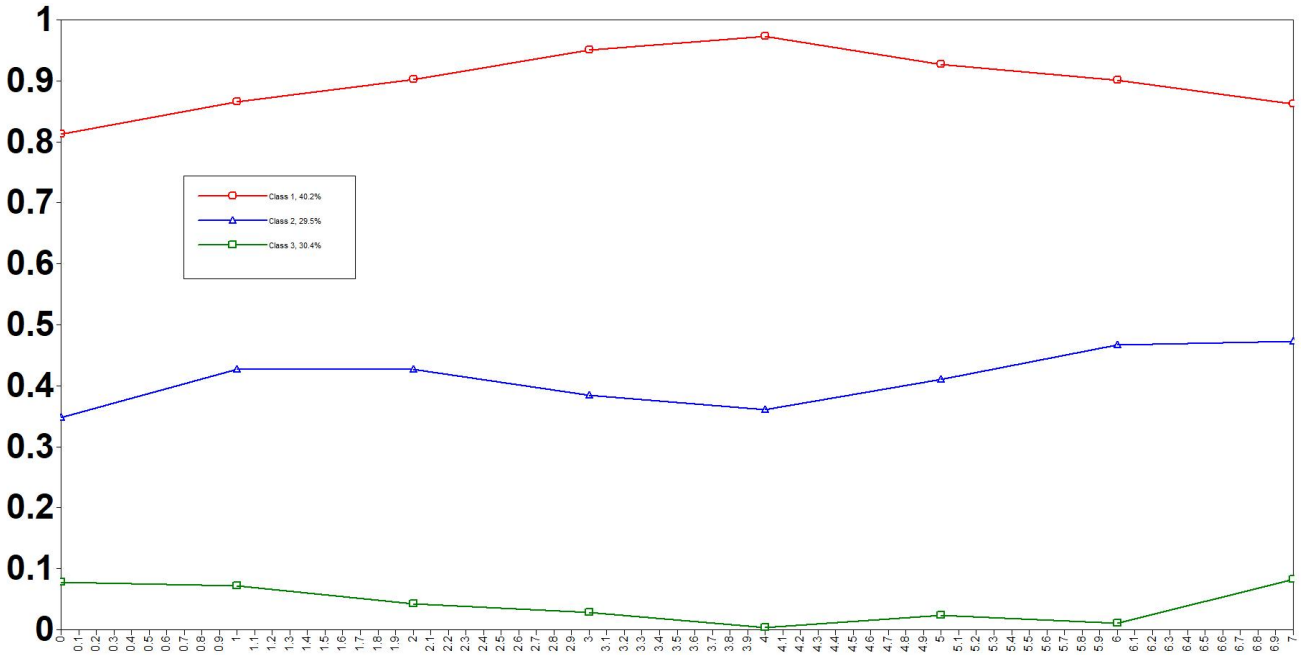
- Parameterization = Rescov
  - Asparouhov & Muthén (2015). Residual associations in latent class and latent transition analysis. Structural Equation Modeling: A Multidisciplinary Journal, 22:2, 169-177

```
ANALYSIS:
        CLASSES = c(2);
        TYPE = MIXTURE;
        ESTIMATOR = MLR;
        PROCESSORS = 8;
        STARTS = 80 20;
        PARAMETERIZATION = RESCOV;

MODEL:
        %OVERALL%
        y2-y8 PWITH y1-y7;

OUTPUT:
        TECH10;

PLOT:
        TYPE = PLOT3;
        SERIES = y1-y8(*);
```

LCA MLR Model Fit: Abstinence

Model	# par's	LL	BIC	# Sig Standardized Residuals: Response Patterns	# Sig Standardized Residuals: Bivariate	Comments
0. Linear Growth AR2, Bayes	18			0	0	Good fit
1. LCA 2c	17	-5203	10528	18 (all 0's)	32	Poor fit
2. LCA 3c	26	-4956	10100	7 (all 0's)	24	Poor fit
3. LCA 4c	35	-4794	9841	4 (19)	2	OK fit
4. LCA 5c	44	-4774	9865	3 (13) 0	0	Good fit
5. LCA 2 rescov	24	-4852	9878	5 (all 0's)	3	Poor fit
6. **LCA 3c rescov**	33	-4787	**9813**	2 (22)	1	Good fit
7. LCA 4c rescov	42	-4763	9830	0	0	Good fit

