Long Longitudinal Data Modeling

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SMEP Sells Award presentation, October 12, 2018

I thank Tihomir Asparouhov, Ellen Hamaker, and Marten Schultzberg for helpful comments and Noah Hastings for excellent assistance

Outline

- A bit of history
- Features of long longitudinal data modeling
- Regression analysis: A smoking cessation example
- Growth/trend analysis
- Longitudinal factor analysis
- Current activities
  - Two-part, two-level longitudinal analysis
  - Modeling cycles by sine-cosine
  - Very long longitudinal data

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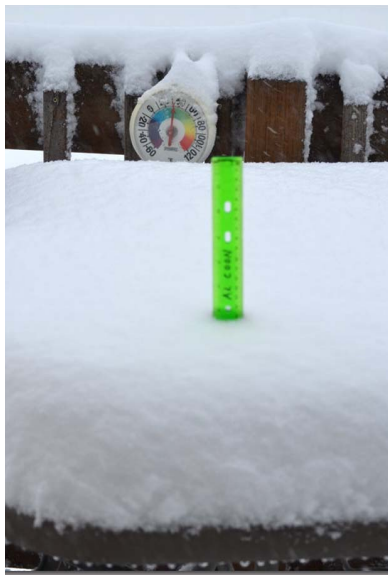
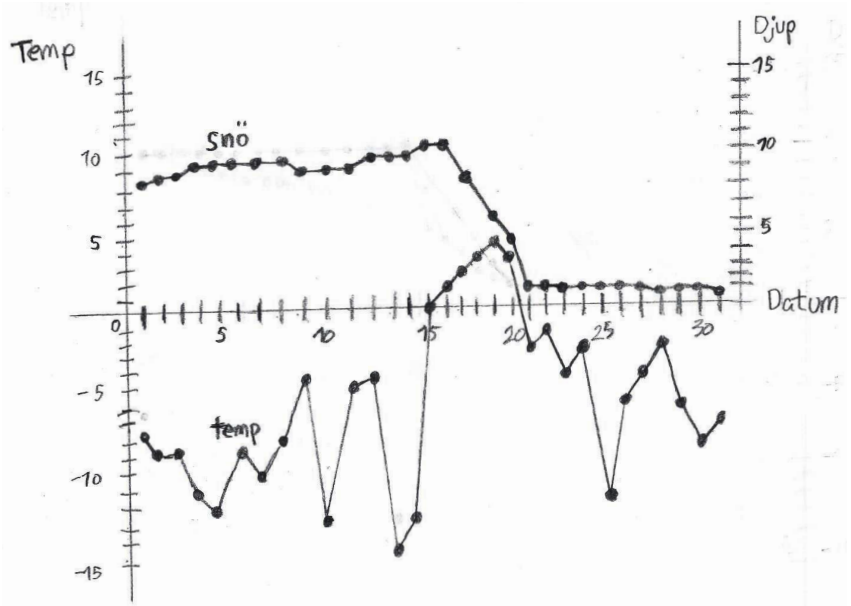
Temperature and Snow Depth
Bivariate Time-Series Data with a Lagged Effect
- Implications for Sledding





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Department of Statistics, Uppsala University, Early 70's

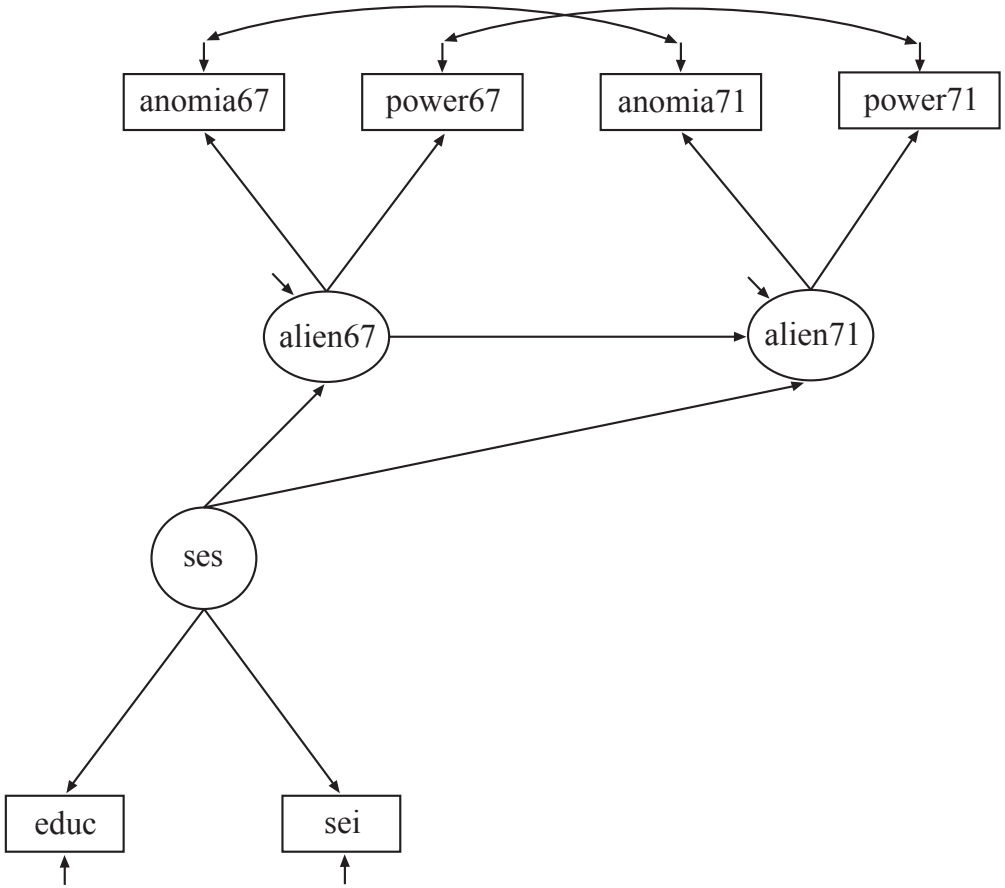
- Bengt's grad school term project related to time-series analysis:
  - Repeated measurements on respiratory problems of 7 dogs
  - Fortran program for ML estimation with autoregressive and heteroscedastic residuals

UPPSALA
UNIVERSITET

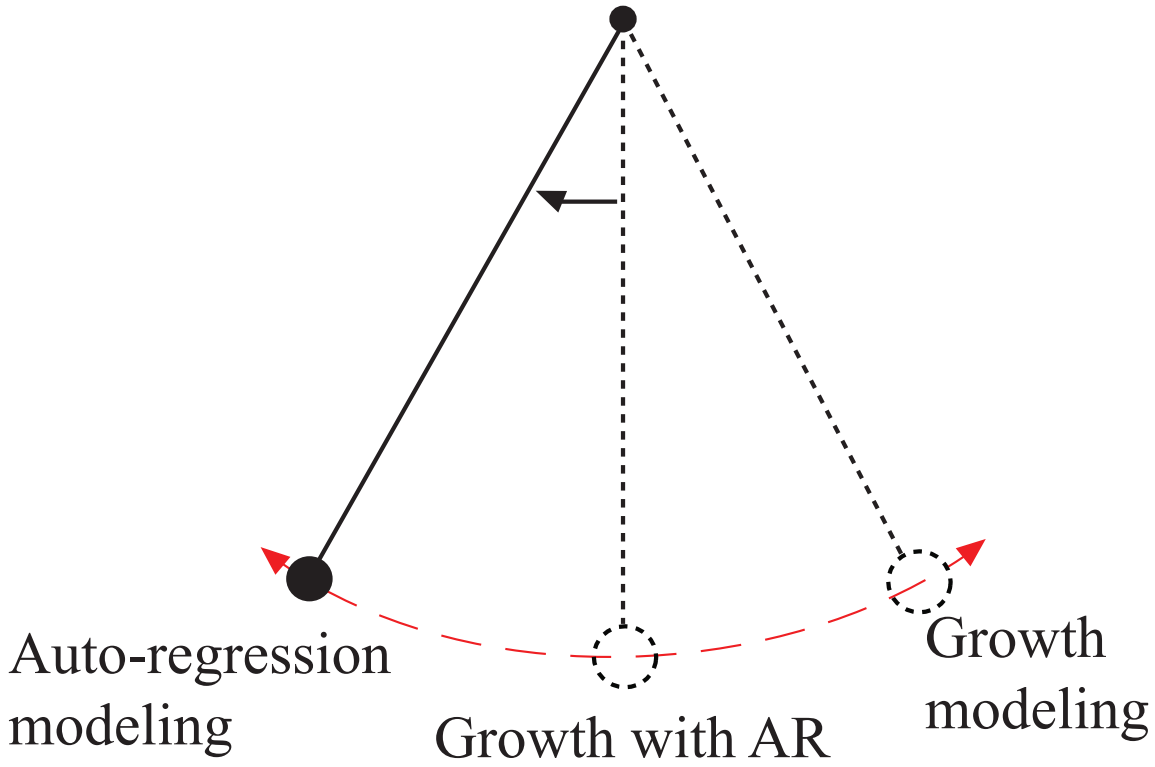
University of Wisconsin - Madison 1974-75

Wheaton, B., Muthén, B., Alwin, D., & Summers, G. (1977). Assessing reliability and stability in panel models. In D. R. Heise (Ed.), Sociological Methodology 1977 (pp. 84 - 136). San Francisco: Jossey-Bass, Inc.

