An Overview of Markov Chain Methods for the Study of Stage-Sequential Developmental Processes

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This article presents an overview of quantitative methodologies for the study of stage-sequential development based on extensions of Markov chain modeling. Four methods are presented that exemplify the flexibility of this approach: the manifest Markov model, the latent Markov model, latent transition analysis, and the mixture latent Markov model. A special case of the mixture latent Markov model, the so-called mover–stayer model, is used in this study. Unconditional and conditional models are estimated for the manifest Markov model and the latent Markov model, where the conditional models include a measure of poverty status. Issues of model specification, estimation, and testing using the Mplus software environment are briefly discussed, and the Mplus input syntax is provided. The author applies these 4 methods to a single example of stage-sequential development in reading competency in the early school years, using data from the Early Childhood Longitudinal Study—Kindergarten Cohort.

Keywords: Markov chain models, categorical variables, reading development

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Central to the goal of testing theories of developmental processes is the extension and application of statistical methodologies that capture as closely as possible change over time. In recent years, there have been significant advances in statistical methods designed to model change over time in continuous variables. The standard methodological framework for the study of intraindividual differences’ change over time in continuously measured variables is growth curve modeling (Meredith & Tisak, 1990; Muthén, 1991; Rogosa, Brandt, & Zimowski, 1982; Singer & Willett, 2003; Willett, 1988; Willett & Sayer, 1994). Growth curve modeling takes as its data source individual empirical growth trajectories. Variation in the empirical growth trajectories can be related to substantively relevant time-varying and/or time-invariant predictors. Growth curve modeling also provides an estimate of the average initial level and average rate of growth taken to be estimates of the growth parameters in a defined population.

In contrast to the notion of growth in continuous variables, another type of question that arises in the study of human development concerns change in qualitative status over time. The notion of change over time in developmental status is not new; important examples such as Piaget’s (1947; 1971) stages of cognitive development or Kohlberg’s (1980) stages of moral development have enjoyed a long and illustrious place in developmental research. In addition to theories of change in cognitive or moral development, researchers have also hypothesized and tested stage-sequential models for the onset and development of substance abuse in early adolescence (e.g., Collins, 2002b) and, of relevance to this article, stage-sequential models for reading development (Chall, 1995; Kaplan & Walpole, 2005).

The core statistical model for the study of change in qualitative status over time is the manifest Markov chain model, which concerns modeling change over time in observed categorical variables.1 Over the last 20 years a number of extensions have been added to the manifest Markov chain model that are argued to be of great relevance to developmental researchers. These include extensions that account for measurement error in the responses, allow greater modeling flexibility for longitudinal data, and allow for the progression through qualitative states to be fundamentally different for unobserved clusters of individuals.

This article reviews a selected set of statistical methodologies available for the study of stage-sequential growth that rest on the formal foundation of a discrete-time/discrete-response manifest Markov chain. Beyond the explication and demonstration of the manifest Markov model, this article considers latent Markov models, latent transition analysis, and mixture Markov models. An empirical example focusing on stage-sequential development in reading proficiency in young children is used throughout the article to provide a substantive context for the application of these methods.

Data Source: The Early Childhood Longitudinal Study

The substantive context for this article concerns the problem of representing the development of reading proficiency as a stage-

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1 In line with the terminology of factor analysis, I use manifest to refer to models for observed variables and latent to refer to model for unobserved variables.
sequential process. This problem has a long intellectual history with seminal ideas emanating from the work of Chall (1995) and Juel (1988). A recent article by Kaplan and Walpole (2005) reviewed the background literature and presented empirical support for a stage-sequential model of reading development and the role of poverty status in moderating stage transition probabilities over time. In the interest of space, that literature review is not summarized here.

This study used the same data source as that used in the Kaplan and Walpole (2005) study. Data were obtained from the Early Childhood Longitudinal Study: Kindergarten Class of 1998–1999 (ECLS-K; National Center for Education Statistics, 2001). The ECLS-K database provides a unique opportunity to study the development of successful reading achievement (which can be defined as the ability to comprehend text) by the end of first grade for children with different levels of entering skill and different potential barriers to success. The ECLS-K data available to address this question include longitudinal measures of literacy achievement for a large and nationally representative sample.

