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Latent Variable Growth Modeling with Multilevel Data

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Abstract

Growth modeling of multilevel data is presented within a latent variable framework that allows analysis with conventional structural equation modeling software. Latent variable modeling of growth considers a vector of observations over time for an individual, reducing the two-level problem to a one-level problem. Analogous to this, three-level data on students, time points, and schools can be modeled by a two-level growth model. An interesting feature of this two-level model is that contrary to recent applications of multilevel latent variable modeling, a mean structure is imposed in addition to the covariance structure. An example using educational achievement data illustrates the methodology.

1 Introduction

Longitudinal studies of growth in educational achievement typically use cluster sampling of students within schools. This gives rise to hierarchical data with three levels: student, time point, and school. With large numbers of students per school, ignoring the clustering of students within schools may give strongly distorted inference even with modest intraclass (school) correlations. While three-level modeling is well established with manifest dependent variables (see, e.g., Bock, 1989; Bryk & Raudenbush, 1992; Goldstein, 1987), less work has been done in the area of latent variables.

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#72

The general framework of latent variable modeling is particularly well suited to growth studies of educational achievement data. Longitudinal educational data is frequently collected at fixed time points corresponding to grades. The relevant time dimension for achievement is grade because this reflects the amount of learning that has taken place. In this way, all individuals have the same value on the time dimension at all measurement occasions. This special case offers a convenient simplification which puts the problem in a conventional latent variable framework.

Formulated as a latent variable growth model, the time dimension is transformed into a multivariate vector and the three-level data hierarchy is reduced to a two-level hierarchy. Two-level latent variable models have been studied in the context of covariance structure models, but the growth model also imposes a mean structure. Relative to conventional three-level modeling, the two-level latent variable growth model formulation offers considerable flexibility in the modeling. Using the Muthén (1989) approach to multilevel data, maximum-likelihood estimation under normality assumptions is carried out with conventional structural equation modeling software.

The methodology is illustrated using mathematics achievement data on students in grades 7 - 10 from the Longitudinal Study of American Youth. Here, an average of about 50 students per school are observed in 50 schools for mathematics achievement scores having intraclass correlations of 0.15-0.20.

Sections 2 and 3 will review growth modeling using a simple random sample of individuals. Subsequent sections add the multilevel complication of analyzing individuals observed within schools.

2 Random coefficient growth modeling: Two-level hierarchical modeling

Random coefficient growth modeling (see, e.g. Laird & Ware, 1982) goes beyond conventional structural equation modeling of longitudinal data and its focus on auto-regressive models (see, e.g. Jöreskog & Sorbom, 1977; Wheaton, Muthén, Alwin & Summers, 1977) to describe individual differences in growth. The modeling is of the following type. Consider the indices

Individual i	:	$i = 1, 2, ..., n$
Time point t	:	$t = 1, 2, ..., T$

and the variables

x_{it}	:	time-related variable (age, grade)
w_{it}	:	time-varying covariate
z_i	:	time-invariant covariate

and the model

$$y_{it} = \alpha_i + \beta_i x_{it} + \gamma_t w_{it} + \zeta_{it} \tag{1}$$

where

$$\begin{cases} \alpha_i = \alpha + \pi_\alpha z_i + \delta_{\alpha i} \\ \beta_i = \beta + \pi_\beta z_i + \delta_{\beta i} \end{cases} \tag{2}$$

Here, ζ_{it} is a residual which is independent of x, $\delta_{\alpha i}$ and $\delta_{\beta i}$, $\delta_{\alpha i}$ and $\delta_{\beta i}$ are correlated, and $\delta_{\alpha i}$ and $\delta_{\beta i}$ are independent of z. Here, the special case of linear growth is described, where α_i is the random intercept or initial status and β_i is the random slope or growth rate. Non-linear growth can also be accomodated.

An important special case that will be the focus of this chapter is where the time-related variable $x_{it} = x_t$. An example of this is educational achievement studies where x_t corresponds to grade. The x_t values are for example 0, 1, 2, ..., $T-1$ for linear growth.