Wheaton et al. 1977



The Pendulum of Longitudinal Modeling



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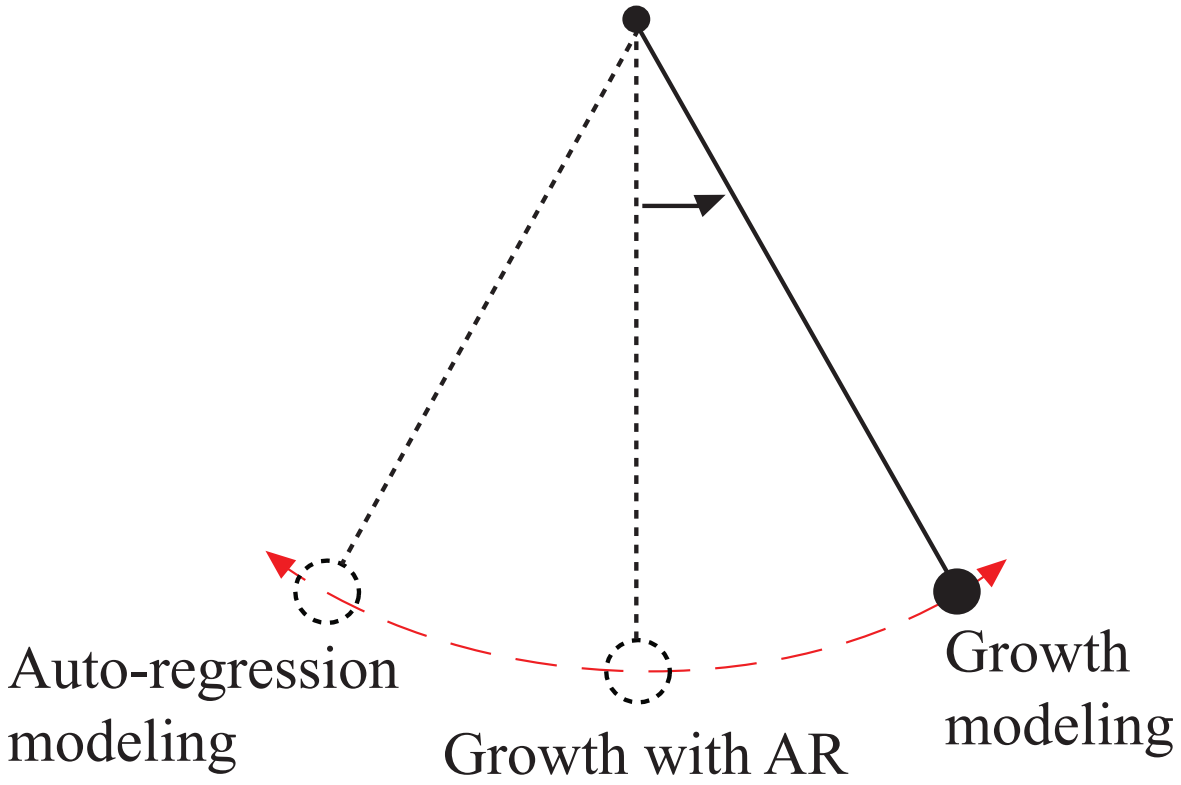
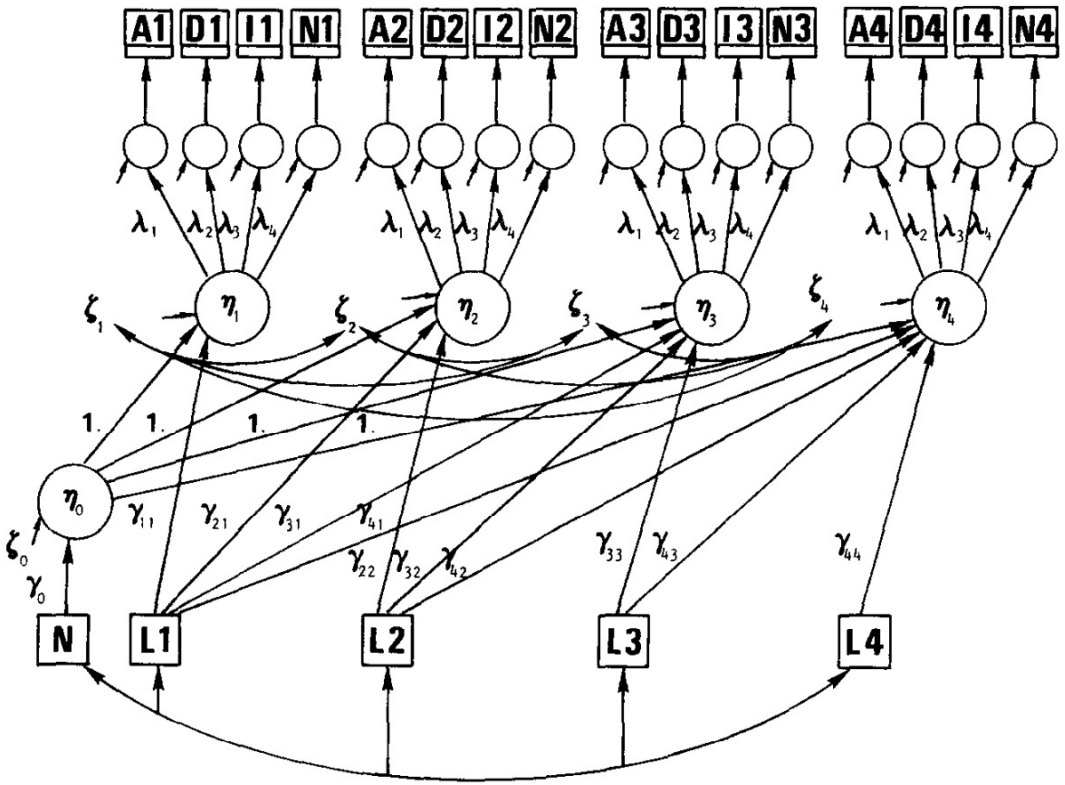


UCLA Early 80's



Muthén (1983). Latent variable structural equation modeling with categorical data. Journal of Econometrics, 22, 43-65.

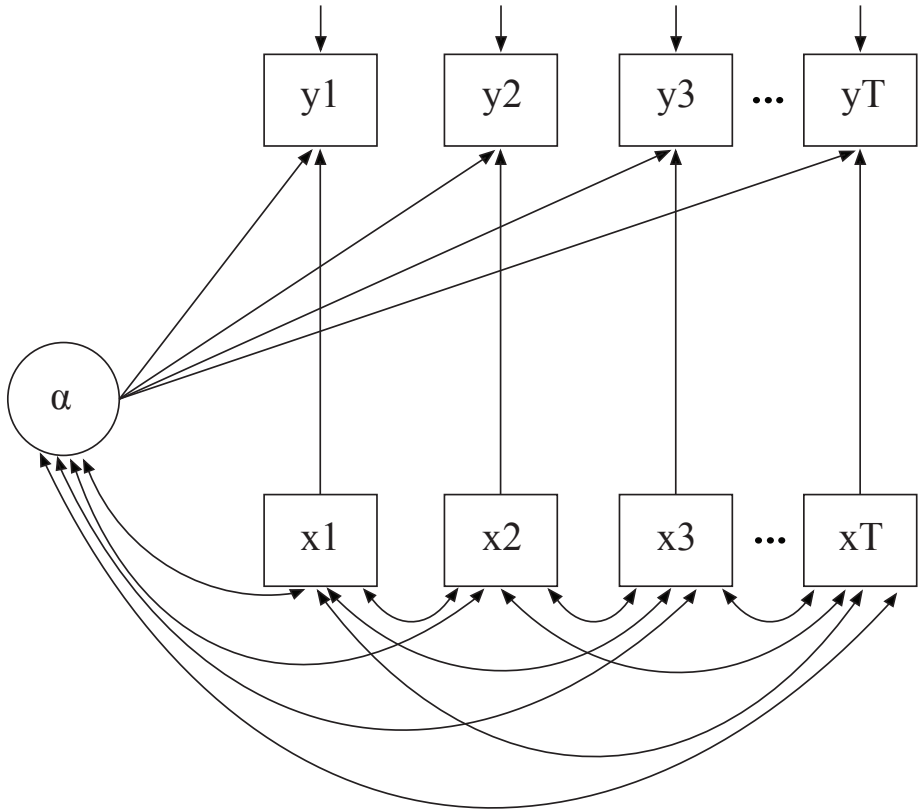
Muthén (1984). A general structural equation model with dichotomous, ordered categorical, and continuous latent variable indicators. Psychometrika, 49, 115-132.

Longitudinal IRT Contributions 30+ Years Later

- Muthén & Asparouhov (2016). Multi-dimensional, multi-level, and multi-timepoint item response modeling. In van der Linden, Handbook of Item Response Theory. Volume One. Models, pp. 527-539. Boca Raton: CRC Press
- FAQ at www.statmodel.com: "Estimator choices with categorical outcomes": WLSMV, ML, and Bayes
- Asparouhov & Muthén (2016). General random effect latent variable modeling: Random subjects, items, contexts, and parameters. In Harring, Stapleton, & Beretvas, (Eds.), Advances in multilevel modeling for educational research: Addressing practical issues found in real-world applications (pp. 163-192). Charlotte, NC: Information Age Publishing, Inc

Fixed Effect vs Random Effect Debate for Panel Data

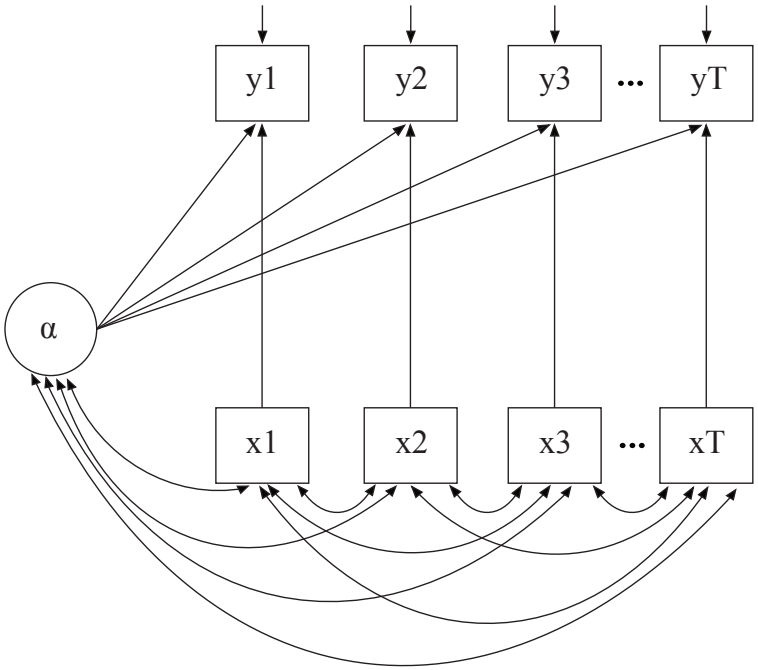
$$y_{it} = \beta_0 + \alpha_i + \beta x_{it} + e_{it}$$



