Penalized Structural Equation Models Supplemental Materials Tihomir Asparouhov and Bengt Muthén

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Figure 1: Alignment of direct effects in MIMIC models

```
MONTECARLO:
    NAMES = y1-y10 x1-x3;
    NOBSERVATIONS = 1000;
    NREPS = 100;
    categorical=y1-y10;
    generate=y1-y10(3);
ANALYSIS: param=theta; conv=0.000001;
MODEL POPULATION:
    [y1$1-y10$1*-1];
    [y1$2-y10$2*0];
    [y1$3-y10$3*1];
    [x1@0 x2@1 x3@2]; x1@3 x2@2 x3@1;
    f1 by y1-y5*1;
    f2 by y6-y10*1;
    f1 on x1*.5 x2*.6 x3*.7;
    f2 on x1*.7 x2*.6 x3*.5;
    y1 on x3*0.3;
    y2 on x1*0.4;
    f1-f2*.7;
    f1 with f2*.25;
    y1-y10*1;
analysis: tolerance=0.001;
MODEL:
    [y1$1-y10$1*-1];
    [y1$2-y10$2*0];
    [y1$3-y10$3*1];
    f1 BY y1-y5;
    f2 BY y6-y10;
    f1 on x1*.5 x2*.6 x3*.7;
    f2 on x1*.7 x2*.6 x3*.5;
    y1 on x3*0.3;
    y2 on x1*0.4;
    y1-y10 ON x1-x3 (d1-d30);
    f1-f2*.7;
    f1 with f2*.25;
```

MODEL PRIORS: d1-d30~ALF(0,10);

Figure 2: Alignment of cross-loadings simulation study: PSEM-ALF

```
MONTECARLO:
        NAMES = y1-y6;
        NOBSERVATIONS = 500;
        NREPS = 100;
MODEL POPULATION:
        f1 by y1-y3*1 y6*0.3;
        f2 by y4-y6*1;
        f1-f2@1;
        f1 with f2*.25;
        y1-y6*.5;
MODEL:
        f1 by y1-y3*1
          y4-y5*0 y6*0.3 (a1-a3);
        f2 by y4-y6*1
          y1-y3*0 (a4-a6);
        f1-f2@1;
        f1 with f2*.25;
        y1-y6*.5;
MODEL PRIORS: a1-a6~ALF(0,1);
```

Figure 3: Alignment of cross-loadings simulation study: ESEM-target

```
MONTECARLO:
        NAMES = y1-y6;
        NOBSERVATIONS = 500;
        NREPS = 100;
MODEL POPULATION:
        f1 by y1-y3*1 y6*0.3;
        f2 by y4-y6*1;
        f1-f2@1;
        f1 with f2*.25;
        y1-y6*.5;
MODEL COVERAGE:
        f1 by y1-y3*1 y6*0.3;
        f2 by y4-y6*1;
        f1-f2@1;
        f1 with f2*.25;
        y1-y6*.5;
analysis: rotation=target;
MODEL:
       f1 by y1-y3 y4-y6~0 (*1);
       f2 by y1-y3~0 y4-y6 (*1);
```

```
MONTECARLO:
        NAMES = u1-u10;
        NOBSERVATIONS = 500;
        NREPS = 100;
MODEL POPULATION:
    f1 by u1-u5*1;
    f2 by u3*0.5 u6-u10*1;
    f1 with f2*0.25;
    u1-u10*1 f1-f2*1;
    u1 with u2*0.5;
    u4 with u8*0.3;
    u9 with u10*0.4;
MODEL:
    f1 by u1-u5*1 u6-u10*0 (*1);
    f2 by u1-u2*0 u3*0.5 u4-u5*0 u6-u10*1 (*1);
    f1 with f2*0.25;
    u1-u10*1;
    u1 with u2*0.5;
    u4 with u8*0.3;
    u9 with u10*0.4;
    u1-u10 with u1-u10 (c1-c45);
MODEL PRIORS: c1-c45~ALF(0,1);
```