3 Latent variable modeling formulation of growth

As shown in Meredith and Tisak (1984, 1990), the random coefficient model of the previous section can be formulated as a latent variable model (for applications in psychology, see McArdle & Epstein, 1987; for applications in education, see Muthén 1993 and Willett & Sayer, 1993; for applications in mental health, see Muthén, 1983, 1991). The basic idea can be simply described as follows. In equation 1, α_i is a latent variable varying across individuals. Assuming the special case of $x_{it} = x_t$, the x variable becomes a constant which multiplies a second latent variable β_i. The equation 1 outcome variable y_{it} for individual i is now reformulated as a $T \times 1$ vector for individual i.

The model may be shown as in the path diagram of Figure 1. Note for example that the constants of x_t are the coefficients for the influence of the β factor on the y variables. This makes it clear that non-linear growth can be accomodated by estimating the x_t coefficients, e.g. holding the first two values fixed at 0 and 1, respectively, for identification purposes.

Given the path diagram in Figure 1, input specifications can be given for conventional structural equation modeling software.

The general SEM framework may be described as

$$\begin{cases} \mathbf{y}^* = \nu + \Lambda\eta + \epsilon \\ B\eta = \kappa + \zeta \end{cases} \tag{3}$$

where the mean and variance of $\mathbf{y}^*$,

$$\begin{cases} E(\mathbf{y}^*) = \nu + \Lambda(I-B)^{-1}\kappa \\ V(\mathbf{y}^*) = \Lambda(I-B)^{-1}\Psi(I-B)^{-1}\Lambda' + \Theta \end{cases} \tag{4}$$

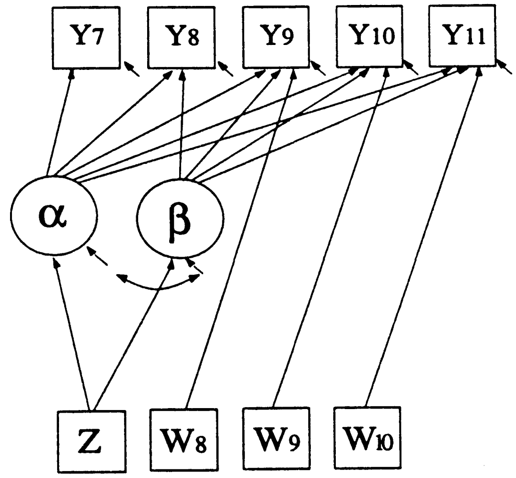

Figure 1: Latent Variable Growth Model Formulation

see, for example, Muthén (1989).

Translating Figure 1 into this general model, we have $\mathbf{y}^{*\prime} = (y_7, \cdots, y_{11}, z,\ w_8,\ w_9,\ w_{10})$, $\eta' = (\mathbf{y}^{*\prime},\ \alpha, \beta)$,

$$\nu = 0,\ \ \Lambda = [\mathrm{I}\ 0],\ \ \epsilon = 0,\ \ B = \begin{bmatrix} B_{11} & 1 & \mathbf{x} \\ B_{21} & 0 & 0 \end{bmatrix} \Psi = \begin{bmatrix} \Psi_{y^*} & 0 \\ 0 & \Psi_{\alpha,\beta} \end{bmatrix}$$

and $\kappa' = (0, \cdots, 0, E(z), E(w_8), E(w_9), E(w_{10}), E(\alpha), E(\beta))$.

The growth model imposes a structure on the mean vector and covariance matrix. It is clear that the Figure 1 model can be easily generalized to applications with multiple indicators of latent variable constructs instead of single outcome measurements y at each time point. The covariates may also be latent variables with multiple indicators. Muthén (1983) showed that binary outcome measures y can also be handled when the $y*$ variables are measured dichotomously. In the continuous variable case, maximum-likelihood estimation is the usual estimator in SEM. Estimates may also be obtained for the individual growth curves by estimating the individual values of the intercept and slope factors α and β. This relates to Empirical Bayes estimation in the conventional growth literature (see, e.g. Bock, 1989).