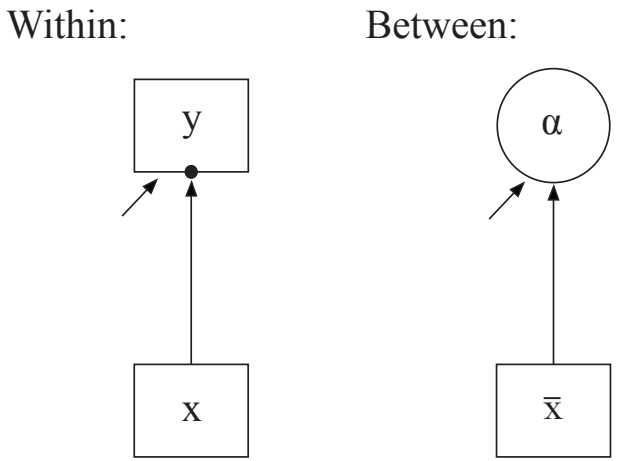
Fixed Effect vs Random Effect Debate: Equivalent Models

$$y_{it} = \beta_0 + \alpha_i + \beta x_{it} + e_{it}$$

Single-level, wide representation:

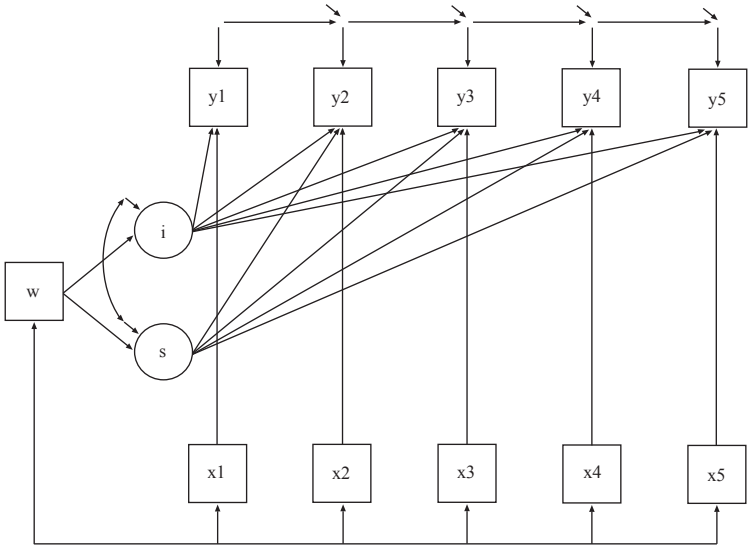


Two-level, long representation:

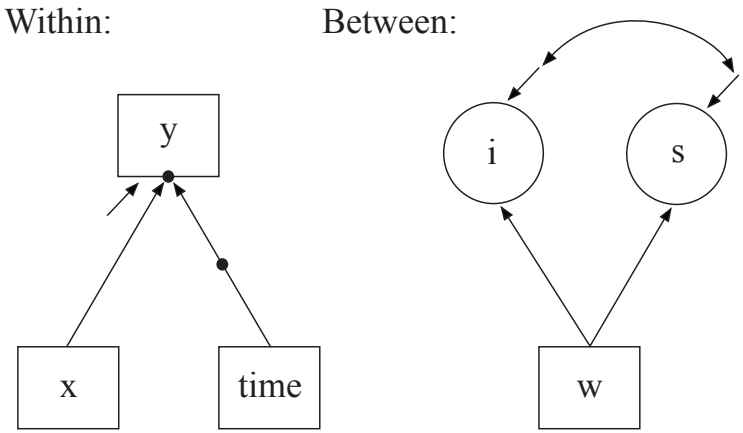


Hamaker & Muthén (2018). The fixed versus random effects debate and how it relates to centering in multilevel modeling. Submitted.

Single-level, wide representation:



Two-level, long representation:



Growth Mixture Modeling for Short Longitudinal Data

- Muthén & Shedden (1999). Finite mixture modeling with mixture outcomes using the EM algorithm. Biometrics, 55, 463-469.
- Muthén, Asparouhov, Hunter & Leuchter (2011). Growth modeling with non-ignorable dropout: Alternative analyses of the STAR*D antidepressant trial. Psychological Methods, 16, 17 - 33
- Muthén & Asparouhov (2015). Growth mixture modeling with non-normal distributions. Statistics in Medicine, 34:6, 1041 1058.
  - Allows for a non-normal within-class distribution using skew-t (using 2 more parameters than the normal distribution). BMI data

Survival Mixture Analysis

- Discrete-time survival:
  - Muthén & Masyn (2005). Discrete-time survival mixture analysis. Journal of Educational and Behavioral Statistics, 30, 27-58
- Continuous-time survival:
  - Asparouhov, Masyn & Muthén (2006). Continuous time survival in latent variable models. Proceedings of the Joint Statistical Meeting in Seattle, August 2006. ASA section on Biometrics, 180-187
  - Muthén, Asparouhov, Boye, Hackshaw & Naegeli (2009). Applications of continuous-time survival in latent variable models for the analysis of oncology randomized clinical trial data using Mplus

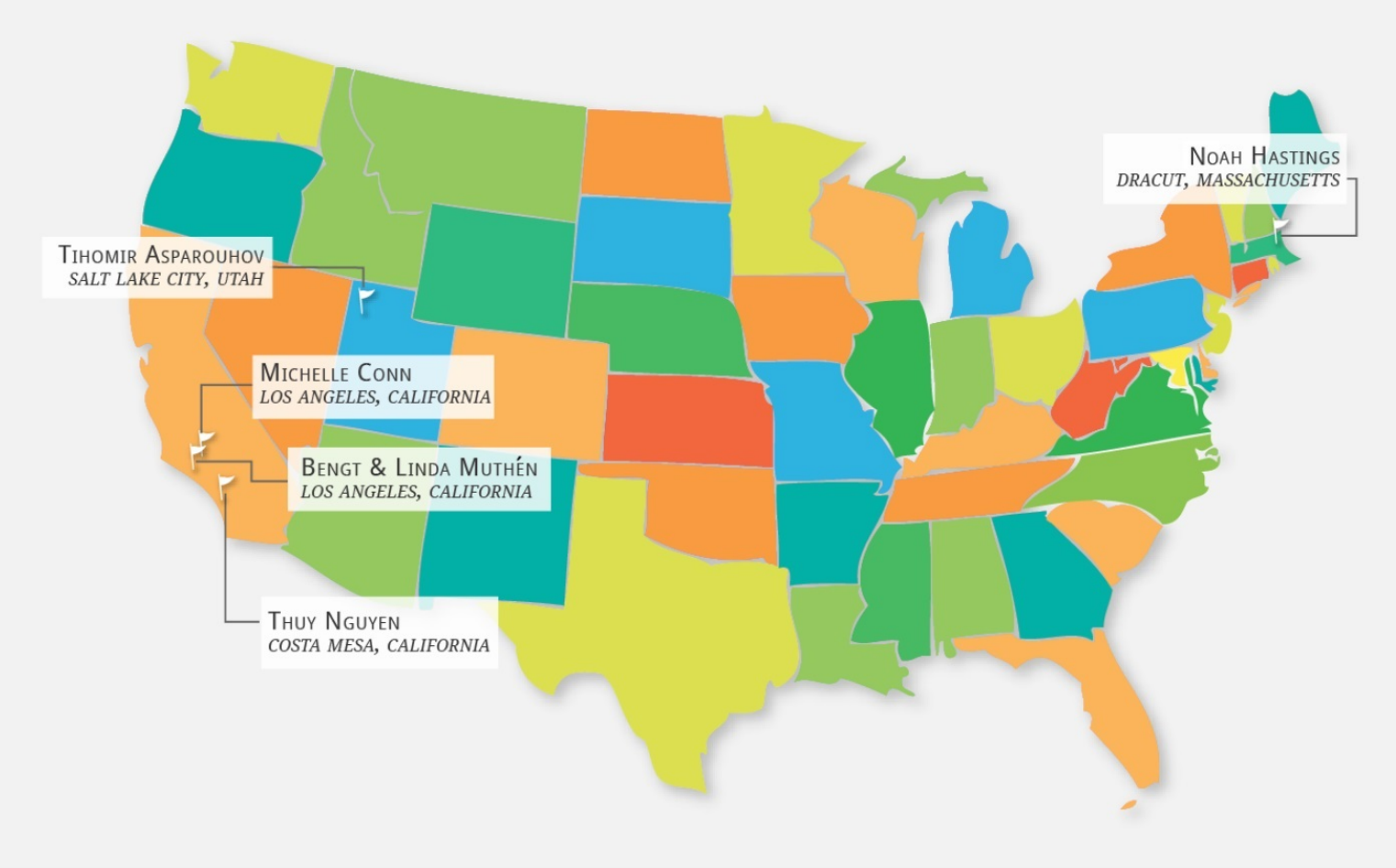
The General Latent Variable Framework of Mplus

- 1995 NIH SBIR: 1998 launch of Mplus
- Merging continuous and categorical latent variables
  - Continuous latent variables:
    - Factors measured by multiple indicators, random effects, frailties, liabilities, latent response variables with missing
  - Categorical latent variables:
    - Latent classes, finite mixtures, latent response variable categories with missing data
- General SEM structure on each of multiple levels for continuous, categorical, count, and censored observed variables
- Estimation by WLS, ML, and Bayes
- Different model types freely combined

General Latent Variable Modeling: Integration of a Multitude of Analyses

- Exploratory factor analysis
- Structural equation modeling
- Item response theory analysis
- Growth modeling
- Latent class analysis
- Latent transition analysis (Hidden Markov modeling)
- Growth mixture modeling
- Survival analysis
- Missing data modeling
- Multilevel analysis
- Complex survey data analysis
- Causal inference
- Time series analysis

The Mplus Team: 4 + 2



Overview papers

- Muthén (2002). Beyond SEM: General latent variable modeling. Behaviormetrika, 29, 81-117
- Muthén (2008). Latent variable hybrids: Overview of old and new models. In Hancock & Samuelsen (Eds.), Advances in latent variable mixture models, pp. 1-24. Information Age Publishing, Inc
- Muthén & Asparouhov (2009). Growth mixture modeling: Analysis with non-Gaussian random effects. In Fitzmaurice, Davidian, Verbeke & Molenberghs, G. (eds.), Longitudinal Data Analysis, pp. 143-165. Boca Raton: Chapman & Hall/CRC Press.