Data used for this article consist of the kindergarten base year (fall 1998/spring 1999) and first grade follow-up (fall 1999/spring 2000) panels of ECLS-K. Only first-time public school kindergarten students who were promoted to and present at the end of first grade were chosen for this study. The sampling design of ECLS-K included a 27% subsample of the total sample at fall of first grade to reduce the cost burden of following the entire sample for four waves but to allow for the study of summer learning loss. Although this dramatically reduced the sample size, I nevertheless included fall of first grade in this study to allow four time points for estimation of the transition probabilities. The sample size for this study is 3,575.

Measures and Instrument Design

The measures used in this study consist of a series of reading assessments designed to measure basic skills that Whitehurst and Lonigan (2002) have identified as particularly salient in the first 2 years of school. Specifically, the reading assessment yields scores for (a) letter recognition, (b) beginning sounds, (c) ending letter sounds, (d) sight words, and (e) words in context.

In addition to the reading scale scores, ECLS-K provides transformations of these scores into probabilities of proficiency as well as dichotomous proficiency scores, which are used in this study. Dichotomous proficiency scores can be calculated because the ECLS-K instrument was designed with clusters of reading assessment items having similar content and difficulty. A child is assumed to have passed a particular skill level if he or she answers at least three out of four items in the skill cluster correctly. A fail score is given if the child incorrectly answered or did not know at least two items within the skill cluster. In the case of exactly two items correct, a pass or a fail score is given if the pattern of passes and fails for the remaining proficiencies could suggest an unambiguous pass or fail.

A measure of poverty is provided in the ECLS-K and is computed by taking income information obtained from the parent survey and comparing it to the 1998 U.S. census poverty thresholds that vary according to household size. A dichotomous variable is provided that indicates whether the child’s household is below \((= 1)\) or above \((= 0)\) the poverty threshold. In the next section, I demonstrate how Markov models can incorporate predictors, using this variable.

Selected Markov Models

This section reviews the selected set of Markov chain models to be applied later to the problem of reading development. A history of Markov modeling can be found in Kaplan and Uribe-Zarain (2005). A hierarchy of Markov models adapted from Langeheine (1994) is shown in Figure 1 and illustrates the models considered in this article.

At the top of the hierarchy is the mixture latent Markov model. This model is the most general, allowing for measurement error in the categorical responses as well as allowing for the possibility of unobserved heterogeneity in the transition probabilities—implying that there are distinct subpopulations that follow their own unique transitions over time. Two special cases derive directly from the mixture latent Markov model: latent transition analysis and latent Markov models. In these two cases, it is assumed that there is a single population described by the transition probabilities but that there is measurement error in the categorical responses. The difference between the two models is subtle and described later. From here, two separate models can be derived: the latent class model and the manifest Markov model. The latent class model allows for measurement error in the categorical outcomes, but the data are not longitudinal. On the other hand, if data are longitudinal and we assume perfect reliability among the categorical measures, this yields the manifest Markov model.

I start at the bottom of the hierarchy with the manifest Markov model and then move to the latent class model. Although the latent class model is applied to cross-sectional data, it is presented here to demonstrate the issue of measurement error and also because it sets the framework for latent Markov models. This is followed by the latent Markov model. From there, I present latent transition analysis, which extends the latent Markov model to handle multiple indicators of an underlying latent state and is arguably better suited for the study of stage-sequential development. The latent transition model is then extended to add a mixture component allowing for specification of so-called mover–stayer models. Throughout this article, I adopt the statistical notation given in Langeheine and Van de Pol (2002).

The Manifest Markov Model

For the manifest Markov model and the remaining models in this article, the data of interest are observed categorical responses. The manifest Markov model consists of a single chain, in which predicting the current state of an individual requires data from the previous occasion only. In line with the example of reading development, consider measuring mastery of ending letter sounds at four discrete time points, where mastery is a dichotomous variable.