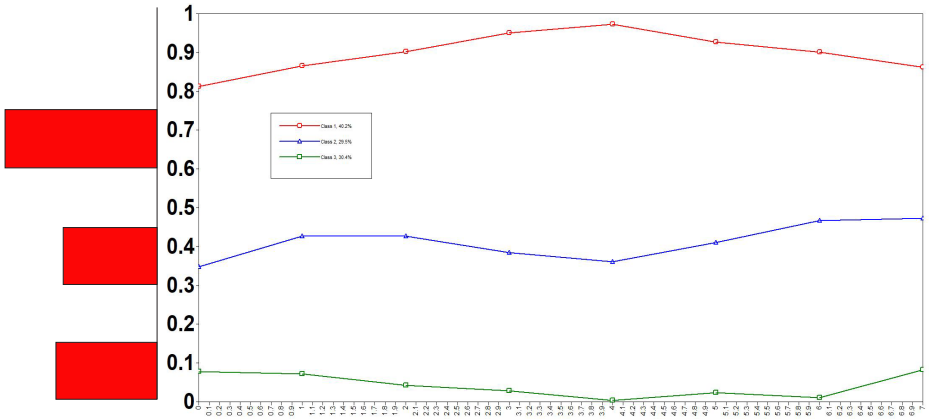
LCA 3c with Rescov: MLR Estimated Curves for Non-Abstinence Probability (Entropy = 0.75)



- Classes: low (30%), medium (29%), high (41%)
- 4-class solution splits the medium class into two

Interpreting the Latent Classes

- Classes: low (30%), medium (29%), high (41%)
- The 3 classes can be seen as a non-parametric representation of a random intercept instead of the normal distribution specification of the RI-AR model and the linear growth model



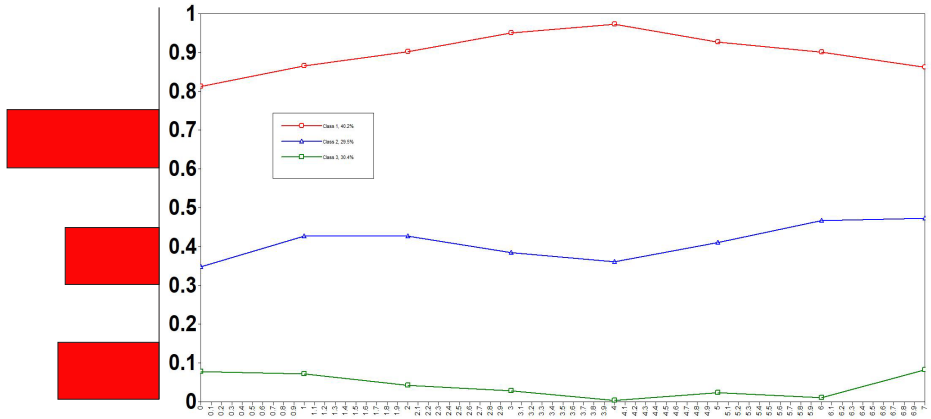
- Are the 3 classes substantively meaningful? - How do they relate to antecedents?
  - 9 treatment groups of size N=150. First group placebo

LCA 3c with Rescov: Mplus Input for Trajectory Classes Related to 9 Treatment Groups Using 8 Dummy Variables

