Assignment 3

For this assignment, I selected Algebra IRT scores from the LSAY data set to measure change in Algebra knowledge from Grade 7 to Grade 12. Table 1 provides descriptive information. Of the original 3116 cases, 782 students had full assessment information at the 6 time points.

	Mean	*Std. Dev.*	*Variance*	*Minimum*	*Maximum*
Grade 7	52.012	10.487	109.977	34.42	75.60
Grade 8	60.895	18.513	342.726	36.17	116.82
Grade 9	72.736	19.450	378.295	35.37	112.76
Grade 10	81.460	20.984	440.344	39.89	114.56
Grade 11	88.699	22.450	504.007	44.69	121.66
Grade 12	92.637	25.595	655.112	38.34	132.57

Table 1. Descriptive statistics for Algebra IRT scores from Grade 7 to Grade 12 (N=782).

There is a positive increase in the mean Algebra score (mean curve not included). The minimum values do not necessarily reflect this general positive increase. This prompted me to take a closer look at a few cases that had a greater amount of variation (see LSAYID=121104 as an example). These types of cases might be considered outliers because their scores tended to bounce around a lot, but without confidence in my substantive understanding (maybe there is something with prior knowledge and how recently the student took an Algebra course) and for the purposes of this assignment, I ignored potential outliers and simply included all cases with complete data.

The variances and range of scores increase over the years. The increase in variance across time indicates that there is a greater range of Algebra scores as students progress. Students might initially start off with little or no knowledge of Algebra and through experiences (i.e. course work), their knowledge of the subject increases at different rates. For example, students do not take the same math courses and even if they do, there are often different tracks or levels of the courses with the same title. In other words, Algebra for students on an honors track might look different than Algebra for students who are not. It might be the same course title (Algebra) but the quality and rigor might not be the same. Performance in math classes or the level/types of subsequent math courses might also be reflected in performance on the Algebra items. A few (or combination) of these reasons could be explored through the use of covariates which helps to account for the increased variance over time. The positive and moderate to high correlations suggest that performance over time is related (Table 2).

	Grade 7	*Grade 8*	*Grade 9*	*Grade 10*	*Grade 11*	*Grade 12*
Grade 7	- - -					
Grade 8	0.534	- - -				
Grade 9	0.482	0.729	- - -			
Grade 10	0.461	0.686	0.769	- - -		
Grade 11	0.455	0.684	0.769	0.854	- - -	
Grade 12	0.465	0.701	0.761	0.775	0.820	- - -

Table 2. Correlations for Algebra IRT-based scale scores from Grade 7 to Grade 10.

In addition to the average IRT scores increasing in a general linear pattern (as viewed in the sample mean plots, not included in write-up), sample plots of the observed individual score trajectories were visually inspected. If students were provided with effective instruction in Algebra, the goal is that performance on the Algebra items would reflect this increased knowledge. This increase in performance on the Algebra items seems to be the case for most students. A sample plot of 10 students (Figure 1) demonstrates the general linear trend and the observed increases in performance.