Figure 5: Alignment of residual correlations and cross-loadings in CFA

```
MONTECARLO:
        NAMES = u1-u10;
        NOBSERVATIONS = 500;
        NREPS = 100;
MODEL POPULATION:
    f1 by u1-u5*1;
    f2 by u3*0.5 u6-u10*1;;
    u1-u10*1 f1-f2*1;
    f1 with f2*0.25;
    u1 with u2*0.5;
    u4 with u8*0.3;
    u9 with u10*0.4;
MODEL:
    f1 by u1-u5*1
          u6-u10*0 (L1-L5);
    f2 by u6-u10*1
          u1-u2*0 u3*0.5 u4-u5*0 (L6-L10);
    u1-u10*1; f1-f2@1;
    f1 with f2*0.25;
    u1 with u2*0.5;
    u4 with u8*0.3;
    u9 with u10*0.4;
    u1-u10 with u1-u10 (c1-c45);
MODEL PRIORS: c1-c45~ALF(0,1); L1-L10~ALF(0,1);
```

```
MONTECARLO:
      NAMES = y1-y4;
      NOBSERVATIONS = 3(1000);
      NGROUPS = 3;
      NREPS = 100;
MODEL POPULATION: f1 BY y1-y4*1; [y1-y4*0]; y1-y4*.5;
MODEL POPULATION-G1: [y4*0.5]; [f1@0]; f1@1;
MODEL POPULATION-G2: f1 BY y4*0.5; [f1*0.4]; f1*1.5;
MODEL POPULATION-G3: [f1*-0.3]; f1*1.2;
MODEL COVERAGE: f1 BY y1-y4*1; y1-y4*.5;
MODEL COVERAGE-G1: f1*1; [y4*0.5];
MODEL COVERAGE-G2: f1 BY y4*0.5; [f1*0.4]; f1*1.5;
MODEL COVERAGE-G3: [f1*-0.3]; f1*1.2;
MODEL: f1 BY y1-y4*1 (L#_1-L#_4);
           [y1-y4*0] (M#_1-M#_4);
MODEL G1: [f1@0]; f1@1;
MODEL PRIOR:
DO(1,4) DIFF(L1_#-L3_#)~ALF(0,1);
DO(1,4) DIFF(M1_#-M3_#)~ALF(0,1);
```