- C ON X carried out using AUXILIARY R3STEP

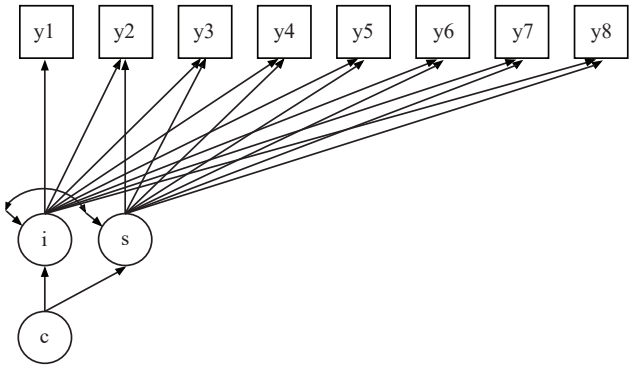
```
                USEVARIABLES = z1-z8 x2-x9;
                CATEGORICAL = z1-z8;
                CLASSES = c(3);
                AUXILIARY = x2-x9(R3STEP);
DEFINE:
                CUTPOINT z1-z8 (0.5);
                ! ccell = 1 is the placebo group
                IF(ccell EQUAL 2)THEN x2=1 ELSE x2=0;
                IF(ccell EQUAL 3)THEN x3=1 ELSE x3=0;
                IF(ccell EQUAL 4)THEN x4=1 ELSE x4=0;
                IF(ccell EQUAL 5)THEN x5=1 ELSE x5=0;
                IF(ccell EQUAL 6)THEN x6=1 ELSE x6=0;
                IF(ccell EQUAL 7)THEN x7=1 ELSE x7=0;
                IF(ccell EQUAL 8)THEN x8=1 ELSE x8=0;
                IF(ccell EQUAL 9)THEN x9=1 ELSE x9=0;
ANALYSIS:
                TYPE = MIXTURE;
                ESTIMATOR = MLR;
                PROCESSORS = 8;
                STARTS = 80 20;
                PARAMETERIZATION = RESCOV;
MODEL:
                %OVERALL%
                z2-z8 PWITH z1-z7;
OUTPUT:
                TECH10;
PLOT:
                TYPE = PLOT3;
                SERIES = z1-z8(*);
```

LCA 3c with Rescov: Trajectory Classes Related to Treatment Groups



- C ON X results using the low class as a comparison class:
  - Two significant effects: The high class regressed on X3 and X4 has negative effects, i.e., lowering the odds of being in the high class relative to being in the low class (can be translated into probabilities as shown in UG chapter 14)
  - X3 = naltrexone, X4 = naltrexone + acamprosate
- LCA-Rescov research question: How to connect to RI-AR modeling and the bivariate RI-CLPM?

Growth Mixture Modeling (GMM)



- Asparouhov-Muthén (2009): `http://www.statmodel.com/download/ChapmanHall06V24.pdf`
- Particular growth curve shapes chosen beforehand or via LCA
- Continuous latent growth factors: i, s, q
  - Growth factor means drive the probability trajectories while thresholds are time- and class-invariant
  - Special case with zero growth factor variances: Latent class growth analysis (LCGA)
- Residual correlations not yet accessible for categorical outcomes:
  - ML with Rescov not available with random effects
  - Bayes with AR difficult for mixtures due to no ML starts (STVALUES=ML) and potential label switching

Section 14 Distal Outcomes

Assessing Treatment Effects: Proximal and Distal Outcomes in Bivariate Cross-Lagged Panel Modeling for Stress and Abstinence in the COMBINE Data

- Proximal outcome (primary endpoint): Week 16 abstinence
  - Multiple-group Bayesian analysis using Knownclass for placebo and treatment groups
  - Bivariate Linear Growth AR2 model for stress and abstinence week 1 - week 16 (T=8) from slide 142
  - Centering the time scores at week 16 to assess primary outcome effect via the means of the random intercept
  - Fixing the random intercept means of the placebo group at zero as a comparison group
- Distal (long-term secondary) outcome: Week 52 abstinence
  - Regressing the week 52 abstinence outcome on the random intercepts with group-invariant slope to find the treatment effect by the comparison of the week 52 abstinence probability for the treatment groups and the placebo group

Mplus Input for Assessing ProximalTreatment Effects at Week 16 Bivariate Linear Growth AR2 Model Treatment Effect Assessed by IY and IZ Means