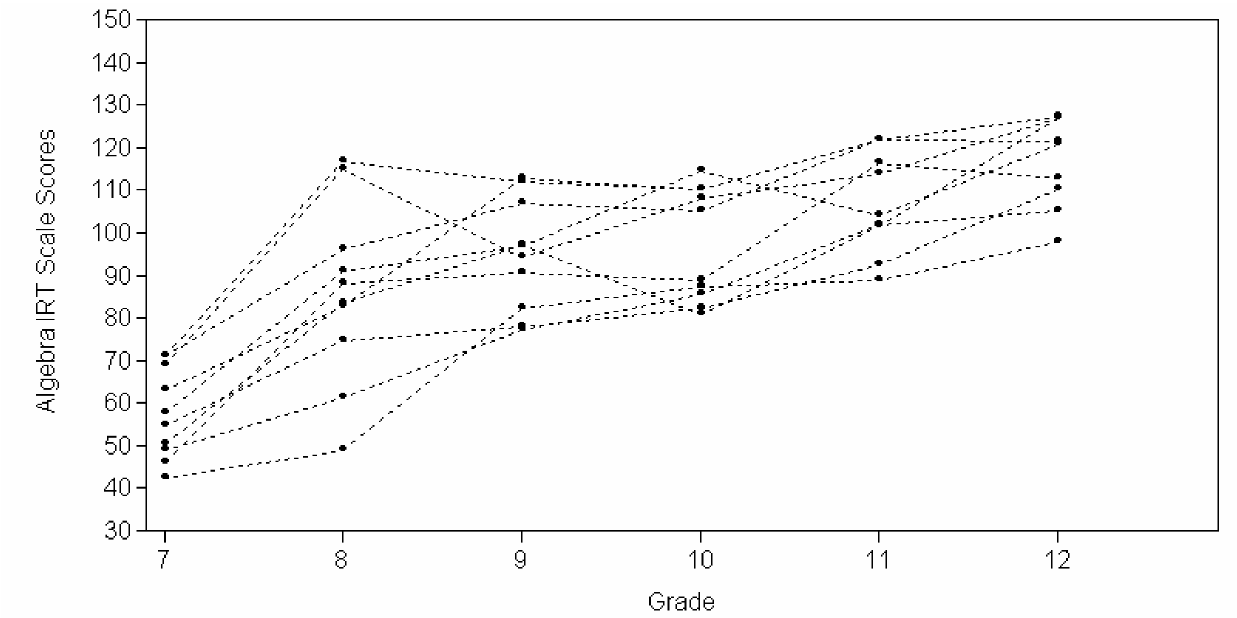

Figure 1. Observed values of Algebra performance for 10 selected students from Grade 7 to Grade 12.

Assuming linear growth, I first fit the growth model without any covariates using fixed time scores (Model 1). The chi-square test of model fit with did not indicate a good fit but this data does not take into account the nested nature of the data so the significant value is interpreted with some caution. However, the CFI and TLI are low which also suggest that this model does not fit the data well. The RMSEA 90% confidence interval does not include 0 and the SRMR is high. In looking to the model modification indices for ways to modify this model, freeing the alg7 and the slope and alg12 and the slope suggest a significant increase in the fit.

In Model 2, these two parameters were free to be estimated (trend with alg7, trend with alg12). The fit for the second model is slightly better. The chi-square values are still significant but are smaller with 2 less degrees of freedom. The CFI and TLI are higher compared to the first model, the AIC, BIC, RMSEA and SRMR are lower than the first model. Thus, compared to the first model, these indices suggest an improvement. In addition, a likelihood ratio chi-square test provides another indication that there is an improvement of fit.

To attempt a more substantively interesting and better fitting model, (and as suggested by the modification indices), I correlated a few of the residuals. Model 3 correlated the residuals for alg8 and alg9, alg10 with alg11 and alg10 and alg12. There might be substantive reasons for the correlation of these residuals. One possible reason is that performance on the Algebra items for a particular year might be related to performance on Algebra items for another year because no new Algebra knowledge was gained in that particular year and there was no other practice with Algebra or Algebra-related content that might influence performance. Maybe in Grades 8 and 9 there is a relationship because students are changing from middle school to high school and are placed in particular courses based on the courses that they've taken and recommendations by middle school teachers as to what "track" they should continue on in high school. Once students get to high school, most will likely take Algebra in the 9th grade. So their performance in Grade 9 isn't related to subsequent years because they might receive effective and direct instruction in Algebra. They might do really well on the Algebra items in 9th grade because they recently learned the material (so errors are not correlated with anything else) but in subsequent years (Grades 10-12), they tend to forget and perform less well on the Algebra items. Perhaps Grade 10 and 11 might be related because the Algebra concepts aren't as fresh in their minds (assuming most students take Algebra in 9th grade). Grade 10 and 12 might also be related to this decline in performance as time progresses. Table 3 and 4 provide fit indices comparing these three models.

	Model 1	Model 2	Model 3
Chi-square value	449.430	228.063	167.178
df	16	14	11
p-value	0.0000	0.0000	0.0000
CFI	0.886	0.944	0.959
TLI	0.893	0.940	0.944
Akaike's Information Criterion (AIC)	37553.473	37336.107	37281.222
Schwarz's Bayesian Criterion (BIC)	37604.754	37397.711	38355.811
RMSEA	0.186	0.140	0.135
SRMR	0.146	0.099	0.091

Table 3. Tests of model fit for Model 1, 2 and 3.

	Chi-square	*Difference*
Model 1	449.430 (16)	
Model 2	228.063 (14)	221.367 (2)*
Model 3	167.178 (11)	60.852 (3)*

Table 4. Difference in chi-square values for Model 1, 2 and 3.

Figure 2 provides a schematic of Model 3. This model seems to have the best fit of the three models and may also provide some substantive meaning.