4 A three-level hierarchical model

We will now add the complication of cluster sampling to the growth modeling. Here, data are obtained on individuals observed within groups. Such hierarchical data are naturally obtained in educational studies where students are sampled within randomly sampled schools. It is well-known that ignoring the hierarchical nature of the data and applying techniques developed for simple random samples distorts the standard errors of estimates and the chi-square test of model fit (see, e.g., Muthén & Satorra, 1993). Standard errors are deflated and chi-square values are inflated. This is particularly pronounced when the intraclass correlations are large and the number of individuals within each group is large. The following latent variable methodology draws on Muthén (1994a).

Conventional random-coefficient modeling for such three-level data is described e.g. in Goldstein (1987) and Bryk and Raudenbush (1992). For

Individual	:	$i = 1, 2, ..., n$
Time	:	$t = 1, 2, ..., T$
Group (School)	:	$g = 1, 2, ..., G$

consider the growth model, again expressed for the special case of $x_{it} = x_t$,

$$y_{itg} = \alpha_{ig} + x_t \beta_{ig} + \zeta_{itg} \tag{5}$$

where for simplicity there are no covariates

$$\begin{cases} \alpha_{ig} = \alpha_g + \delta_{\alpha ig} \\ \beta_{ig} = \beta_g + \delta_{\beta ig} \end{cases} \tag{6}$$

$$\begin{cases} \alpha_g = \alpha + \delta_{\alpha g} \\ \beta_g = \beta + \delta_{\beta g} \end{cases} \tag{7}$$

5 A two-level formulation of multilevel growth

In the case of growth modeling using a simple random sample of individuals, it was possible to translate the growth model from a two-level model to a one-level model by considering a $T \times 1$ vector of outcome variable for each individual. Analogously, we may reduce the three-level model of the previous section to two levels as follows.

$$\mathbf{y}_{ig} = \begin{pmatrix} y_{i1g} \\ \vdots \\ y_{iTg} \end{pmatrix} = [1\ \mathbf{x}] \begin{pmatrix} \alpha_{ig} \\ \beta_{ig} \end{pmatrix} + \zeta_{ig} \tag{8}$$

which may be expressed in five terms

$$\mathbf{y}_{ig} = [1\ \mathbf{x}] \begin{pmatrix} \alpha \\ \beta \end{pmatrix} + [1\ \mathbf{x}] \begin{pmatrix} \delta_{\alpha g} \\ \delta_{\beta g} \end{pmatrix} + \zeta^*_g + [1\ \mathbf{x}] \begin{pmatrix} \delta_{\alpha ig} \\ \delta_{\beta ig} \end{pmatrix} + \zeta^*_{ig} \tag{9}$$

The first term represents the mean as a function of the mean of the initial status and the mean of the growth rate. The second and third terms correspond to between-group (school) variation. The fourth and fifth terms correspond to within-group variation.

6 Latent variable modeling of two-level growth data

Assume $g = 1, 2, ..., G$ independently observed groups with $i = 1, 2, ..., N_g$ individual observations within group g. Let z and y represent group- and individual-level variables, respectively. Arrange the data vector for which independent observations are obtained as

$$\mathbf{d}'_g = (\mathbf{z}'_g, \mathbf{y}'_{g1}, \mathbf{y}'_{g2}, ..., \mathbf{y}'_{gN_g}) \tag{10}$$

where we note that the length of $\mathbf{d}_g$ varies across groups. The mean vector and covariance matrix are,

$$\mu'_{\mathbf{d}_g} = [\mu'_{\mathbf{z}}, \mathbf{1}'_{N_g} \otimes \mu'_{\mathbf{y}}] \tag{11}$$

$$\Sigma_{\mathbf{d}_g} = \begin{pmatrix} \Sigma_{\mathbf{zz}} & \text{symmetric} \\ \mathbf{1}_{N_g} \otimes \Sigma_{\mathbf{yz}} & \mathbf{I}_{N_g} \otimes \Sigma_w + \mathbf{1}_{N_g}\mathbf{1}'_{N_g} \otimes \Sigma_B \end{pmatrix} \tag{12}$$