The manifest Markov model can be written as

$$P_{ijkl} = \delta^{i,j,k}_{l} \xi_{i}^{0} \eta_{j}^{0} \zeta_{k}^{0} \xi_{i}^{1} \eta_{j}^{1} \zeta_{k}^{1} \xi_{i}^{2} \eta_{j}^{2} \zeta_{k}^{2} \xi_{i}^{3} \eta_{j}^{3} \zeta_{k}^{3}$$

where \(P_{ijkl}\) is the model-based expected proportion of respondents in the defined population in cell \((i, j, k, l)\). The subscripts, \(i, j, k,\) and \(l\), are the manifest categories for Times 1, 2, 3, and 4, with \(i = 1 \ldots I, j = 1 \ldots J, k = 1 \ldots K,\) and \(l = 1 \ldots L\). In this study, there are two categorical responses for \(i, j, k,\) and \(l\)—namely,
mastery or nonmastery of ending letter sounds. Thus, I = J = K = L = 2. The parameter \( \delta_i \) is the observed proportion of individuals at Time 1 who have or have not mastered ending letter sounds and corresponds to the initial marginal distribution of mastery and nonmastery. The parameters \( \tau_{ij}^{21}, \tau_{ij}^{32}, \) and \( \tau_{ij}^{43} \) are the transition probabilities. Specifically, the parameter \( \tau_{ij}^{21} \) represents the transition probability from Time 1 to Time 2 for those in category \( j \), given that they were in category \( i \) at the beginning of the study. In substantive terms, \( \tau_{ij}^{21} \) provides the probability that a child will have mastered ending letter sounds at Time 2, given that he or she mastered ending letter sounds at Time 1. Because these transition probabilities must sum to 1.0, the transition probability matrix will also provide the probabilities associated with moving from the nonmastery class at Time 1 to the mastery class at Time 2. Of course, the transition probability matrix will also provide information about movement from mastery at Time 1 to nonmastery at Time 2. In a similar fashion, the parameter \( \tau_{ij}^{32} \) represents the transition probability from Time 2 to Time 3 for those in category \( j \), given that they were in category \( i \) at the previous time point. Finally, the parameter \( \tau_{ij}^{43} \) is the transition probability from Time 3 to Time 4 for those in category \( j \), given that they were in category \( i \) at the previous time point.

The modeling of transition probabilities is quite flexible. In this example, it may be desirable to impose the restriction that once a child has mastered ending letter sounds at time \( t \), he or she cannot move to nonmastery. This type of modeling imposes a strict Guttman response pattern to the data. Also, the manifest Markov model can be specified to allow transition probabilities to be constant over time or to allow transition probabilities to differ over time. The former is referred to as a stationary Markov chain, whereas the latter is referred to as a nonstationary Markov chain.

The Latent Class Model

The manifest Markov model assumes that the responses are measured without error. In some cases, this might be an unreasonable assumption, and thus it might be desirable to attempt to handle measurement error in Markov models explicitly. Latent class analysis serves as the underlying measurement model for latent Markov models and their extensions; it was introduced by Lazarsfeld and Henry (1968; see also Clogg, 1995) for the purpose of deriving latent attitude variables from responses to dichotomous survey items. In a traditional latent class analysis, it is assumed that an individual belongs to one and only one latent class and that given an individual’s latent class membership, the observed responses are independent of one another—referred to as the local independence assumption. The latent classes are, in essence, categorical factors arising from the pattern of response frequencies to

![Figure 1](image-url)
categorical items, in which the response frequencies play a role similar to that of the correlation matrix in factor analysis (Collins, Hyatt, & Graham, 2000). The analog of factor loadings are parameters that estimate the probability of a particular response on the manifest indicator given membership in the latent class. Unlike continuous latent variables (i.e., factors), categorical latent variables (latent classes) divide individuals into mutually independent groups.

The latent class model can be written as follows. Let

\[ P_{ijkl} = \sum_{a=1}^{A} \delta_{ia} \rho_{1a} \rho_{2a} \rho_{3a} \rho_{4a} \]  

(2)

where here \( \delta_{ia} \) is the proportion of individuals in latent class \( a \). The parameters \( \rho_{1a} \), \( \rho_{2a} \), \( \rho_{3a} \), and \( \rho_{4a} \) are the response probabilities for items \( i, j, k, \) and \( l \), respectively, conditional on membership in latent class \( a \).

The application of latent class analysis is quite similar to the application of factor analysis. That is, an investigator would hypothesize a priori a latent categorical variable with \( A \) latent classes. Under the hypothesis of \( A \) latent classes, the model is fit to the observed categorical data. The pattern of the response probabilities is used to name the categorical latent variable and the classes. Various measures of model fit (described later) can be used to test the hypothesis that the model based on \( A \) latent classes reproduces the observed categorical responses.