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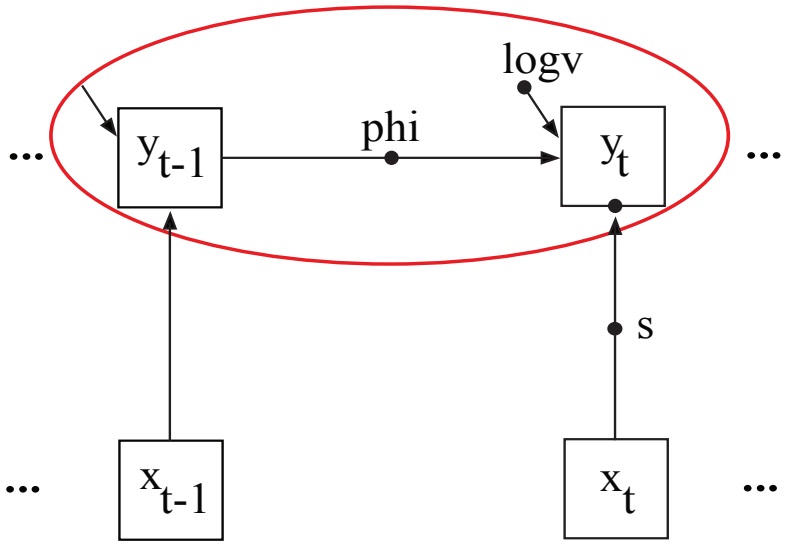
Intensive Longitudinal Data: EMA, ESM, AA, Diary

- Many time points: T = 50 - 1000
  - Wide format analysis not feasible/practical
  - Long format analysis needed
- Measurements are typically close in time
  - Random effects are not sufficient to represent correlation across time for subjects
  - Auto-regression needed as well
- Two-level and cross-classified time series analysis (Dynamic SEM; DSEM, RDSEM)
  - General SEM structure on each level
  - Random effects for intercepts, slopes, ARs, and variances
  - As T and N increase, increasingly more flexible models can be estimated
  - Bayesian estimation needed

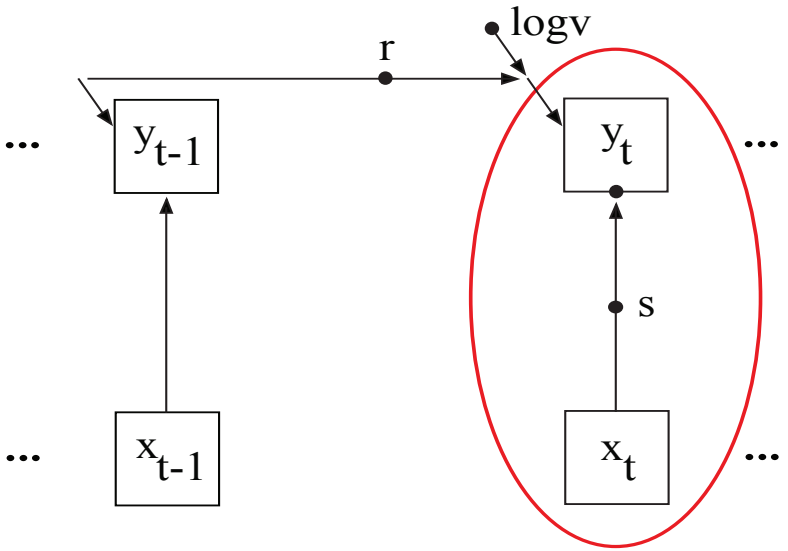
Our Recent Papers on Long Longitudinal Data Analysis: Posted at statmodel.com/TimeSeries with Topic 12-13 Videos

- Asparouhov, Hamaker & Muthén (2017). Dynamic latent class analysis. Structural Equation Modeling, 24:2, 257-269
- Asparouhov, Hamaker & Muthén (2018). Dynamic structural equation models. Structural Equation Modeling, 25:3, 359-388
- Hamaker, Asparouhov, Brose, Schmiedek & Muthén (2018). At the frontiers of modeling intensive longitudinal data: Dynamic structural equation models for the affective measurements from the COGITO study. Multivariate Behavioral Research
- Schultzberg & Muthén (2018). Number of subjects and time points needed for multilevel time series analysis: A simulation study of dynamic structural equation modeling. Structural Equation Modeling, 25:4, 495-515
- Asparouhov & Muthén (2018). Latent variable centering of predictors and mediators in multilevel and time-series models. Accepted for publication in Structural Equation Modeling
- Asparouhov & Muthén (2018). Comparison of models for the analysis of intensive longitudinal data. Submitted for publication

Auto-Regression for the Outcome or the Residual?



- **Focus on y_t regressed on y_{t-1}**
- Hamaker et al., MBR (2018): Daily measurements of negative and positive affect over 100 days
- Autoregressive parameter indicating "how quickly a person restores equilibrium after being perturbed": inertia
- Time series tradition (our term: DSEM)



- **Focus on y_t regressed on x_t**
- Liu & West, J. Personality (2015): Daily diary study over 60 days
- Stress during the day influencing alcohol consumption that evening
- Multilevel tradition (our term: RDSEM)

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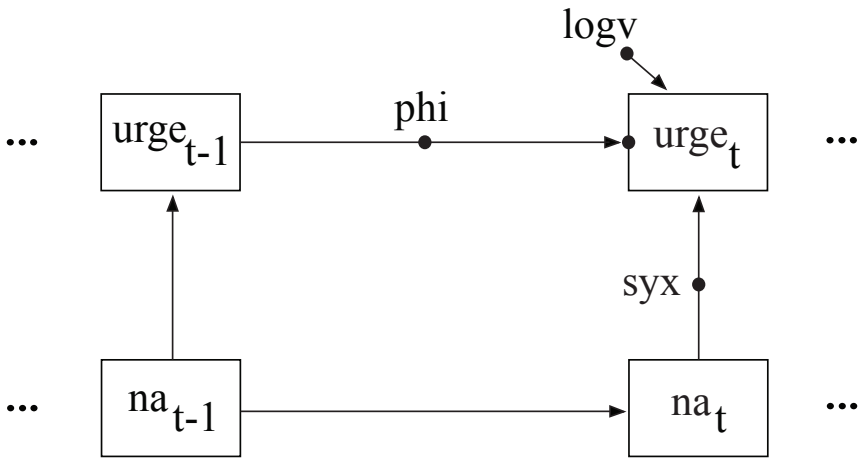
EMA Example: Smoking Urge Data

- Shiffman smoking cessation data
- N = 230, T $\approx$ 150: Random prompts from Personal Digital Assistant (hand held PC) approx. 5 times per day for a month
- Variables: Smoking urge (0-10 scale), negative affect (unhappy, irritable, miserable, tense, discontent, frustrated-angry, sad), gender, age, quit/relapse

Two-Level Time Series Analysis: Regression of Smoking Urge on Negative Affect (na) Using 4 Random Effects

Within:



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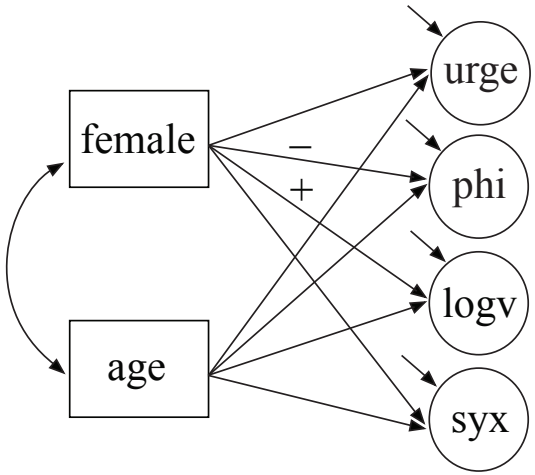

Bayesian Analysis: Advantages over ML

- Bayes with non-informative priors - a powerful computing algorithm :
  - Analyses are often less computationally demanding, for example, when maximum-likelihood requires high-dimensional numerical integration due to many latent variables (factors, random effects)
  - In cases where maximum-likelihood computations are prohibitive, Bayes with non-informative priors can be viewed as a computing algorithm that would give essentially the same results as maximum-likelihood if maximum-likelihood estimation were computationally feasible
    - New types of models can be analyzed where the maximum-likelihood approach is not feasible (e.g. multilevel time series models with many random effects)
- Bayes with informative parameter priors - a better reflection of hypotheses based on previous studies

Bayesian Analysis

- Learning Bayesian analysis in the early 90's via mixture modeling using BUGS - too slow, switched to ML
- Arminger & Muthén (1998). A Bayesian approach to nonlinear latent variable models using the Gibbs sampler and the Metropolis-Hastings algorithm. Psychometrika, 63, 271-300
- Muthén & Asparouhov (2012). Bayesian SEM: A more flexible representation of substantive theory. Psychological Methods, 17, 313-335
  - Zero-mean, small-variance priors instead of fixed zeros
  - Strictness of the factor model hypothesis: EFA $<$ BSEM $<$ CFA
  - Reduces the CFA risk of over-estimating factor correlations
  - Produces a counterpart to ML's modification indices (Lagrange multipliers)
- Bayes papers posted at statmodel.com/papers: Bayesian Analysis

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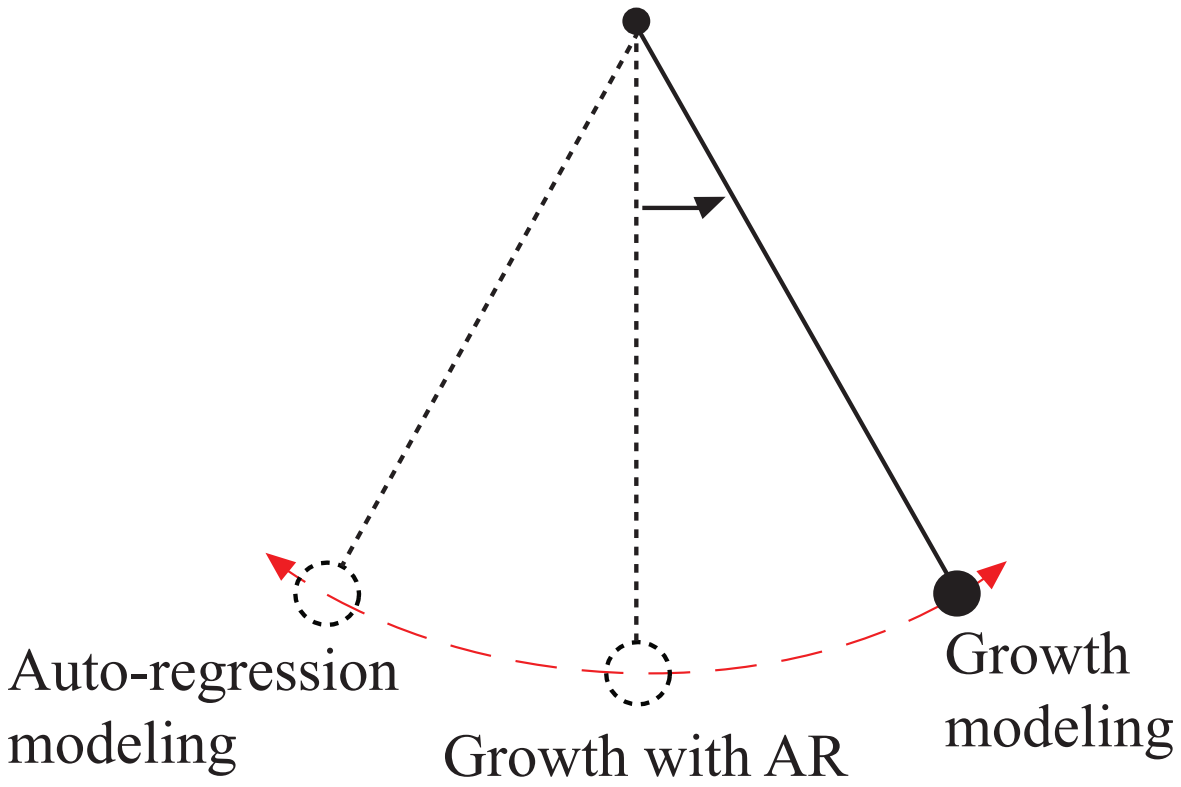
The Pendulum of Longitudinal Modeling