- 8 treatment groups and placebo group are represented as 9 classes using Knownclass in a Type=Mixture analysis

```
                    USEVARIABLES = y1-y8 z1-z8;
                    CATEGORICAL = z1-z8;
                    CLASSES = c(9);
                    KNOWNCLASS = c(cCell = 1-9);
DEFINE:
                    CUTPOINT z1-z8 (0.5);
ANALYSIS:
                    TYPE = MIXTURE;
                    ESTIMATOR = BAYES;
                    BITERATIONS = (5000);
                    THIN = 10;
                    PROCESSORS = 8;
```

Mplus Input Continued: Assessing ProximalTreatment Effects at Week 16 Treatment Effect Assessed by IY and IZ Means

```
MODEL:
    %OVERALL%
    iy sy | y1@-1.5 y2@-1.4 y3@-1.2 y4@-1 y5@-
    .8 y6@-.6 y7@-.4 y8@0;
    iz sz | z1@-1.5 z2@-1.4 z3@-1.2 z4@-1 z5@-.8
    z6@-.6 z7@-.4 z8@0;
    y2^-y8^ PON y1^-y7^;
    y3^-y8^ PON y1^-y6^;
    z2^-z8^ PON z1^-z7^;
    z3^-z8^ PON z1^-z6^;
    z1-z8@1;
    y2^-y8^ PON z1^-z7^;
    z2^-z8^ PON y1^-y7^;
    y1^-y8^ PWITH z1^-z8^;
    sz@0.1;
    sz WITH iz@0;
    sz WITH iy@0;
    sz WITH sy@0;
    ! changing the default:
    [y1-y8] (int);
    %c#1% ! Placebo group
    [iy@0 iz@0];
    %c#9%
    ! Avoiding mixture default
    ! of zero means in last class:
    [iy-sz];
OUTPUT:
    STANDARDIZED  RESIDUAL
    TECH1 TECH8 TECH10;
PLOT:
    TYPE = PLOT3;
```

Assessing Proximal Treatment Effects: Results

- The means for the IZ abstinence random intercept show significant treatment effects on week 16 abstinence for 4 groups:
  - 3: naltrexone
  - 4: naltrexone + acamprosate
  - 5: placebo + behavioral intervention
  - 8: naltrexone + acamprosate + behavioral intervention
- No significant treatment effect on the IY stress random intercept means

Mplus Input for Assessing Treatment Effects on Week 52

```
! z9 is week 52 abstinence
USEVARIABLES = y1-y8 z1-z9;
CATEGORICAL = z1-z9;
CLASSES = c(9);
KNOWNCLASS = c(cCell = 1-9);
DEFINE:
CUTPOINT z1-z9 (0.5);
ANALYSIS:
ESTIMATOR = BAYES;
BITERATIONS = (2000);
THIN = 100;
PROCESSORS = 8;
TYPE = MIXTURE;
MODEL:
%OVERALL%
iy sy | y1@-1.5 y2@-1.4 y3@-1.2 y4@-1 y5@-.8 y6@-.6 y7@-.4 y8@0;
iz sz | z1@-1.5 z2@-1.4 z3@-1.2 z4@-1 z5@-.8 z6@-.6 z7@-.4 z8@0;
y2^-y8^ PON y1^-y7^;
y3^-y8^ PON y1^-y6^;
z2^-z8^ PON z1^-z7^;
z3^-z8^ PON z1^-z6^;
z1-z8@1;
y2^-y8^ PON z1^-z7^;
z2^-z8^ PON y1^-y7^;
y1^-y8^ PWITH z1^-z8^;
sz@0.1;
sz WITH iz@0;
sz WITH iy@0;
sz WITH sy@0;
[y1-y8] (int);
z9 ON iz (b);
[iy@0];
[sy] (slopey);
[sz] (slopez);
```