MONTECARLO: NAMES = y1-y4;NOBSERVATIONS = 3(1000);NGROUPS = 3; NREPS = 100;MODEL POPULATION: f1 BY y1-y4*1; y1-y4*.5; MODEL POPULATION-G1: [f1@0]; f1@1; [y1*-1 y2*1 y3*-0.5 y4*0.5]; MODEL POPULATION-G2: f1 BY y4*0.5; f1*1.5; [y1*1 y2*-1 y3*0.5 y4*-0.5]; MODEL POPULATION-G3: f1*1.2; [y1-y4*1]; MODEL COVERAGE: f1 BY y1-y4*1; y1-y4*.5; MODEL COVERAGE-G1: f1@1; [y1*-1 y2*1 y3*-0.5 y4*0.5]; MODEL COVERAGE-G2: f1 BY y4*0.5; f1*1.5; [y1*1 y2*-1 y3*0.5 y4*-0.5]; MODEL COVERAGE-G3: f1*1.2; [y1-y4*1]; MODEL: f1 BY y1-y4*1 (L#_1-L#_4); [y1-y4*0] (M#_1-M#_4); [f1@0]; MODEL G1: f1@1; MODEL PRIOR: DO(1,4) DIFF(L1_#-L3_#)~ALF(0,1);

```
MONTECARLO:
    NAMES = y11-y13 y21-y23 y31-y33 y41-y43 Y51-Y53;
    NOBSERVATIONS = 1000;
    NREPS = 100;
analysis: alignment=fixed; tolerance=0.001;
MODEL POPULATION:
    f1 BY y11-y13*1 ;
    f2 BY y21*1 y22*0.5 y23*1;
    f3 BY y31-y33*1;
    f4 BY y41-y43*1;
    f5 BY y51-y53*1;
    [y42*0.5];
    f1-f5 with f1-f5*0.5;
    f1*1 f2*0.9 f3*1.4 f4*1.5 f5*2;
    [f1*0 f2*0.3 f3*0.6 f4*0.9 f5*1.2];
    y11-y53*1;
MODEL:
    f1 BY y11-y13*1 ;
    f2 BY y21*1 y22*0.5 y23*1;
    f3 BY y31-y33*1;
    f4 BY y41-y43*1;
    f5 BY y51-y53*1;
    [y42*0.5];
    f1-f5 with f1-f5*0.5;
    f1*1 f2*0.9 f3*1.4 f4*1.5 f5*2;
    [f1*0 f2*0.3 f3*0.6 f4*0.9 f5*1.2];
    y11-y53*1;
MODEL t1: f1 by y11-y13;
MODEL t2: f2 by y21-y23;
MODEL t3: f3 by y31-y33;
MODEL t4: f4 by y41-y43;
MODEL t5: f5 by y51-y53;
```

```
MONTECARLO:
   NAMES = y11-y13 y21-y23 y31-y33 y41-y43 Y51-Y53;
    NOBSERVATIONS = 1000;
   NREPS = 100;
MODEL POPULATION:
    f1 BY y11-y13*1;
   f2 BY y21*1 y22*0.5 y23*1;
   f3 BY y31-y33*1;
    f4 BY y41-y43*1;
    f5 BY y51-y53*1;
    [y42*0.5];
    f1-f5*1;
    [f1-f5*0];
   y11-y53*1;
    i s | f1@0 f2@1 f3@2 f4@3 f5@4;
    [i*0 s*0.3];
    i*1 s*0.3;
    i with s*-0.2;
MODEL:
    f1 BY y11-y13*1 (a11-a13);
    f2 BY y21*1 y22*0.5 y23*1 (a21-a23);
    f3 BY y31-y33*1 (a31-a33);
    f4 BY y41-y43*1 (a41-a43);
    f5 BY y51-y53*1 (a51-a53);
    [y42*0.5];
    [y11-y53] (n1-n15);
    f1-f5*1;
    [f1-f5@0];
   y11-y53*1;
    i s | f1@0 f2@1 f3@2 f4@3 f5@4;
    [i@0 s*0.3];
    i@1 s*0.3;
    i with s*-0.2;
MODEL PRIOR:
   DIFF(a11 a21 a31 a41 a51)~ALF(0,1);
    DIFF(a12 a22 a32 a42 a52)~ALF(0,1);
    DIFF(a13 a23 a33 a43 a53)~ALF(0,1);
    DIFF(n1 n4 n7 n10 n13)~ALF(0,1);
    DIFF(n2 n5 n8 n11 n14)~ALF(0,1);
    DIFF(n3 n6 n9 n12 n15)~ALF(0,1);
```

```
MONTECARLO:
   NAMES = y1-y20;
   NOBSERVATIONS = 1000;
   NREPS = 100;
MODEL POPULATION:
   f1 BY y1*0.7 y2*0.3 y3-y4*0.4 y5*0.3;
   f2 BY y5*0.6 y6*0.7 y7*0.5 y8*0.5 y9*1 y10*0.5;
   f3 BY y11*0.7 y12*0.3 y13-y15*0.4;
   f4 BY y15*0.6 y16*0.7 y17*0.5 y18*0.5 y19*1 y20*0.5;
   f1-f4@1;
   y1-y4*0.35 y5-y8*0.42 y9-y10*0.51;
   y11-y14*0.35 y15-y18*0.42 y19-y20*0.51;
    f1 with f2*0.4; f1 with f3*0.1;
   f1 with f4*0.3; f2 with f3*0.3;
   f2 with f4*0.2; f3 with f4*0.2;
MODEL:
   f1 BY y1*0.7 y2*0.3 y3-y4*0.4 y5*0.3 y6-y20*0 (a1-a20);
    f2 BY y1-y4*0 y5*0.6 y6*0.7 y7*0.5 y8*0.5 y9*1 y10*0.5 y11-y20*0 (a21-a40);
    f3 BY y1-y10*0 y11*0.7 y12*0.3 y13-y14*0.4 y15*0.4 y16-y20*0 (a41-a60);
   f4 BY y1-y14*0 y15*0.6 y16*0.7 y17*0.5 y18*0.5 y19*1 y20*0.5 (a61-a80);
   f1-f4@1;
   y1-y4*0.35 y5-y8*0.42 y9-y10*0.51;
   y11-y14*0.35 y15-y18*0.42 y19-y20*0.51;
   f1 with f2*0.4; f1 with f3*0.1;
   f1 with f4*0.3; f2 with f3*0.3;
   f2 with f4*0.2; f3 with f4*0.2;
MODEL PRIORS:
a1-a80~Geomin(4,.1);
```