Cross-Classified Longitudinal Analysis

- Two between-level cluster variables: subject crossed with time (one observation for a given subject at a given time point)
- Generalization of the two-level model providing more flexibility: random effects can vary across not only subject but also across time

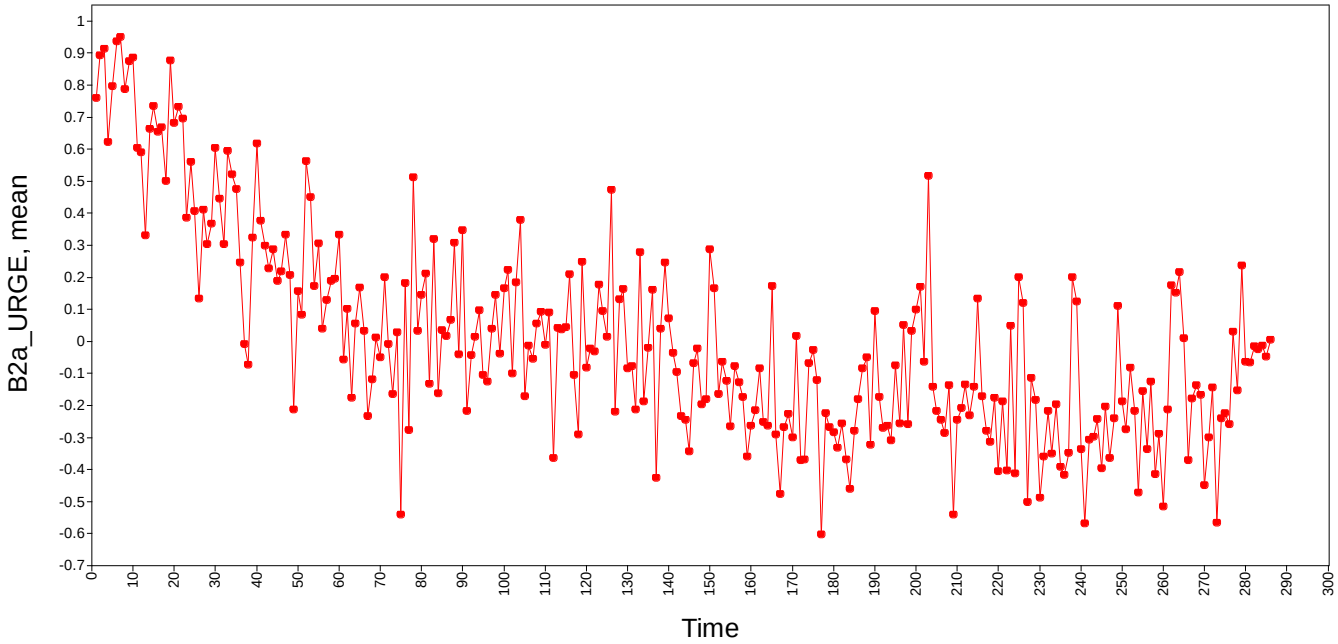
Consider the two-level model with a random intercept/mean:

$$y_{it} = \underbrace{\alpha + \alpha_i}_{\text{Between subject}} + \underbrace{\beta\, y_{w,it-1} + \varepsilon_{it}}_{\text{Within subject}}. \quad (1)$$

The corresponding cross-classified model is:

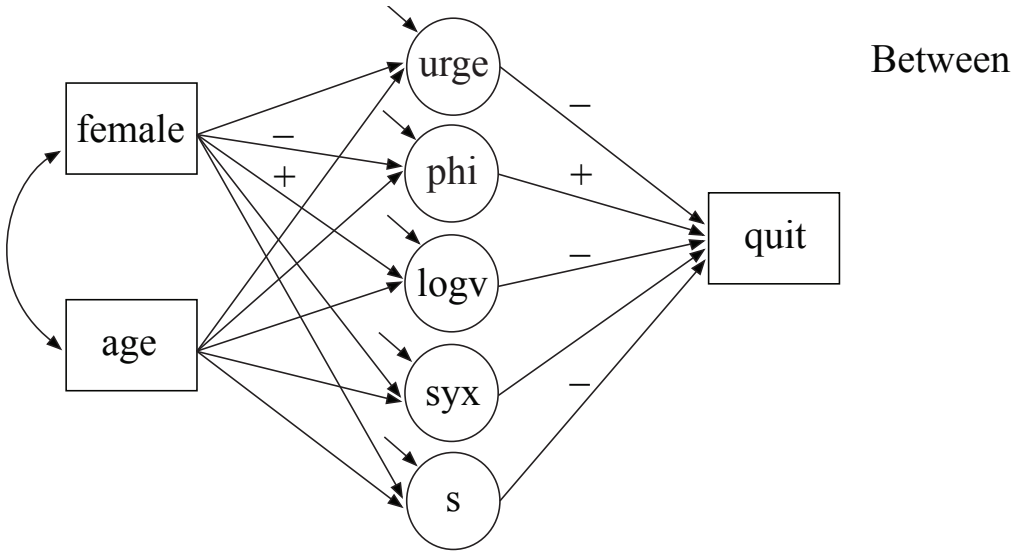
$$y_{it} = \underbrace{\alpha + \alpha_i}_{\text{Between subject}} + \underbrace{\alpha_t}_{\text{Between time}} + \underbrace{\beta\, y_{w,it-1} + \varepsilon_{it}}_{\text{Within subject}}. \quad (2)$$

Cross-Classified Analysis: Finding a Trend
Plot of Time-Varying Random Effects for Smoking Urge



- Analysis used cross-classified modeling
- The trend can be modeled according to some functional form
  - In a cross-classified analysis
  - In a two-level analysis

Smoking Urge Data: Two-Level Analysis Adding a Trend for Urge and a Binary Dependent Variable on Between



Quit (binary) regressed on random effects:

- higher urge gives lower quit probability
- higher autocorrelation gives higher quit probability
- higher residual variance gives lower quit probability
- higher trend slope gives lower quit probability

Counterfactually-Defined Indirect Effects

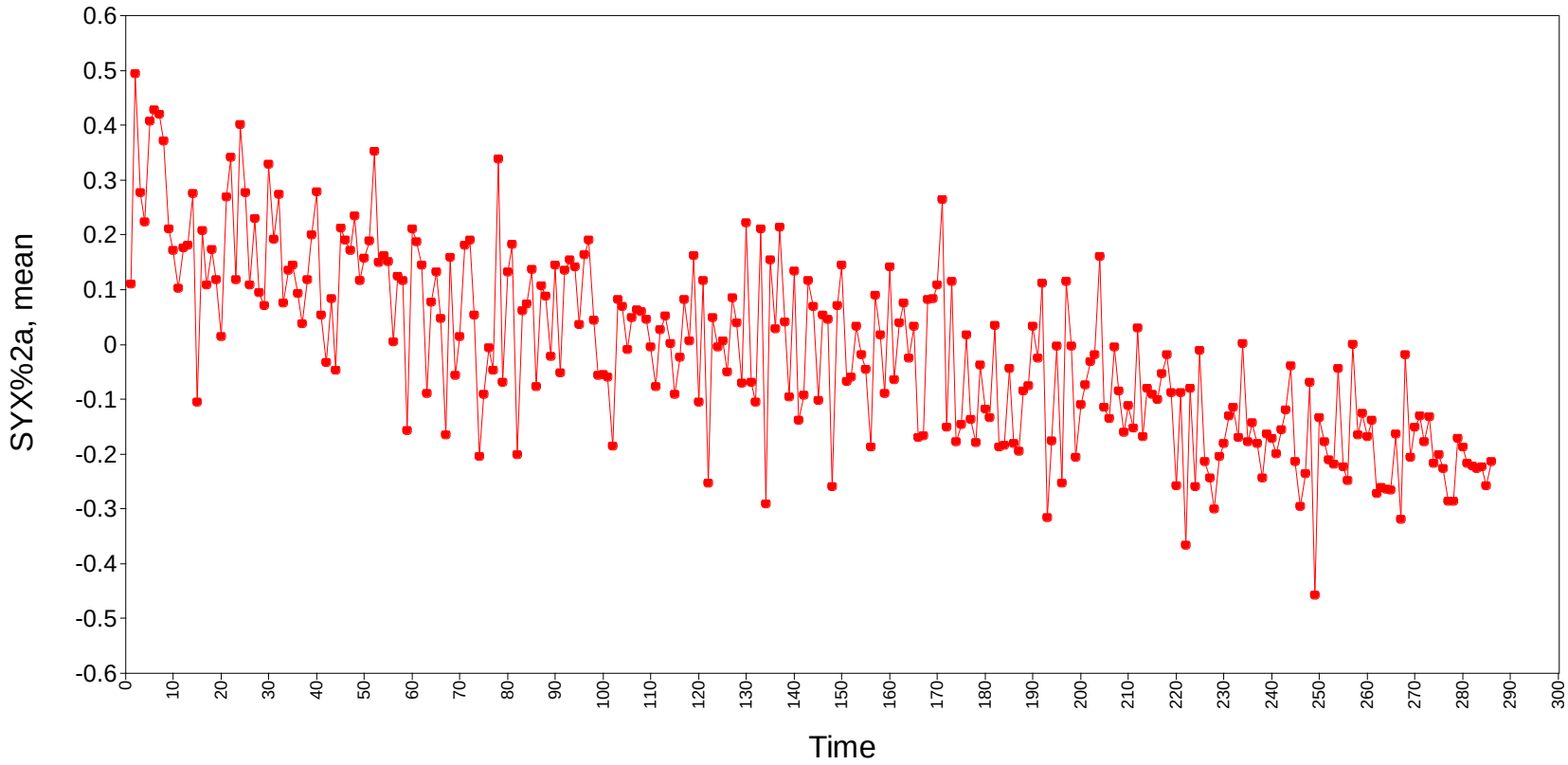
- The binary quit outcome calls for counterfactually-defined (causal) indirect and direct effects
- Muthén & Asparouhov (2015). Causal effects in mediation modeling: An introduction with applications to latent variables. Structural Equation Modeling: A Multidisciplinary Journal, 22(1), 12-23
- Muthén, B., Muthén, L. & Asparouhov, T. (2016). Regression And Mediation Analysis Using Mplus. - Chapters 4 and 8
- In this example, mediation is on level 2 with multiple latent mediators

Cross-Classified Analysis: Time-Varying Effect Modeling (TVEM)

- Cross-classified modeling allows parameters to change over time
- An example is a regression slope
  - Does the influence of negative affect on smoking urge show a decline over time?