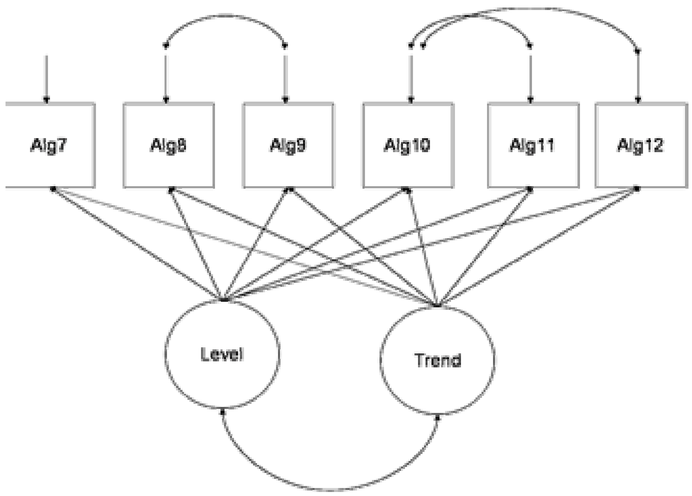

Figure 2. This model (Model 3) includes correlated residuals between Grade 8 and Grade 9, Grade 10 and Grade 11, and Grade 10 and 12 and slopes for alg7 and alg12 are freely estimated.

The R^2 values for the observed variables are moderate to high, ranging from .485 (Alg8) to .852 (Alg11). This provides an indication of how well the variables are observed by the latent variables.

There is a significant, positive relationship between the intercept (level) and slope (trend) which indicates that students who start off high tend to grow faster. For students who start off low, their growth rate does not tend to be very fast, compared to students who start of higher in 8^{th} grade. The growth between time point 0 and 1 (Grade 8 and Grade 9) is 8.507. Grade 8 (initial starting point) is estimated to be 63.660 which is close to the observed or expected intercept (Grade 8) of 60.895. The variation of the intercepts around the mean is significant and high (139.605). The variation of the slope between Grade 8 and 9 is also significant and high (13.888). Table 5 compares the estimated values for each grade level with an intercept of 63.660 and slope of 8.507. There is not much difference between the two, which is a good indication that the model fit the data. Growth appears to slow down from Grade 11 to Grade 12 (<1) and seems to speed up from Grade 7 to 8 (>1). These two parameters were freely estimated do seem to depart slightly from linearity.

Grade	*Time Score*	*Estimated Means = intercept + slope(time score)*	*Observed Means*
7	-1.413	63.660 + 8.507(-1.413) = 51.639	52.012
8	0.000	63.660 + 8.507(0) = 63.660	60.895
9	1.000	63.660 + 8.507(1) = 72.167	72.736
10	2.000	63.660 + 8.507(2) = 80.674	81.600
11	3.000	63.660 + 8.507(3) = 89.181	88.699
12	3.418	63.660 + 8.507(3.418) = 92.734	92.637

Table 5. Estimated means from Model 3 and observed means for 782 students.

In addition to the improvement in model fit and the estimated and observe means being fairly close, one still might wonder what else could help to explain the model. The next step would be to add covariates to the model to help explain the variation across individuals beyond the growth factors. A time-invariant covariate such as gender, ethnicity or home resources varies across individuals but does not vary over time. A time-variant covariates such as course-taking patterns varies across individuals and varies over time.

I tried to find a time-variant covariate that could contribute substantively and statistically to the story of Algebra knowledge. I didn't go through all of the possible covariates, but tried a few covariates that I thought might be meaningful (gender, math and science home resources, mothers level of education). None of the variables that I selected seemed to contribute much to the improved fit of the model. In fact, in some cases, the inclusion worsened the fit of the model. The next step would be to attempt time-varying covariates, like the highest level of math course taken at each grade.

There is still much room for improvement in the final model (Model 3), which the inclusion of covariates. Though I tried a few covariates, I was unable to find any that seemed to improve the fit of the model. This is not to say that the inclusion of covariates won't fit the model, just that I was unable to find covariates that had some intuitive and statistical value. The model still indicates a positive increase on Algebra items. With two departures, the trend is linear. Two time scores were free (Grade 7 and Grade 12). These free time scores indicates that the mean of the growth factor is not a constant rate of change over all time points. Instead, the mean of the slope growth is the rate of change for a time score change of one.

If we are confident that these items capture knowledge of Algebra, we would infer that there is an increase in knowledge as reflected in performance on these items, over time. There is variation between individuals in terms of where they start off and how they progress. Students who have higher initial starting points (high performance in Grade 8) tend to grow faster than students who have lower initial starting points. This is good news for students who come with prior knowledge of Algebra. However, for students without the same prior knowledge, their growth is not as fast. These students do grow or increase in their knowledge of Algebra, but just not at the same rate as students who come with prior knowledge.