Assuming multivariate normality of $\mathbf{d}_g$, the ML estimator minimizes the function

$$F = \sum_{g=1}^{G} \{\log|\Sigma_{\mathbf{d}_g}| + (\mathbf{d}_g - \mu_{\mathbf{d}_g})'\Sigma_{\mathbf{d}_g}^{-1}(\mathbf{d}_g - \mu_{\mathbf{d}_g})\} \tag{13}$$

which may be simplified as (cf. McDonald & Goldstein, 1989; Muthén, 1989, 1990)

$$\begin{aligned} F = \; & \textstyle\sum_d^D G_d\{\ln|\Sigma_{B_d}| + tr[\Sigma_{B_d}^{-1}(S_{B_d} + N_d(\bar{v}_d - \mu)(\bar{v}_d - \mu)')]\} \\ & + (N - G)\{\ln|\Sigma_W| + tr[\Sigma_W^{-1} S_{PW}]\} \end{aligned} \tag{14}$$

where

$$\Sigma_{B_d} = \begin{pmatrix} N_d\Sigma_{zz} & \text{symmetric} \\ N_d\Sigma_{yz} & \Sigma_W + N_d\Sigma_B \end{pmatrix}$$

$$S_{B_d} = N_d G_d^{-1} \sum_{k=1}^{G_d} \begin{pmatrix} z_{dk} - \bar{z}_d \\ \bar{y}_{dk} - \bar{y}_d \end{pmatrix} [(z_{dk} - \bar{z}_d)'(\bar{y}_{dk} - \bar{y}_d)']$$

$$\bar{v}_d - \mu = \begin{pmatrix} \bar{z}_d - \mu_z \\ \bar{y}_d - \mu_y \end{pmatrix}$$

$$S_{PW} = (N - G)^{-1} \sum_{g=1}^{G} \sum_{i=1}^{N_g} (y_{gi} - \bar{y}_g)(y_{gi} - \bar{y}_g)'$$

Here, D denotes the number of groups of a distinct size, d is an index denoting a distinct group size category with group size N_d, G_d denotes the number of groups of that size, S_{B_d} denotes a between-group sample covariance matrix, and S_{PW} is the usual pooled-within sample covariance matrix.

Muthén (1989,1990) pointed out that the minimization of the ML fitting function defined by equation 14 can be carried out by conventional structural equation modeling software, apart from a slight modification due to the possibility of singular sample covariance matrices for groups with small G_d values. A multiple-group analysis is carried out for $D+1$ groups, the first D groups having sample size G_d and the last group having sample size $N-G$. Equality constraints are imposed across the groups for the elements of the parameter arrays μ, Σ_{zz}, Σ_{yz}, Σ_B, and Σ_W. To obtain the correct chi-square test of model fit, a separate H_1 analysis needs to be done (see Muthén, 1990 for details).

Muthén (1989, 1990) also suggested an ad hoc estimator which considered only two groups,

$$
\begin{aligned}
F' = \; & G\{\ln|\Sigma_{B_c}| + tr[\Sigma_{B_c}^{-1}(S_B + c(\bar{v}-\mu)(\bar{v}-\mu)')]\} \\
& +(N-G)\{\ln|\Sigma_W| + tr[\Sigma_W^{-1}S_{PW}]\}
\end{aligned} \tag{15}
$$

where the definition of the terms simplifies relative to equation 14 due to ignoring the variation in group size, dropping the d subcript, and using $D=1$, $G_d=G$, and $N_d=c$, where c is the average group size (see Muthén, 1990 for details). When data are balanced, i.e. the group size is constant for all groups, this gives the ML estimator. Experience with the ad hoc estimator for unbalanced data indicates that the estimates, and also the standard errors and chi-square test of model fit, are quite close to those obtained by the true ML estimator. This experience, however, is limited to models that do not have a mean structure and is therefore not directly applicable to growth models.