Although latent class analysis is a useful procedure in its own right, in the context of latent Markov models and their extensions it is helpful to run a series of cross-sectional latent class analyses to examine the consistency of the latent class structure over time.

**The Latent Markov Model**

Returning to the problem of estimating stage-sequential development, a disadvantage of the manifest Markov model described earlier is that it assumes that the observed categorical responses are perfectly reliable measures of a true latent state. In the context of the ending letter sounds example, this would imply that the observed categorical responses measure the true mastery or nonmastery of ending letter sounds. Rather, it may be more reasonable to assume that the observed responses are fallible measures of an unobservable latent state and it is the study of transitions across true latent states that are of interest.

To take an example that is developed later, consider the mastery or nonmastery of ending letter sounds at Time 1. The manifest behavior is the response on the ending letter sound assessment, yielding a mastery or nonmastery score. Under the assumption that the ending letter sound assessment is perfectly reliable, this would imply that those who have truly mastered the skill would have received a mastery score, and those that did not would receive a nonmastery score. However, the problem could be conceptualized somewhat differently. Namely, we could postulate a true ending letter sound skill, but a child who truly possesses the skill might still be scored as having not mastered the skill because of measurement error. Latent class analysis can address the problem of measurement error at any given point in time by estimating the existence of latent classes, but when merged with manifest Markov models, we can study change over time at the latent level.

The latent Markov model was developed by Wiggins (1973) to address the problem of measurement error in observed categorical responses and as a result to obtain transition probabilities at the latent level. The latent Markov model can be written as

\[ P_{ijkl} = \sum_{b=1}^{A} \sum_{c=1}^{A} \sum_{d=1}^{A} \delta_{ib} \rho_{1b} \delta_{jc} \rho_{2j} \delta_{kc} \rho_{3k} \delta_{lc} \rho_{4l} \]  

(3)

where the parameters in Equation 2 take on slightly different meanings from those in Equation 1. In particular, the parameter \( \delta_{ia} \) represents a latent distribution having \( A \) latent states. The linkage of the latent states to manifest responses is accomplished by the response probabilities \( \rho \). The response probabilities thus play a role analogous to that of factor loadings in factor analysis. Accordingly, \( \rho_{ij}^a \) refers to the response probability associated with category \( i \) given membership in latent state \( a \). The parameter \( \rho_{ij}^a \) is interpreted as the response probability associated with category \( j \), given membership in latent state \( b \) at Time 2. Remaining response probabilities are similarly interpreted.

It should be noted that what has been described so far is the latent class model discussed earlier. The goal, of course, is to measure change over time at the latent level. Thus, as with the manifest Markov model, the transition from Time 1 to Time 2 in latent state membership is captured by \( \tau_{ij} \) in Equation 2. At Time 2, the latent state is measured by the response probabilities \( \rho_{ij}^a \).

Remaining response and transition probabilities are analogously interpreted. Note that Equation 2 reveals that if the response probabilities were all 1.0 (indicating perfect measurement of the latent variable), then Equation 2 would essentially reduce to Equation 1—the manifest Markov model.

**Latent Transition Analysis**

Although the application of Markov models for the analysis of psychological variables goes back to Anderson (1954; as cited in Collins & Wugalter, 1992), most applications focused on single manifest measures. However, as with the early work in the factor analysis of intelligence tests (e.g., Spearman, 1904), it was recognized that many important psychological variables are latent—in the sense of not being directly observed but possibly measured by numerous manifest indicators. The advantages to measuring multiple latent variables via multiple indicators are the known benefits with regard to reliability and validity. Therefore, it might be more realistic to specify multiple manifest categorical indicators of the categorical latent variable and combine them with Markov chain models. The combination of multiple indicator categorical latent variable models and Markov chain models provides the foundation for the latent transition analysis of stage-sequential dynamic latent variables.

