Further Practical Issues

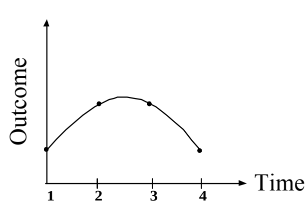
1

Specifying Time Scores For Quadratic Growth Models

Quadratic growth model

$$y_{ti} = \eta_{0i} + \eta_{1i}\, x_t + \eta_{2i}\; x_t^2 + \varepsilon_{ti}$$

- Need three latent variables to describe a quadratic growth model: Intercept, linear slope, quadratic slope



- Linear slope time scores: 0 1 2 3
  0 .1 .2 .3
- Quadratic slope time scores: 0 1 4 9
  0 .01 .04 .09

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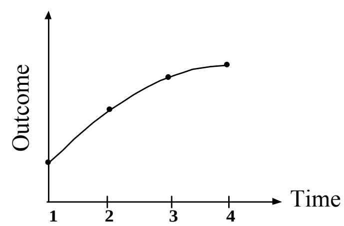
Specifying Time Scores For Non-Linear Growth Models With Fixed Time Scores

Non-Linear Growth Models with Fixed Time scores

- Need two latent variables to describe a non-linear growth model: Intercept and slope

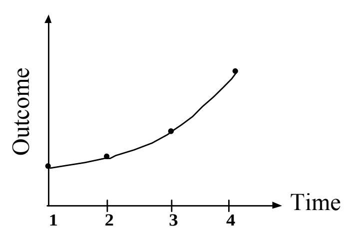
Growth model with a logarithmic growth curve--ln(t)



Time scores: 0 0.69 1.10 1.39

Specifying Time Scores For Non-Linear Growth Models With Fixed Time Scores (Continued)

Growth model with an exponential growth curve–
exp(t-1) - 1



Time scores: 0 1.72 6.39 19.09

Piecewise Growth Modeling

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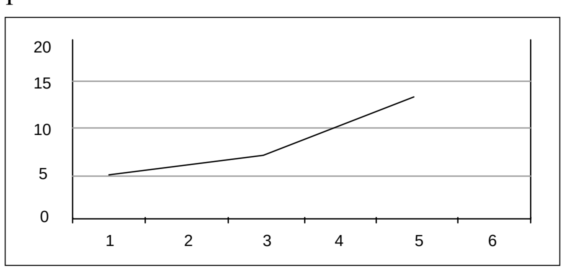
Piecewise Growth Modeling

- Can be used to represent different phases of development
- Can be used to capture non-linear growth
- Each piece has its own growth factor(s)
- Each piece can have its own coefficients for covariates



One intercept growth factor, two slope growth factors

0	1	2	2	2	2	Time scores piece 1
0	0	0	1	2	3	Time scores piece 2

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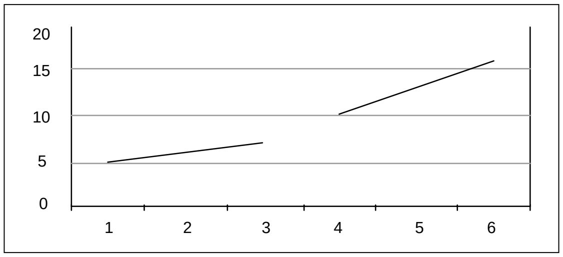
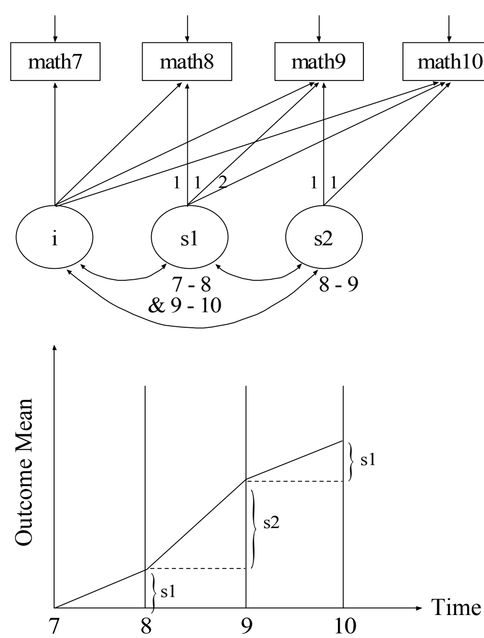
Piecewise Growth Modeling (Continued)



Two intercept growth factors, two slope growth factors

0 1 2 Time scores piece 1

0 1 2 Time scores piece 2



Input For LSAY Piecewise Growth Model With Covariates

```
MODEL:      i s1 | math7@0 math8@1 math9@1 math10@2;
            i s2 | math7@0 math8@0 math9@1 math10@1;
            i s1 s2 ON mothed homeres;
```