Trend in Slope for Urge Regressed on Negative Affect

- syx is the slope of urge regressed on negative affect



- syx regressed on time gives a significant negative slope: The effect of negative affect on smoking urge is reduced over time

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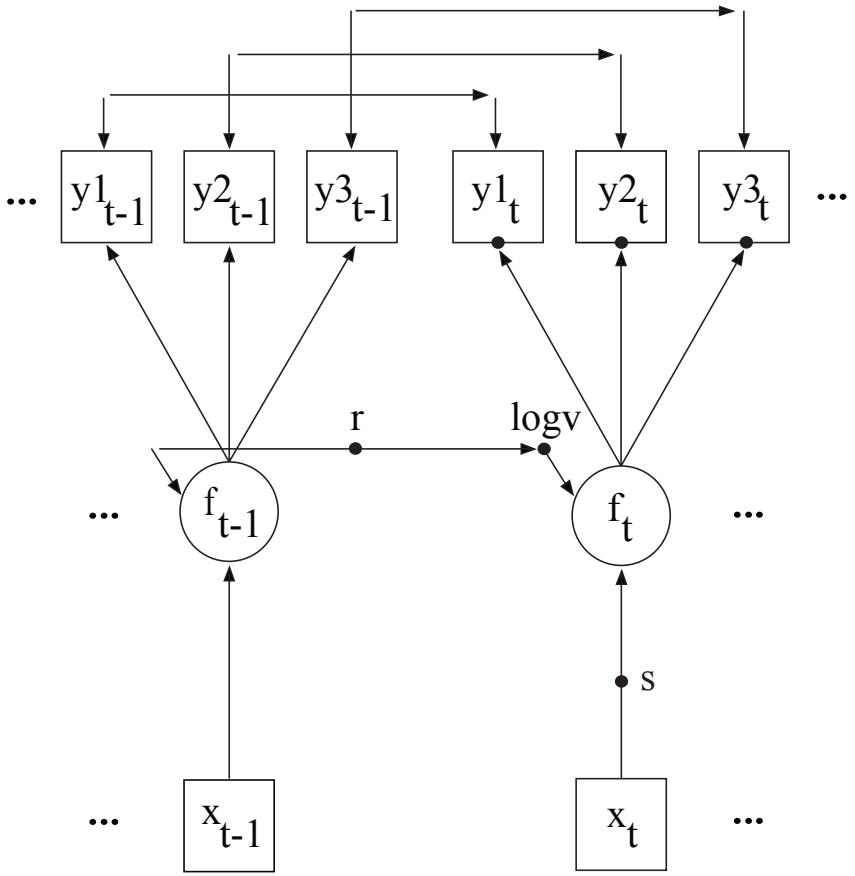
Two-Level Time Series Factor Analysis

Within:



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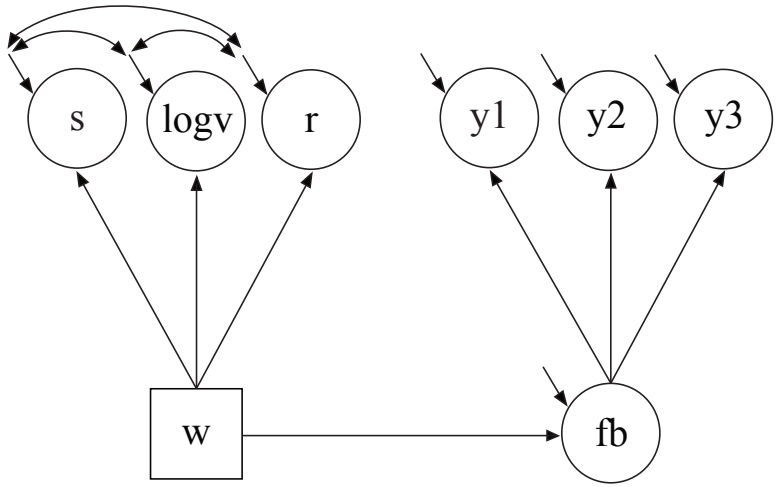
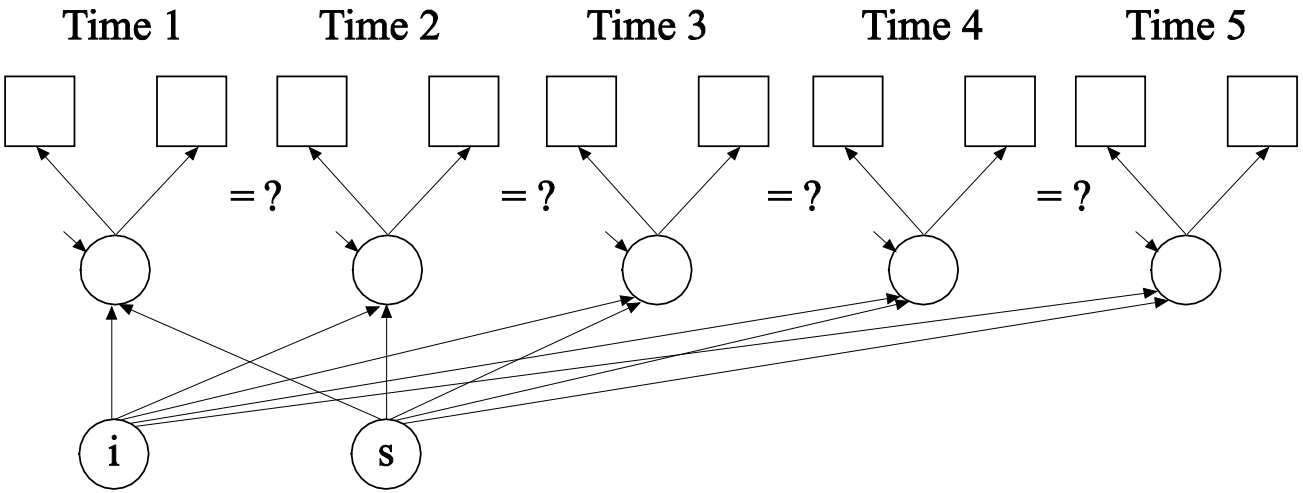


Measurement Invariance in Longitudinal Factor Analysis

- An old dilemma
- A new solution

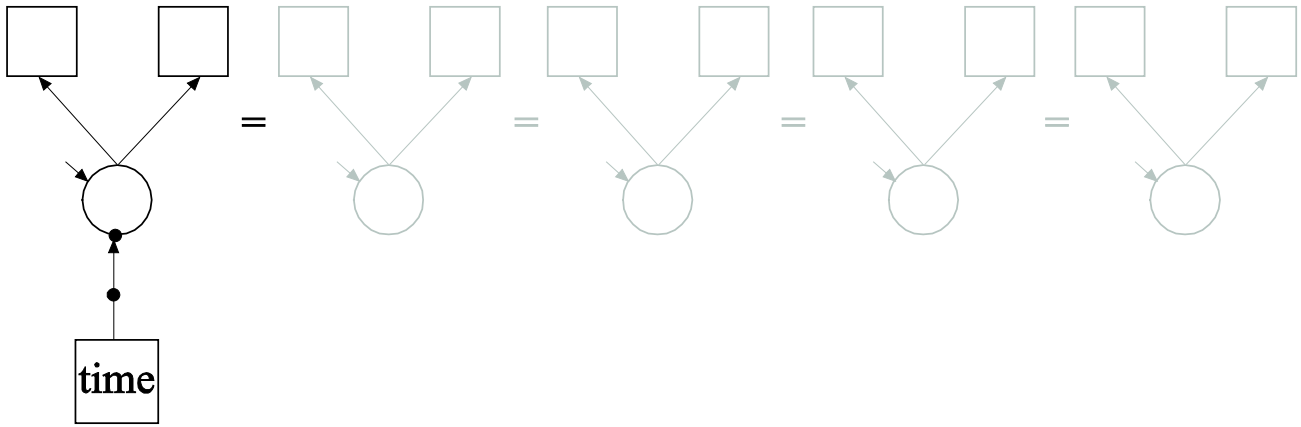
Categorical Items, Single-Level, Wide Format Approach



Single-level analysis with $p \times T = 2 \times 5 = 10$ variables, $T = 5$ factors.