In line with Collins and Flaherty (2002) and in the context of the Kaplan and Walpole (2005) study, consider the current reading example in which the data provide information on the mastery of five different skills, such as phonemic awareness, beginning sounds, ending letter sounds, sight reading, and reading words in...
context. At any given point in time, a child has mastered or not mastered one or more of these skills. It is reasonable in this example to postulate a model that specifies that these reading skills are related in such a way that mastery of a later skill implies mastery of all preceding skills. At each time point, the child’s latent class membership defines his or her latent status. The model specifies a particular type of change over time in latent status. This is defined by Collins and Flaherty (2002) as a “model of stage-sequential development, and the skill acquisition process is a stage-sequential dynamic latent variable” (p. 289). It is important to point out that there is no fundamental difference between latent transition analysis and latent Markov chain modeling. The difference is practical, with latent transition analysis being perhaps better suited conceptually for the study of change in developmental status.

The model form for latent transition analysis uses Equation 2 except that model estimation is undertaken with multiple indicators of the latent categorical variable. The appropriate measurement model for categorical latent variables is the latent class model.

### Mixture Latent Markov Model (the Mover–Stayer Model)

A limitation of the models described so far is that they assume that the sample of observations arises from a single population that can be characterized by a single Markov chain (latent or otherwise) and one set of parameters—albeit perhaps different for certain manifest groups such as those children living above or below the poverty level. It is possible, however, that the population is composed of a finite and unobserved mixture of subpopulations characterized by qualitatively different Markov chains. To the extent that the population consists of finite mixtures of subpopulations, a one-size-fits-all application of the Markov chain model can lead to biased estimates of the parameters of the model as well as incorrect substantive conclusions regarding the nature of the developmental process in question. A reasonable strategy for addressing this problem involves combining Markov-chain-based models under the assumption of a mixture distribution (see McLachlan & Peel, 2000, for an excellent overview of finite mixture modeling). This is referred to as the mixture latent Markov model.3

An important special case of the mixture latent Markov model is referred to as the mover–stayer model (Blumen, Kogan, & McCarthy, 1955). In the mover–stayer model there exists a latent class of individuals who transition across stages over time (movers) and a latent class that does not transition across stages (stayers). In the context of reading development, the stayers are those who never move beyond, say, mastery of letter recognition. Variants of the mover–stayer model have been considered by Van de Pol and Langeheine (1989; see also Moolenaar, 1998).

The mixture latent Markov model can be written as

\[
\mathbf{P}_{s} = \sum_{i=1}^{S} \sum_{a=1}^{A} \sum_{b=1}^{B} \sum_{c=1}^{C} \sum_{d=1}^{D} \pi_i \delta_a^i \theta_{abb}^2 \theta_{cdd}^3 \theta_{ddd}^4
\]

where \( \pi_i \) represents the proportion of observations in Markov chain \( s \) \( (s = 1, 2, \ldots, 5) \) and the remaining parameters are interpreted as in Equation 2, with the exception that they are conditioned on membership in Markov chain \( s \).

The model in Equation 4 is the most general of those considered in this article, with the preceding models being derived as special cases. For example, with \( s = 1 \) Equation 4 reduces to the latent Markov model in Equation 2. Also, with \( s = 1 \) and no transition probabilities, the model in Equation 4 reduces to the latent class model of Equation 3.

### Identification, Estimation, and Testing

In this section, I briefly discuss the problem of identification, estimation, and model testing in Markov chain models. Identification refers to the problem of being able to obtain a unique estimate of model parameters from observed data. In some cases, models are not identified, in which case there exist an infinite number of possible estimates of the model parameters. In some cases, there exists one possible estimate of the model parameters. Finally, in some cases there are a finite number of estimates of the model parameters and methods are devised to find the estimate that satisfies some optimal properties. As with the problem of identification in factor analysis and structural equation models, identification in Markov models is achieved by placing restrictions on model parameters (see, e.g., Kaplan, 2000, for a discussion of identification in factor analysis and structural equation modeling).

With regard to manifest Markov chains, identification is not an issue. All parameters can be obtained directly from manifest categorical responses. In the context of latent Markov chain models with a single indicator, the situation is somewhat more difficult. Specifically, identification is achieved by restricting the response probabilities to be invariant over time. As noted by Langeheine (1994), this restriction simply means that measurement error is assumed to be equal over time. For four or more time points, it is only required that the first and last set of response frequencies be invariant.