Alternative language:

```
MODEL:      i BY math7-math10@1;
            s1 BY math7@0 math8@1 math9@1 math10@2;
            s2 BY math7@0 math8@0 math9@1 math10@1;
            [math7-math10@0];
            [i s1 s2];
            i s1 s2 ON mothed homeres;
```


Output Excerpts LSAY Piecewise Growth Model With Covariates

n = 935

Tests of Model Fit

```
CHI-SQUARE TEST OF MODEL FIT

          Value                         11.721
          Degrees of Freedom                 3
          P-Value                        .0083

RMSEA (ROOT MEAN SQUARE ERROR OF APPROXIMATION)

          Estimate                        .056
          90 Percent C.I.                 .025   .091
          Probability RMSEA <= .05        .331
```

Output Excerpts LSAY Piecewise Growth Model With Covariates (Continued)

Selected Estimates

	Estimates	S.E.	Est./S.E.	Std	StdYX
I ON					
MOTHED	2.127	.284	7.488	.266	.256
HOMERES	1.389	.185	7.524	.174	.257
S1 ON					
MOTHED	-.126	.147	-.858	-.113	-.109
HOMERES	.091	.096	.950	.081	.120
S2 ON					
MOTHED	.436	.191	2.285	.185	.178
HOMERES	.289	.124	2.329	.123	.181

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Growth Model With Individually-Varying Times Of Observation And Random Slopes For Time-Varying Covariates

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Growth Modeling In Multilevel Terms

Time point t, individual i (two-level modeling, no clustering):

y_{ti} : repeated measures of the outcome, e.g. math achievement
a_{1ti} : time-related variable; e.g. grade 7-10
a_{2ti} : time-varying covariate, e.g. math course taking
x_i : time-invariant covariate, e.g. grade 7 expectations

Two-level analysis with individually-varying times of observation and random slopes for time-varying covariates:

$$Level\ 1: y_{ti} = \pi_{0i} + \pi_{1i}\, a_{1ti} + \pi_{2ti}\, a_{2ti} + e_{ti}, \quad (55)$$

$$Level\ 2: \begin{cases} \pi_{0i} = \beta_{00} + \beta_{01}\, x_i + r_{0i}, \\ \pi_{1i} = \beta_{10} + \beta_{11}\, x_i + r_{1i}, \\ \pi_{2i} = \beta_{20} + \beta_{21}\, x_i + r_{2i}. \end{cases} \quad (56)$$

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Growth Modeling In Multilevel Terms (Continued)

Time scores a_{1ti} read in as data (not loading parameters).

- π_{2ti} possible with time-varying random slope variances
- Flexible correlation structure for $V(e) = \Theta$ (T x T)
- Regressions among random coefficients possible, e.g.

$$\pi_{1i} = \beta_{10} + \gamma_1\, \pi_{0i} + \beta_{11}\, x_i + r_{1i}, \quad (57)$$
$$\pi_{2i} = \beta_{20} + \gamma_2\, \pi_{0i} + \beta_{21}\, x_i + r_{2i}. \quad (58)$$

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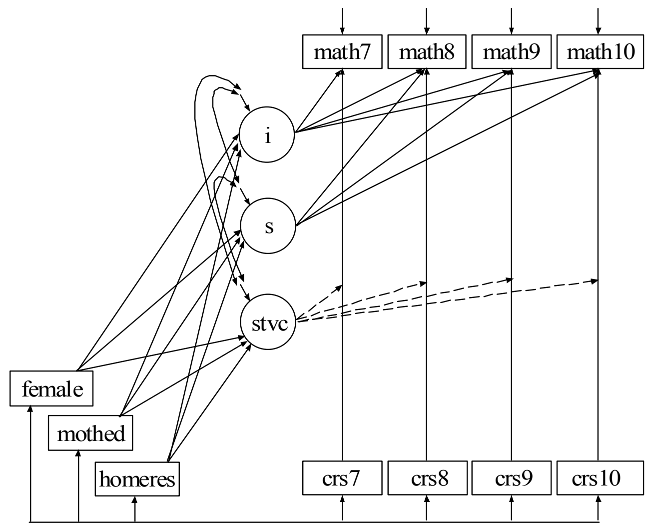

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Input For Growth Model With Individually Varying Times Of Observation

```
TITLE:      Growth model with individually varying times of
            observation and random slopes
DATA:       FILE IS lsaynew.dat;
            FORMAT IS 3F8.0 F8.4 8F8.2 3F8.0;
VARIABLE:   NAMES ARE math7 math8 math9 math10 crs7 crs8 crs9
            crs10 female mothed homeres a7-a10;

            ! crs7-crs10 = highest math course taken during each
            ! grade (0=no course, 1=low, basic, 2=average, 3=high.
            ! 4=pre-algebra, 5=algebra I, 6=geometry,
            ! 7=algebra II, 8=pre-calc, 9=calculus)

            MISSING ARE ALL (9999);
            CENTER = GRANDMEAN (crs7-crs10 mothed homeres);
            TSCORES = a7-a10;
```