- ML hard and impossible as T increases (numerical integration)
- WLSMV possible but hard when $p \times T$ increases and biased unless attrition is MCAR or multiple imputation is done first
- Bayes possible
- Searching for partial measurement invariance is cumbersome

Categorical Items, Two-Level, Long Format Approach



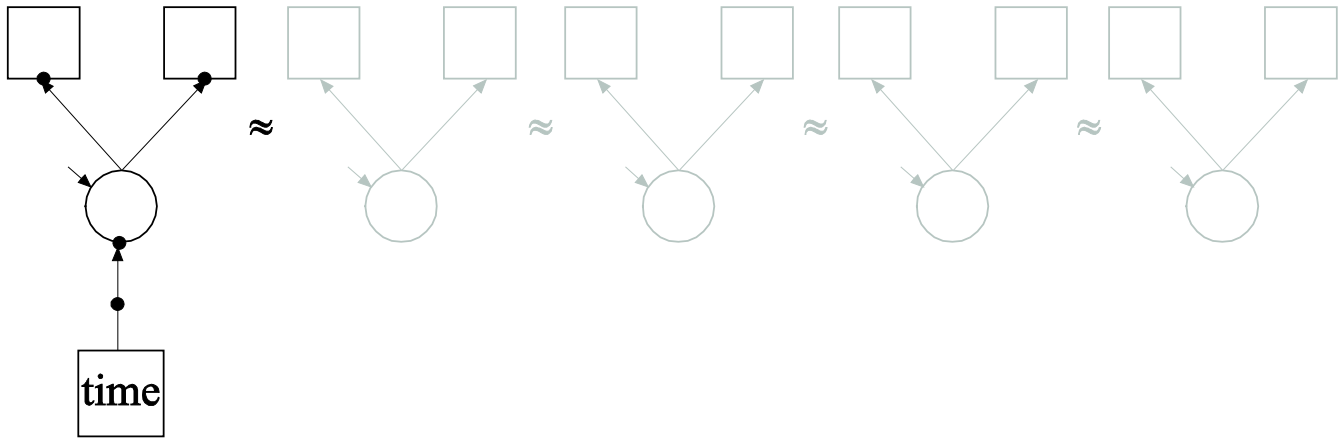
Two-level analysis with $p = 2$ variables, 1 within-factor, 2-between factors, **assuming full measurement invariance across time**.

- ML feasible
- WLSMV feasible (2-level WLSMV)
- Bayes feasible

Measurement Invariance Across Time

- New solution, time is a random mode
- A long format, cross-classified approach
  - Best of both worlds: Keeping the limited number of variables of the two-level approach without having to assume full measurement invariance across time

Cross-Classified, Long Format Approach



- Clusters are person and time
- Bayes cross-classified random effects analysis with random measurement intercepts varying across person **and time**

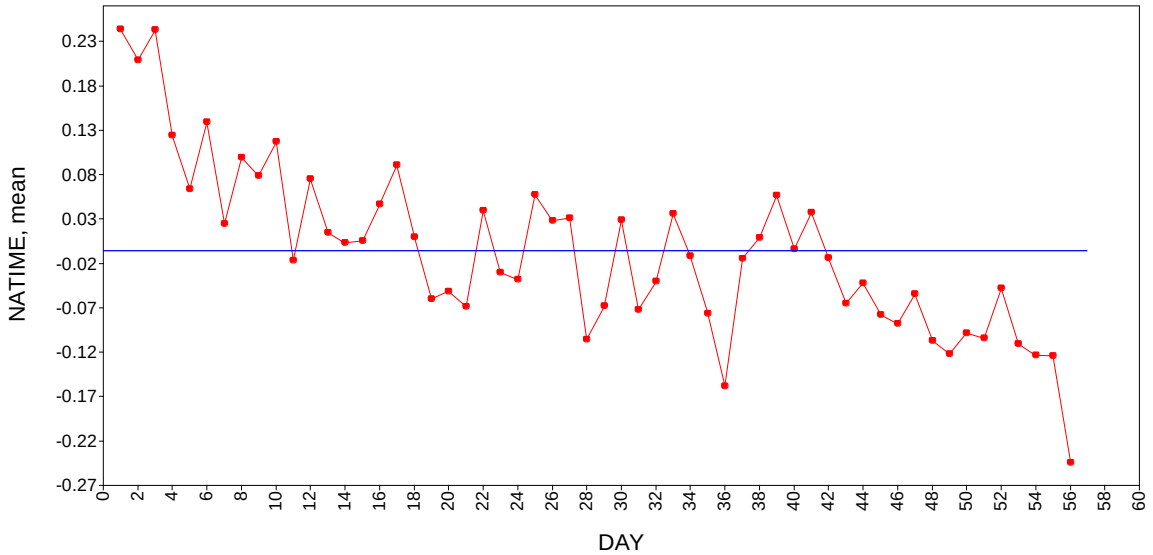
Example: Item Factor Analysis (IRT) Using 10 Negative Affect Items

- Data from the older cohort of the Notre Dame Study of Health & Well-being (Bergeman): N = 270, T = 56 (daily measures on consecutive days)
- Wang, Hamaker, Bergeman (2012). Investigating inter-individual differences in short-term intra-individual variability. *Psychological Methods*
- Predictors and distal outcomes of negative affect development over the 56 days
- 10 NA items (5-cat scale): afraid, ashamed, guilty, hostile, scared, upset, irritable, jittery, nervous, distressed (average score used in article). Wide format would have 56*10 variables
- Question format: Today I felt... (1 = Not at all, ..., 5 = Extremely)
- 1-factor DAFS model with ordinal factor indicators

Results of Cross-Classified Factor Analysis with One NA Factor for 10 Ordinal Items

$$na\,factor_{it} = \underbrace{\alpha + \alpha_i}_{\text{Between subject}} + \underbrace{\alpha_t}_{\text{Between time}} + \underbrace{\beta\, y_{w,it-1} + \varepsilon_{it}}_{\text{Within subject}}.$$

- The factor score plot for the na_time factor (on the between day level) shows a drop of 40% of the total factor SD over the 56 days:



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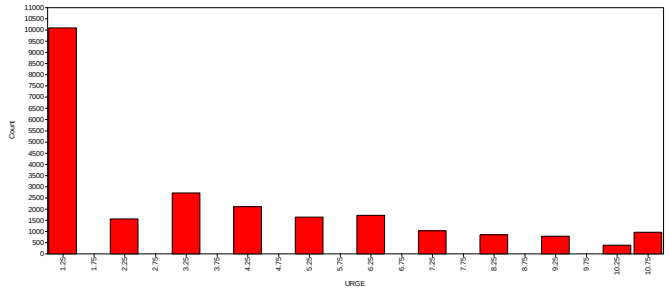
Current Activities

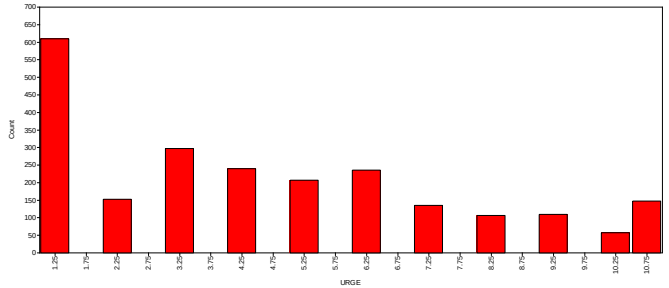
- Two-part, two-level longitudinal analysis
- Modeling cycles by sine-cosine
- Very long longitudinal data

Long Longitudinal Analysis with Strong Floor Effects

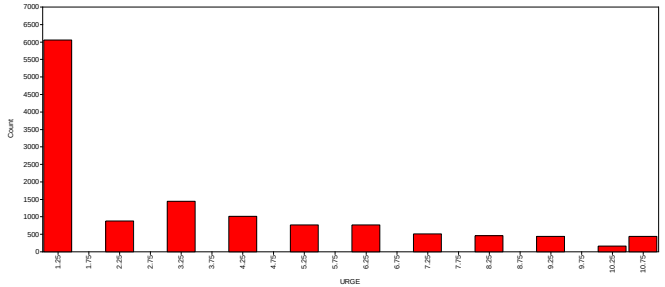
Overall: 42% at the floor value (smoking urge in cessation study)



Early: 27% at the floor value



Late: 47% at the floor value



Two-Part Modeling of Floor Effects

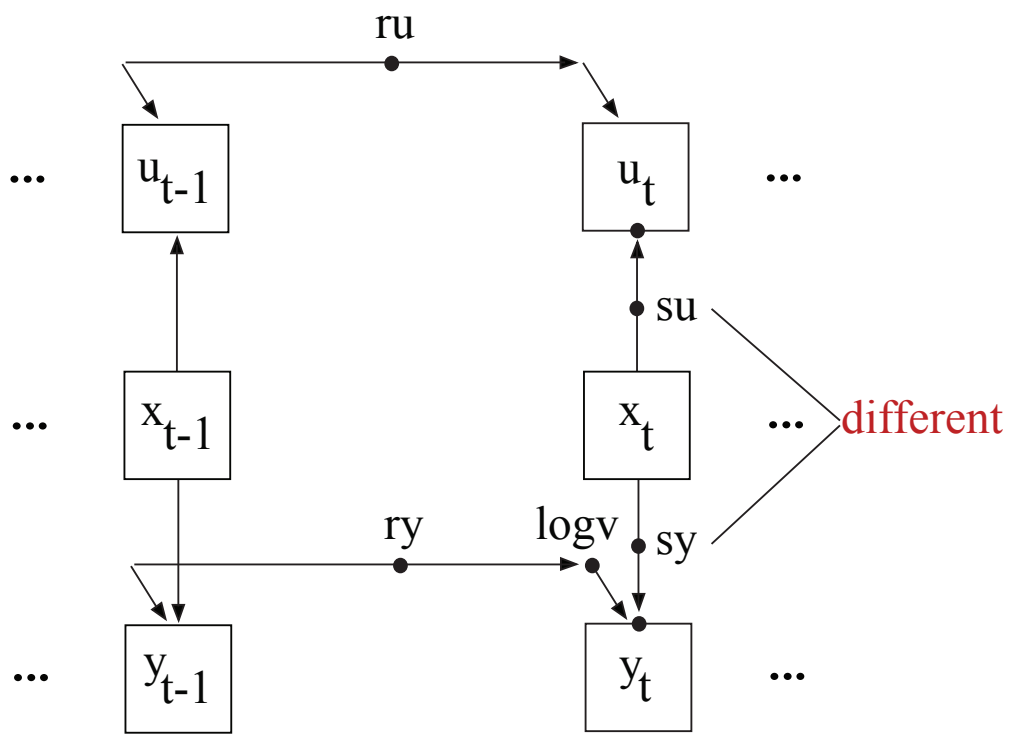
- Olsen & Schafer (2001). Two-part random-effects model for semicontinuous longitudinal data. JASA
- Kim & Muthén (2009). Two-part factor mixture modeling: Application to an aggressive behavior measurement instrument. Structural Equation Modeling, 16, 602-624
- Categorical DSEM and RDSEM: Asparouhov, Hamaker & Muthén (2018). Dynamic structural equation models. Structural Equation Modeling, 25:3, 359-388