Identification in the mixture case adds the additional complication that even though the number of parameters is fewer than the data points, the parameters may not be identified. This will depend on the number of different chains being specified. For example, with the mover–stayer model, identification should not be a problem in practice, assuming other conditions of identification are met. This is because mover–stayer models are “strong,” requiring adherence of the model to a simple structure with relatively few parameters. On the other hand, with “weak” models consisting of many chains and many parameters to be estimated, the model may not be identified (Langeheine & Van de Pol, 2002).

Model identification is, of course, a necessary condition for model estimation. It is beyond the scope of this article to provide the details regarding parameter estimation. Suffice it to say that the general approach to estimation of the model parameters has used maximum likelihood estimation via the expectation-maximization algorithm (Dempster, Laird, & Rubin, 1977). This is the approach implemented in Mplus (Muthén & Muthén, 2006).

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2 The ECLS-K indeed provides data on these five skills.

3 It should be noted that finite mixture modeling has been applied to continuous growth curve models under the name general growth mixture models (Muthén, 2004). These models have been applied to problems in the development of reading competencies (Kaplan, 2002) and math competencies (Jordan, Kaplan, Nabors-Ohl, & Locuniak, 2006).
After obtaining estimates of model parameters, the next step is to assess whether the specified model fits the data. In the context of Markov chain models and latent class extensions, model fit is assessed by comparing the observed response proportions against the response proportions predicted by the model. Two statistical tests are available for assessing the fit of the model on the basis of comparing observed versus predicted response proportions. The first is the classic Pearson chi-square statistic. As an example from the latent class framework, the Pearson chi-square test can be written as

\[ \chi^2 = \sum_{ijkl} \frac{(F_{ijkl} - f_{ijkl})^2}{f_{ijkl}} \]  

(5)

where \( F_{ijkl} \) are the observed frequencies of the IJKL contingency table and \( f_{ijkl} \) are the expected cell counts. The degrees of freedom are obtained by subtracting the number of parameters to be estimated from the total number of cells of the contingency table that are free to vary.

In addition to the Pearson chi-square test, a likelihood ratio statistic can be obtained that is asymptotically distributed as chi-square, in which the degrees of freedom are calculated in the same manner as the Pearson chi-square test. In cases where there are sizable disagreements between the Pearson chi-square test and the likelihood ratio chi-square test, it is likely due to the occurrence of sparse cells—that is, patterns of responses where there are very few observations. The Mplus software program allows inspection of the frequency of each response pattern and provides standardized residuals to aid in identifying the location of the problem.

Finally, two types of information criteria are provided for aiding in model selection and apply a penalty function for specifying and testing a complex (less parsimonious) model. The first is the Akaike information criterion (AIC; Akaike, 1973), defined as

\[ \text{AIC} = \chi^2 - 2df \]  

(6)

and the second is the Bayesian information criterion (BIC), defined as

\[ \text{BIC} = \chi^2 - df[\ln(N)], \]  

(7)

where \( N \) is the sample size. In both cases, these indices can be used for model comparisons, and the model with lowest AIC or BIC value is preferred. The BIC penalizes for the addition of free parameters more severely than does the AIC.

Applications to Stage-Sequential Reading Development

In this section, each Markov model previously described will be applied to the problem of stage-sequential development in reading. Corresponding tables provide the goodness-of-fit statistics and are presented for completeness, but no attempt will be made to modify models for the purposes of selecting a best-fitting model. The applications are solely for the purpose of illustrating the methodologies. A more detailed substantive study of reading development as a stage-sequential process can be found in Kaplan and Walpole (2005).

In what follows, I add poverty status for the manifest Markov model and the latent Markov model, but in the interest of space I estimate the latent transition model and mixture Markov model without poverty status added. For a detailed account of poverty effects on stage sequential reading development, see Kaplan and Walpole (2005).

Application of the Manifest Markov Model

Supplemental Appendix A provides the Mplus input used to estimate the manifest Markov model. Table 1 presents the results of the nonstationary manifest Markov model applied to the development of competency in ending letter sounds. For the total sample, it can be seen that over time the probabilities associated with moving from nonmastery to mastery of ending letter sounds change. For example, at the beginning of kindergarten and the beginning of first grade, the proportion of students who have not mastered beginning sounds and the proportion who then go on to master ending letter sounds is relatively constant. However, the transition from nonmastery of ending letter sounds to mastery of ending letter sounds is much greater from the beginning of first grade to the end of first grade. Nevertheless, approximately 25% of the sample who did not master ending letter sounds at the beginning of first grade did not appear to have mastered ending letter sounds by the end of first grade.