Figure 12: PSEM-ESEM single data input file

```
variable: NAMES = y1-y20;
data: FILE=PSEMESEM1.dat;
analysis: iter=10000;
MODEL:
    f1 BY y1-y20*1 (a1-a20);
    f2 BY y1-y20*1 (a21-a40);
    f3 BY y1-y20*1 (a41-a60);
    f4 BY y1-y20*1 (a61-a80);
    f1-f4@1;
MODEL PRIORS: a1-a80~Geomin(4,.1);
```

```
MONTECARLO:
    NAMES = y1-y20;
    NOBSERVATIONS = 2000;
   NREPS = 100;
MODEL POPULATION:
    f1 BY y1*0.7 y2*1.3 y3-y4*0.8 y5*0.3;
    f2 BY y5*0.6 y6*0.7 y7*0.5 y8*0.8 y9*1 y10*0.5 ;
    f3 BY y11*0.7 y12*1 y13-y15*0.4;
    f4 BY y15*0.6 y16*0.7 y17*1 y18*0.5 y19*1 y20*0.5;
   f1-f4@1;
    y1-y20*1;
    f0 by f1*0.4 f2*0.5 f3*0.5 f4*0.6; f0@1;
MODEL:
    f1 BY y1*0.7 y2*1.3 y3-y4*0.8 y5*0.3 y6-y20*0(a1-a20);
    f2 BY y1-y4*0 y5*0.6 y6*0.7 y7*0.5 y8*0.8 y9*1 y10*0.5 y11-y20*0 (a21-a40);
    f3 BY y1-y10*0 y11*0.7 y12*1 y13-y14*0.4 y15*0.4 y16-y20*0 (a41-a60);
    f4 BY y1-y14*0 y15*0.6 y16*0.7 y17*1 y18*0.5 y19*1 y20*0.5 (a61-a80);
    f1-f4@1;
   y1-y20*1;
    f0 by f1*0.4 f2*0.5 f3*0.5 f4*0.6; f0@1;
MODEL PRIORS: a1-a80~Geomin(4,0.1,0.001);
```