In line with Muthén (1989, 1990), Figure 2 shows a path diagram which is useful in implementing the estimation using F or F'. The figure corresponds to the case of no covariates given in equations 5 - 7 and 9. It shows how the covariance structure

$$
\Sigma_W + N_d\Sigma_B \tag{16}
$$

can be represented by latent variables, introducing a latent between-level variable for each outcome variable y. These latent between-level variables may also be related to observed between-level variables z_g. The between-level α and β factors correspond to the δ_{α_g} and δ_{β_g} residuals of equation 7. The within-level α and β factors correspond to the $\delta_{\alpha_{ig}}$ and $\delta_{\beta_{ig}}$ residuals of equation 6. From equation 9 it is clear that the the influence from these two factors is the same on the between side as it is on the within side. In Figure 2, the Σ_B structure is identical to the Σ_W structure. A strength of the latent variable approach is that this equality assumption can easily be relaxed. For example, it may not be necessary include between-group variation in the growth rate.

Specific to the growth model is the mean structure imposed on μ in equation 14, where μ represents the means of group- and individual-level variables. In the specific growth model shown in Figure 2, the mean structure arises from the five observed variable means being expressed as functions of the means of the α and β factors, here applied on the between side, see equation 9. Equation 14 indicates that the means need to be included on the between side of Figure 2 given that the mean term of F is scaled by

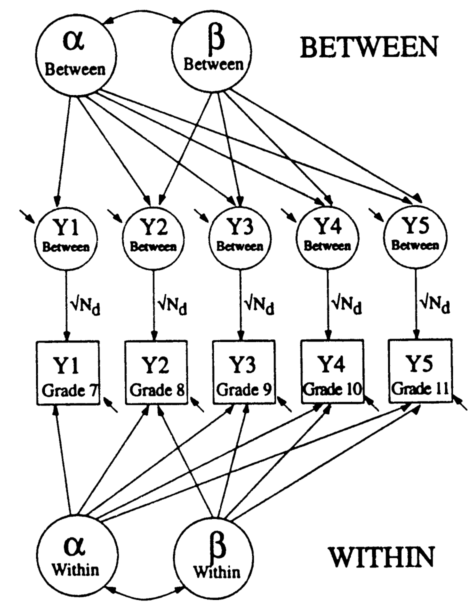

Figure 2: Two-level Latent Variable Growth Model Formulation

N_d, while the means on the within side are fixed at zero. This implies that dummy zero means are entered for the within group. The degrees of freedom for the chi-square test of model fit obtained in conventional software then needs to be reduced by the number of y variables.

Further details and references on latent variable modeling with two-level data are given in Muthén (1994b), also giving suggestions for analysis strategies. Software is available from the author for calculating the necessary sample statistics, including intraclass correlations.

7 Analysis of longitudinal achievement data

Mathematics achievement data from four time points will be used to illustrate the methodology described above. Data are from grades 7-10 of The Longitudinal Study of American Youth (LSAY) and were collected every Fall starting in 1987. The data we will analyze consists of a total sample of 2,488 students in 50 schools. The average school size is therefore 49. The intraclass correlations for math achievement for the four grades are estimated as 0.19, 0.16, 0.16, and 0.14.

The ML estimator of equation 14 will be reported. As a comparison, the ad hoc estimator of equation 15 will also be given for the final model. As a further comparison, the conventional ML estimator assuming a simple random sample will also be reported.

As an initial model, the linear growth model using $x_t = 0, 1, 2, 3$ was used, resulting in a chi-square value of 18.69 with 5 df ($p < 0.01$). It is interesting to note that conventional modeling gives a higher chi-square value with fewer degrees of freedom, 41.85 with 2 df. This illustrates the inflation of chi-square values when ignoring the clustering so that proper models might be inadvertently rejected.

To investigate if the misfit is due to the assumption of equal covariance structure on the between and within sides, the Σ_B structure was relaxed and an unrestricted Σ_B matrix was used (the latent between-level variables are then freely correlated). This improved the fit, but the improvement was not significant on the 1% level (the chi-square value was 9.17 with 2 df and $p = 0.01$; the chi-square difference test has a chi-square of 9.52 with 3 df, $p > 0.02$).