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Input For Growth Model With Individually Varying Times Of Observation (Continued)

```
DEFINE:      math7 = math7/10;
             math8 = math8/10;
             math9 = math9/10;
             math10 = math10/10;
ANALYSIS:    TYPE = RANDOM MISSING;
             ESTIMATOR = ML;
             MCONVERGENCE = .001;
MODEL:       i s | math7-math10 AT a7-a10;
             stvc | math7 ON crs7;
             stvc | math8 ON crs8;
             stvc | math9 ON crs9;
             stvc | math10 ON crs10;
             i ON female mothed homeres;
             s ON female mothed homeres;
             stvc ON female mothed homeres;
             i WITH s;
             stvc WITH i;
             stvc WITH s;
OUTPUT:      TECH8;
```

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Output Excerpts For Growth Model With Individually Varying Times Of Observation And Random Slopes For Time-Varying Covariates

n = 2271

Tests of Model Fit

```
Loglikelihood

          H0 Value                      -8199.311

Information Criteria

          Number of Free Parameters            22
          Akaike (AIC)                  16442.623
          Bayesian (BIC)                16568.638
          Sample-Size Adjusted BIC      16498.740
             (n* = (n + 2) / 24)
```

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Output Excerpts For Growth Model With Individually Varying Times Of Observation And Random Slopes For Time-Varying Covariates (Continued)

Model Results

	Estimates	S.E.	Est./S.E.
I ON			
FEMALE	0.187	0.036	5.247
MOTHED	0.187	0.018	10.231
HOMERES	0.159	0.011	14.194
S ON			
FEMALE	-0.025	0.012	-2.017
MOTHED	0.015	0.006	2.429
HOMERES	0.019	0.004	4.835
STVC ON			
FEMALE	-0.008	0.013	-0.590
MOTHED	0.003	0.007	0.429
HOMERES	0.009	0.004	2.167
I WITH			
S	0.038	0.006	6.445
STVC WITH			
I	0.011	0.005	2.087
S	0.004	0.002	2.033

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Output Excerpts For Growth Model With Individually Varying Times Of Observation And Random Slopes For Time-Varying Covariates (Continued)

Intercepts			
MATH7	0.000	0.000	0.000
MATH8	0.000	0.000	0.000
MATH9	0.000	0.000	0.000
MATH10	0.000	0.000	0.000
I	4.992	0.025	198.456
S	0.417	0.009	47.275
STVC	0.113	0.010	11.416
Residual Variances			
MATH7	0.185	0.011	16.464
MATH8	0.178	0.008	22.232
MATH9	0.156	0.008	18.497
MATH10	0.169	0.014	12.500
I	0.570	0.023	25.087
S	0.036	0.003	12.064
STVC	0.012	0.002	5.055

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Random Slopes

- In single-level modeling random slopes β_i describe variation across individuals *i*,

$$y_i = \alpha_i + \beta_i x_i + \varepsilon_i, \quad (100)$$
$$\alpha_i = \alpha + \zeta_{0i}, \quad (101)$$
$$\beta_i = \beta + \zeta_{1i}, \quad (102)$$

Resulting in heteroscedastic residual variances

$$V(y_i \mid x_i) = V(\beta_i)\ x_i^2 + \theta \quad (103)$$

- In two-level modeling random slopes β_j describe variation across clusters *j*

$$y_{ij} = a_j + \beta_j x_{ij} + \varepsilon_{ij}, \quad (104)$$
$$a_j = a + \zeta_{0j}, \quad (105)$$
$$\beta_j = \beta + \zeta_{1j}. \quad (106)$$

A small variance for a random slope typically leads to slow convergence of the ML-EM iterations. This suggests respecifying the slope as fixed.

Mplus allows random slopes for predictors that are
- Observed covariates
- Observed dependent variables
- Continuous latent variables

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Computational Issues For Growth Models

- Decreasing variances of the observed variables over time may make the modeling more difficult
- Scale of observed variables – keep on a similar scale
- Convergence – often related to starting values or the type of model being estimated
  - Program stops because maximum number of iterations has been reached
    - If no negative residual variances, either increase the number of iterations or use the preliminary parameter estimates as starting values
    - If there are large negative residual variances, try better starting values
  - Program stops before the maximum number of iterations has been reached
    - Check if variables are on a similar scale
    - Try new starting values
- Starting values – the most important parameters to give starting values to are residual variances and the intercept growth factor mean
- Convergence for models using the | symbol
  - Non-convergence may be caused by zero random slope variances which indicates that the slopes should be fixed rather than random

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