MONTECARLO: NAMES = y1-y20; NOBSERVATIONS = 1000; NREPS = 100;MODEL POPULATION: g1 BY y1*.7 y2*.3 y3-y4*.4 y5-y10*1 y11*.2 y12*.3; g2 BY y11-y20*1; f1 BY y5*.6 y6*.7 y7*.5 y18*.4 y19*.3 y20*.4; f2 BY y11*.7 y12*.3 y13-y14*.4 y1-y4*1 y20*.2; f3 BY y15*.6 y16*.7 y17*.5 y8*.5 y9*1 y10*.5; y1-y4*.35 y5-y8*.42 y9-y10*.51; y11-y14*.35 y15-y18*.42 y19-y20*.51; f1-f3@1; f1-f3 with f1-f3@0; g1-g2@1; g1-g2 with f1-f3@0; g1 with g2*.4; MODEL: g1 BY y1*.7 y2*.3 y3-y4*.4 y5-y10*1 y11*.2 y12*.3 y13-y20*0 (a1-a20); g2 BY y1-y10*0 y11-y20*1 (a21-a40); f1 BY y1-y4*0 y5*.6 y6*.7 y7*.5 y8-y17*0 y18*.4 y19*.3 y20*.4 (a41-a60); f2 BY y1-y4*1 y5-y10*0 y11*.7 y12*.3 y13-y14*.4 y15-y19*0 y20*.2 (a61-a80); f3 BY y1-y7*0 y8*.5 y9*1 y10*.5 y11-y14*0 y15*.6 y16*.7 y17*.5 y18-y20*0 (a81-a100); y1-y4*.35 y5-y8*.42 y9-y10*.51; y11-y14*.35 y15-y18*.42 y19-y20*.51; f1-f3@1; f1-f3 with f1-f3@0; g1-g2@1; g1-g2 with f1-f3@0; g1 with g2*.4;

```
MODEL PRIORS: a1-a100~Geomin(5,1,.001);
```

```
montecarlo:
        names = y1-y6;
        ngroups = 2;
        nobs = 200 \ 100;
        nreps = 100;
model population:
        f1 by y1-y3*1; y1-y3@1;
        f2 by y4-y6*1 y1*0.3; y4-y6@1;
        f1-f2@1;
model population-g2:
        f1*1.3; [f1*0.4];
        f2*1.4; [f2*0.9];
model:
        f1 by y1-y3*1 Y4-Y6*0 (a1-a6);
        f2 by y1*0.3 Y2-Y3*0 y4-y6*1 (a7-a12);
        f1 with f2@0; f1-f2@1; y1-y6*1;
model g2:
        f1*1.3; [f1*0.4];
        f2*1.4; [f2*0.9];
model prior: a1-a12~Geomin(2,1,.001)
```

```
montecarlo:
    names = y1-y6;
    ngroups = 2;
    nobs = 300 300;
    nreps = 100;
model population:
    f1 by y1-y3*1; y1-y3@1;
    f2 by y4-y6*1 y1*0.3; y4-y6@1;
    f1-f2@1; f1 with f2*0.3;
model population-g2:
    f1*1.3 f2*1.4; f1 with f2*0.1;
    [f1*0.4 f2*0.9];
    f2 by Y4*0.5; ! non-invariant loading
    [y6*0.5]; ! non-invariant intercept
model:
    f1 by y1-y3*1 Y4-Y6*0 (a1-a6);
    f2 by y1*0.3 Y2-Y3*0 y4-y6*1 (a7-a12);
    f1 with f2*0.3; f1-f2@1; y1-y6*1;
model g2:
    f1*1.3 f2*1.4; f1 with f2*0.1;
    [f1*0.4 f2*0.9];
    f2 by Y4*0.5; ! non-invariant loading
    [y6*0.5]; ! non-invariant intercept
model prior: a1-a12~Geomin(2,1,.001);
```

TITLE: ELGM with 4 factors and T=12 DATA: FILE IS 1.dat; VARIABLE: NAMES ARE y1-y12; MODEL: f1-f4 BY y1-y12 (*1); [f1-f4 y1-y12@0]; -----TITLE: ELGM-null with 4 factors and T=12 DATA: FILE IS 1.dat; NAMES ARE y1-y12; VARIABLE: MODEL: f1 BY y1-y12*1; f2 BY y2-y12*1; f3 BY y3-y12*1; f4 BY y4-y12*1; [f1-f4 y1-y12@0]; f1-f4 with f1-f4@0; f1-f4@1; -----TITLE: ELGM null with auto-correlation, 4 factors and T=12 DATA: FILE IS 1.dat; VARIABLE: NAMES ARE y1-y12; ANALYSIS: STARTS=30; iteration=10000; f1 BY y1-y12*1; f2 BY y2-y12*1; f3 BY y3-y12*1; MODEL: f4 BY y4-y12*1; [f1-f4 y1-y12@0]; f1-f4 with f1-f4@0; f1-f4@1; y2^-y12^ pon y1^-y11^ (r);