It was decided to retain the assumption of equal covariance structure on the between and within sides and instead relax the linearity assumption, $x_t = 0, 1, 2, 3$ by letting the last two values be free to be estimated. This non-linear growth model gave a strongly significant improvement over the initial model (the chi-square value was 6.27 with 3 df and p=0.10; the chi-square difference value was 12.43 with 2 df and $p < 0.001$). The estimates from this model are given in Table 1 below.

Table 1 gives estimates from three procedures: using the incorrect, conventional ML estimator, using the correct multilevel ML estimator, and using the ad hoc multilevel estimator. Growth scores refers to the values of x_t.

The multilevel ML approach shows that the estimated growth score for grade 9 is 2.505 which is larger than the linear growth value of 2.0. This means that growth is accelerated during grade eight when many new topics are introduced. The growth rate mean is positive as expected. The variation in both the initial status and the growth rate are significantly different from zero on the within (student) level, but that is not the case for the growth rate variation on the between (school) level. This indicates that schools vary more with respect to the intake of students than how the students develop in achievement over time. Variation in intake may depend on socio-economic neighborhood factors. The estimates show that about 18 % of the total variation in initial status is due to across-school variation. This is in line with the observed intraclass correlations. The within-level correlation between initial status and growth rate is 0.43.

Comparing the conventional ML approach with the multilevel ML approach, it is seen that the parameter estimates of the growth score and the growth rate mean are close. As expected, however, the standard errors for these estimates are too low for the conventional approach. The parameter estimates of initial status and growth rate variances and covariance are quite different. The conventional analysis estimates the total variance, adding between and within variation. The correlation between initial status and growth rate is 0.33, which is lower than the multilevel value of 0.43 for the within level.

Table 1: Estimates for three approaches to multilevel growth modeling

	Conventional ML Analysis		ML Multilevel Analysis		Ad Hoc Multilevel Analysis	
	Estimate	SE	Estimate	SE	Estimate	SE
Initial Status Mean	51.311	0.073	51.125	0.120	51.343	0.129
Growth scores						
Grade 8	1.00*	-	1.00*	-	1.00*	-
Grade 9	2.499	0.095	2.508	0.184	2.541	0.202
Grade10	3.636	0.153	3.635	0.290	3.691	0.318
Growth rate mean	2.443	0.112	2.433	0.229	2.400	0.244
Variance of						
Initial Status	87.032	4.795	66.202	4.277	66.248	4.241
Growth Rate	1.678	0.601	1.519	0.553	1.465	0.536
Covariance of						
Initial Status, Growth Rate	3.928	1.406	4.352	1.357	4.338	1.337
Residual variance of y						
Grade 7	12.614	4.127	15.118	3.858	15.272	3.827
Grade 8	15.964	2.210	18.964	2.085	19.099	2.099
Grade 9	25.956	2.844	22.460	2.839	22.474	2.886
Grade 10	43.231	5.393	38.983	5.531	38.833	5.683
Between Variance of						
Initial status			14.444	3.348	15.406	3.560
Growth Rate			0.094	0.068	0.112	0.077
Between Covariance of						
Initial status, Growth Rate			0.839	0.339	0.909	0.369
Between Residual variance of y						
Grade 7			1.133	0.439	1.342	0.501
Grade 8			0.718	0.278	0.850	0.317
Grade 9			0.220	0.232	0.331	0.271
Grade 10			0.563	0.443	0.766	0.513

*Fixed

Comparing the ad hoc multilevel approach with the ML multilevel approach, it is seen that the parameter estimates are on the whole reasonably close. For example, the ratio of between to total variance in initial status is 19 % compared to 18 % and the within-level correlation between initial status and growth rate is 0.44 compared to 0.43. The standard errors also seem close enough to serve as a rough approximations. The approximation to the chi-square value is 6.51 compared to 6.27 for the ML multilevel approach.

8 Conclusions

This chapter has shown that for an important special case it is possible to use the framework of latent variable modeling to carry out quite general growth modeling of three-level data. Maximum-likelihood estimation under normality assumptions can be carried out with existing structural equation modeling software. A simpler, ad hoc, estimator appears to work well and may be useful at least for initial model exploration. This provides a useful tool for educational research with longitudinal data on students observed within schools and other data with similar structures.

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