Mplus Input Continued

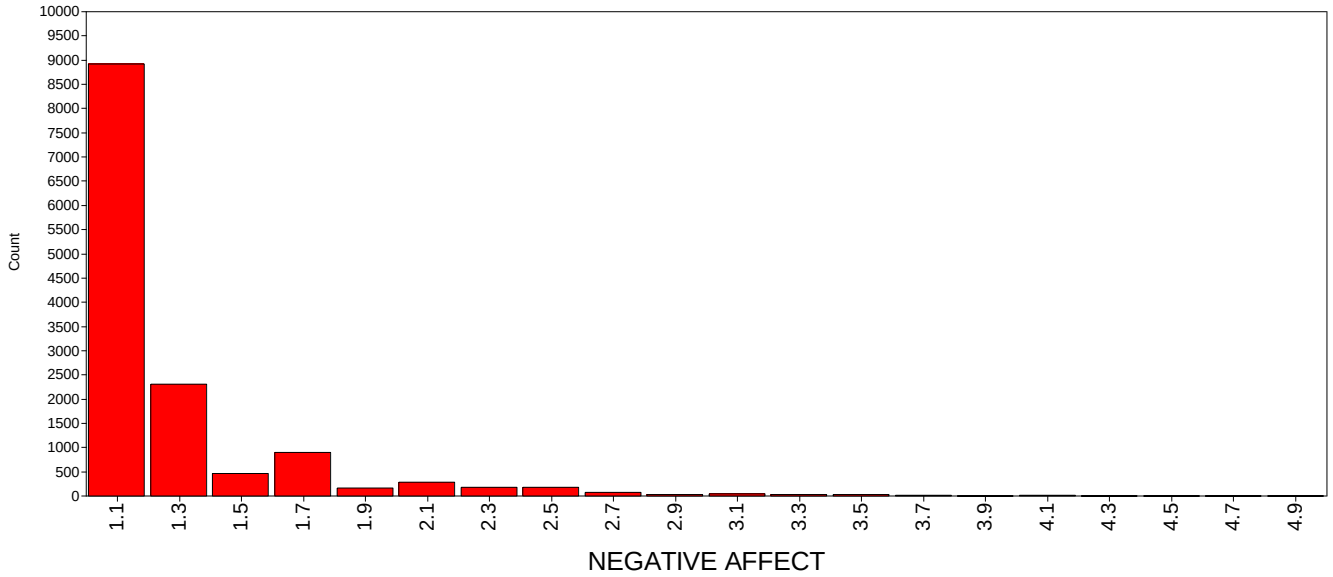
```
%c#1%
[iy@0 iz@0];
[z9$1] (t1);
%c#2%
[z9$1] (t2);
[iz] (m2);
%c#3%
[z9$1] (t3);
[iz] (m3);
%c#4%
[z9$1] (t4);
[iz] (m4);
%c#5%
[z9$1] (t5);
[iz] (m5);
%c#6%
[z9$1] (t6);
[iz] (m6);
%c#7%
[z9$1] (t7);
[iz] (m7);
%c#8%
[z9$1] (t8);
[iz] (m8);
%c#9%
[z9$1] (t9);
[iz] (m9);
MODEL
CONSTRAINT:
NEW(d2-d9);
! probit for Z9,
! = -t + b*m:
! probit for placebo
! = -t1 (m1=0)
! probit diffs for tx - placebo:
d2 = -t2+b*m2+t1;
d3 = -t3+b*m3+t1;
d4 = -t4+b*m4+t1;
d5 = -t5+b*m5+t1;
d6 = -t6+b*m6+t1;
d7 = -t7+b*m7+t1;
d8 = -t8+b*m8+t1;
d9 = -t9+b*m9+t1;
```

Assessing Distal Treatment Effects: Results

- No significant treatment effects at follow-up week 52:
  - The Z9* means (d2 - d9 in Model Constraint) for the treatment groups are not significantly different from that of the placebo group
- The random intercept for abstinence is a significant predictor of Z9
  - A higher random intercept corresponds to a higher probability of non-abstinence
- Extensions:
  - Two-part ordinal both for growth part and distal outcome
  - Using pre-treatment data and a pre-treatment baseline random intercept, it is possible to investigate treatment-baseline interaction

Other Non-Continuous Outcomes

- Other non-continuous outcome types can be accomodated such as semi-continuous variables
  - Data set 3 negative affect score summing over a set of items:



- Can be analyzed by Bayes using censored or two-part RI-AR models and the new hat approach. Right tail of the distribution treated as continuous

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