Figure 18: PSEM-ELGM linear growth simulation study

```
MONTECARLO:
NAMES = y1-y8;
NOBSERVATIONS = 1000;
NREPS = 100;
ANALYSIS: conv=0.000001; iter=100000;
MODEL POPULATION:
y1-y8*1;
I BY Y1*1 Y2*1.03 Y3*1.06 Y4*0.89 Y5*0.94 Y6*0.97 Y7*1 Y8*1.06 (a1-a8);
S BY Y1*0 Y2*0.1 Y3*0.19 Y4*0.29 Y5*0.48 Y6*0.58 Y7*0.68 Y8*0.86 (b1-b8);
I WITH S*0.3; [I*1. S*0.5]; I@1 S@1;
[y2*0.2 y6*0.4];
y2^-y8^ pon y1^-y7^*0.3(r);
MODEL:
y1-y8*1;
I BY Y1*1 Y2*1.03 Y3*1.06 Y4*0.89 Y5*0.94 Y6*0.97 Y7*1 Y8*1.06 (b1-b8);
S BY Y1*0 Y2*0.1 Y3*0.19 Y4*0.29 Y5*0.48 Y6*0.58 Y7*0.68 Y8*0.86 (g1-g8);
I WITH S*0.3; [I*1. S*0.5]; I@1 S@1;
[y1*0 y2*0.2 y3-y5*0 y6*0.4 y7-y8*0] (a1-a8);
y2^-y8^ pon y1^-y7^*0.3(r);
MODEL PRIOR:
a1-a8~ALF(0,1);
DIFF(b1-b8)~ALF(0,1);
```

g1~ALF(0,1);

Figure 19: Height empirical example

DATA: file=1.dat; VARIABLE: names=y1-y14; ANALYSIS: iter=100000; starts=30; MODEL: [y1-y14] (a1-a14); f1 by y1-y14*1 (b1-b14); f2 by y1-y14*1 (g1-g14); y2^-y14^ pon y1^-y13^(r); f1-f2@1; [f1 f2]; MODEL CONSTRAINT: new(d1-d13); d1=b2-b1; d2=b3-b2; d3=b4-b3; d4=b5-b4; d5=b6-b5; d6=b7-b6; d7=b8-b7; d8=b9-b8; d9=b10-b9; d10=b11-b10; d11=b12-b11; d12=b13-b12; d13=b14-b13; MODEL PRIOR: a1-a14~ALF(0,1); DIFF(d1-d13)~ALF(0,1); g1~ALF(0,1);

Figure 20: PSEM-ELGM simulation study: mimicking the height data

```
MONTECARLO:
NAMES = y1-y10;
NOBSERVATIONS = 1000;
NREPS = 100;
ANALYSIS: conv=0.00001; iter=100000;
MODEL POPULATION:
[y1-y9*0 y10*-0.2];
F1 BY Y1*1 Y2*1.1 Y3*1.2 Y4*1.3 Y5*1.4 Y6*1.5 Y7*1.6 Y8*1.7 Y9*1.8 y10*1.9;
F2 BY Y1-Y5*0 Y6*0.4 Y7*0.8 Y8*1.2 y9*0.8 y10*0.4;
F1 WITH F2*0.3; [F1*1. F2*0.5]; F1@1 F2@1;
y2^-y10^ pon y1^-y9^*0.5(r);
y1-y10*1;
MODEL:
[y1-y9*0 y10*-0.2] (a1-a10);
F1 BY Y1*1 Y2*1.1 Y3*1.2 Y4*1.3 Y5*1.4 Y6*1.5 Y7*1.6 Y8*1.7 Y9*1.8 y10*1.9 (b1-b10);
F2 BY Y1-Y5*0 Y6*0.4 Y7*0.8 Y8*1.2 y9*0.8 y10*0.4 (g1-g10);
F1 WITH F2*0.3; [F1*1. F2*0.5]; F1@1 F2@1;
y2^-y10^ pon y1^-y9^*0.5(r);
y1-y10*1;
MODEL CONSTRAINT:
new(d1-d9);
d1=b2-b1;
d2=b3-b2;
d3=b4-b3;
d4=b5-b4;
d5=b6-b5;
d6=b7-b6;
d7=b8-b7;
d8=b9-b8;
d9=b10-b9;
MODEL PRIOR:
a1-a10~ALF(0,1);
DIFF(d1-d9)~ALF(0,1);
g1~ALF(0,1);
```