- Transform the variable into 2 variables:
  - - A binary u and a continuous y (DATA TWOPART)

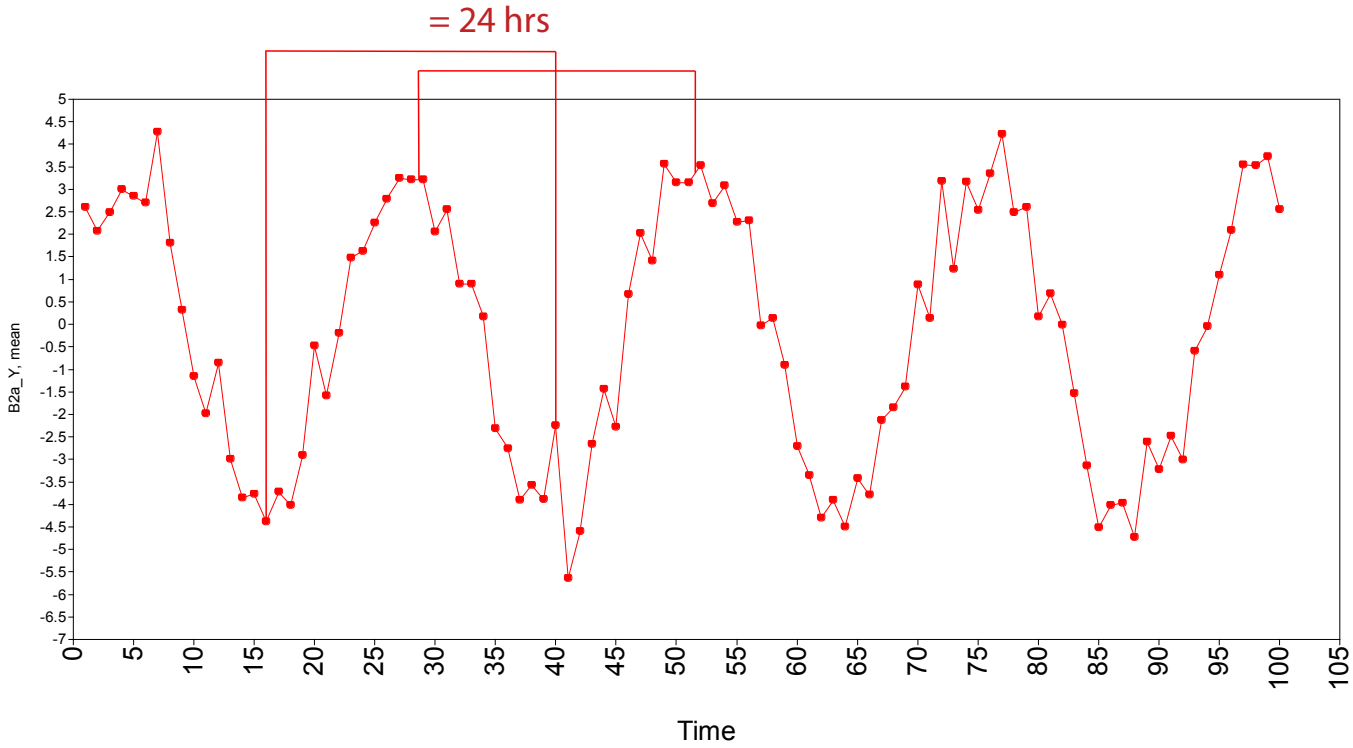
- u = 0 if at the floor: y is missing
- u = 1 if not at the floor: y is observed

- Probit model for u
- Log normal model for y

Two-Part, Two-Level Regression Modeling Binary and Continuous Outcome



Modeling Cycles: Dummies, Splines, Sine-Cos, Free Form



- Biological cycles
  - 24-hour cycles: Circadian rhythm such as heart rate
- Behavioral cycles
  - Weekly drinking pattern
- Environmental cycles
  - Monthly temperature fluctuations

Cyclic Formulas

$$f(t) = A\ cos\ (2\pi\ \omega\ t + \phi)$$
$$= \underbrace{-A\ sin\ \phi}_{\beta_1}\ \underbrace{sin\ (2\pi\ \omega\ t)}_{x_1} + \underbrace{A\ cos\ \phi}_{\beta_2}\ \underbrace{cos\ (2\pi\ \omega\ t)}_{x_2}$$

$$Amplitude = A = \sqrt{\beta_1^2 + \beta_2^2}$$
$$Phase = \phi = tan^{-1}(-\beta_1/\beta_2)$$

- ω is a frequency index defined as cycles per unit. Using $\omega = 1/24 = 0.04167$ gives 24-hour cycles
- Multiple $f(t)$ components with different cycles per unit can be used. Spectral analysis finds the components of the cycles
- Two-level or cross-classified analysis with random effects

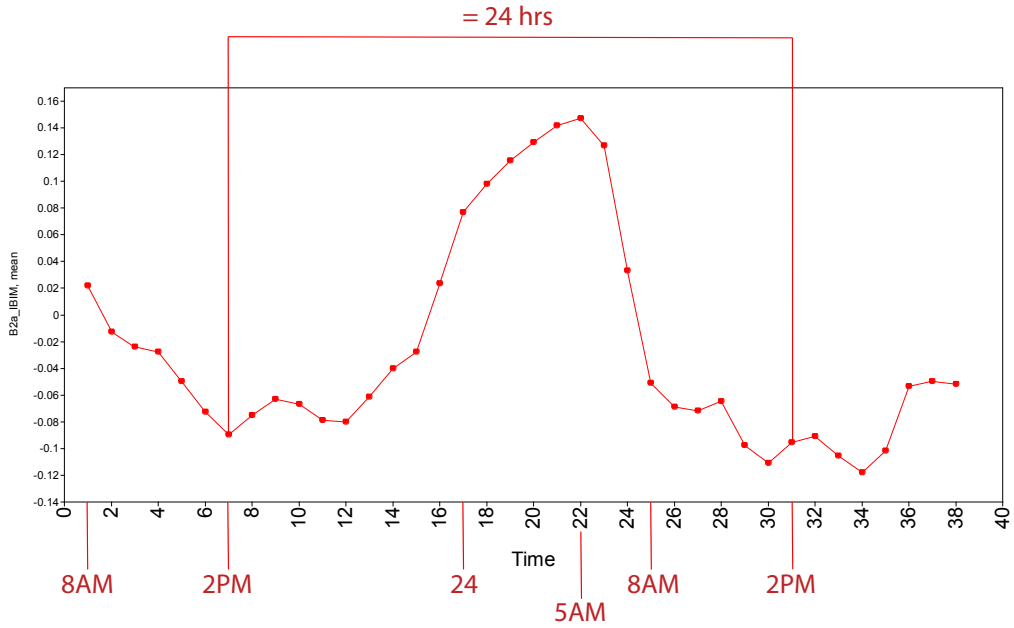
Example: Circadian Analysis of Heart Beat Data

- Data:
  - N = 162, T = 38 (hourly measures used here)
  - Outcome: ibi (time in between heart beats - high is good)
  - Covariates: Gender, smoking, sports
- Model:
  - Single-component two-level with random effects

Background:

Houtveen, Hamaker, van Doornen (2010). Using multilevel path analysis in analyzing 24-h ambulatory physiological recordings applied to medically unexplained symptoms. Psychophysiology, 47, 570-578.

Estimated In-Between Heart Beats Random Effects



- Slopes for sine and cosine both significant with little variation across subjects
- Random effects across subjects for residual variance and auto-regression:
  - Exercise increases time in between heart beats and increases residual variance
  - Smoking decreases time in between heart beats, decreases residual variance, and increases auto-regression

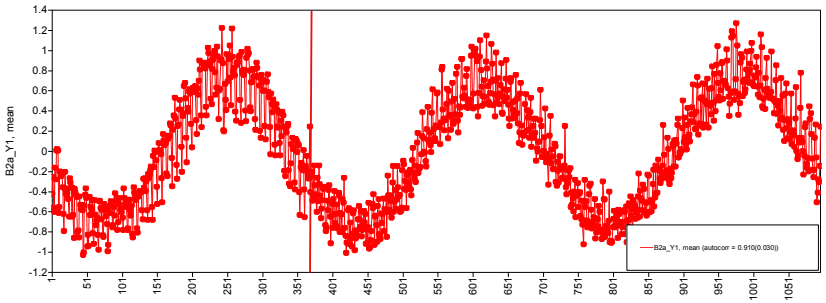
Cyclic Modeling: Two-Level Model as a Measurement Instrument for N=1 Analysis

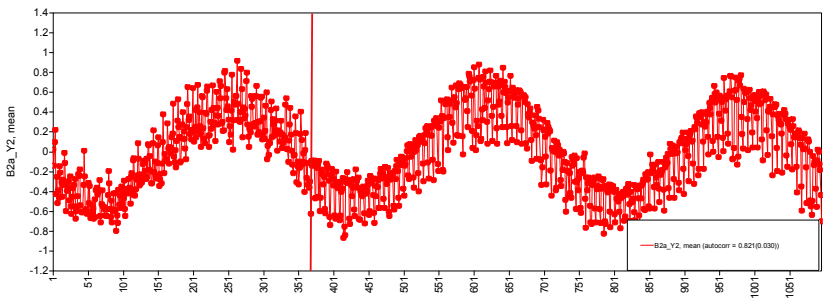
- Ambulatory measurement of blood pressure
- T = 48: Every 30 minutes for 24 hours
- N = 886
- 2-component sine-cosine model with 4 random slopes
- Madden et al. (2018). Morning surge in blood pressure using a random-effects multiple-component cosinor model. Statistics in Medicine.

- Problem: How do you estimate an individual's curve for this complex model from only T = 48?
- Potential solution: Do a two-level analysis with N=1

Very Long Longitudinal Data: T= 1096

- Electricity consumption of firms measured daily (and hourly) over 3 years: T=1096 (Schultzberg, 2018)
  - Intervention: change in tariff
  - N=184 intervention group (N= 800 Control group; not used here)
  - Pre-intervention data for 1 year, post-intervention data for 2 years
  - Sine-cosine cross-classified model





- Significant drop in amplitude after the intervention (marked by a vertical line)
- In the left part of the figure, the curve after the intervention shows the predicted development in the absence of the intervention

Very Long Longitudinal Data

Schultzberg & Muthén (2018). Number of subjects and time points needed for multilevel time series analysis: A simulation study of dynamic structural equation modeling. Structural Equation Modeling. 25:4, 495-515

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