When poverty status is added to the model and estimated transition probabilities are computed for each group, a higher proportion of children below the poverty level do not transition from nonmastery to mastery of ending letter sounds from one time point to the other. For example, for those children living below the poverty level who have not mastered ending letter sounds at the beginning of first grade, 64% will transition to mastery by the end of first grade. This compares to 80% of the children living above the poverty level who will transition to mastery of ending letter sounds during the same time.

Application of the Latent Markov Model

Table 2 compares the transition probabilities for the manifest Markov model and the latent Markov model under the assumption of a stationary Markov chain. Only one chain is presented because the homogeneity assumption, by definition, fixes the transition probabilities to be equal over time. The Mplus syntax for the stationary Markov model is given in supplemental Appendix B, and the syntax for the stationary latent Markov model is given in supplemental Appendix C. The results show small but noticeable differences in the transition probabilities when taking into account measurement error in the manifest categorical responses. Here again we see that for children living below the poverty level, the transition rates are much lower than for those children living above the poverty level.

Application of Latent Transition Analysis

Using all five subtests of the reading assessment in ECLS-K, this section demonstrates a latent transition analysis. It should be noted that calculation of transition probabilities for each group was accomplished by transforming the regression coefficients from the Markov model to probabilities via the expression \[ 1/(1 + e^{-a + bX + ct}), \] where \( a \) is the intercept, \( b \) is the slope relating class membership at time \( t \) to class membership at time \( t - 1 \), and \( c \) is the slope relating class membership to poverty status. Appropriate regression coefficients can be obtained directly from the Mplus program.
noted that a specific form of the latent transition model was estimated—namely, a model that assumes no forgetting or loss of previous skills. This type of model, referred to as a longitudinal Guttman process, was used in a detailed study of stage-sequential reading development by Kaplan and Walpole (2005).

The analysis begins with the estimation of a series of latent class models for each time point separately. Preliminary analyses suggest that a three-class solution appears to provide the best overall fit and explanation of the observed response frequencies.

Table 1

<table>
<thead>
<tr>
<th>Mastery</th>
<th>Total sample</th>
<th>Below poverty</th>
<th>Above poverty</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Ending letter sounds Time 1 (rows) by ending letter sounds Time 2 (columns)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>.55</td>
<td>.45</td>
<td>.78</td>
</tr>
<tr>
<td>2</td>
<td>.10</td>
<td>.90</td>
<td>.26</td>
</tr>
<tr>
<td>Ending letter sounds Time 2 (rows) by ending letter sounds Time 3 (columns)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>.57</td>
<td>.43</td>
<td>.73</td>
</tr>
<tr>
<td>2</td>
<td>.10</td>
<td>.90</td>
<td>.20</td>
</tr>
<tr>
<td>Ending letter sounds Time 3 (rows) by ending letter sounds Time 4 (columns)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>.25</td>
<td>.75</td>
<td>.36</td>
</tr>
<tr>
<td>2</td>
<td>.03</td>
<td>.97</td>
<td>.05</td>
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Goodness-of-fit tests

<table>
<thead>
<tr>
<th>Total sample</th>
<th>With poverty covariate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi^2(8, N = 3,575) = 133.77, p &lt; .05$</td>
<td>$\chi^2(8, N = 3,575) = 150.23, p &lt; .05$</td>
</tr>
<tr>
<td>$\chi_{LR}^2(8, N = 3,575) = 112.54, p &lt; .05$</td>
<td>$\chi_{LR}^2(8, N = 3,575) = 127.71, p &lt; .05$</td>
</tr>
<tr>
<td>BIC = 13,363.49</td>
<td>BIC = 13,070.38</td>
</tr>
</tbody>
</table>

Note. 1 = nonmastery; 2 = mastery; $\chi^2$ refers to the Pearson chi-square test; $\chi_{LR}^2$ refers to the likelihood-ratio chi-square test; BIC = Bayesian information criterion.

The analysis begins with the estimation of a series of latent class models for each time point separately. Preliminary analyses suggest that a three-class solution appears to provide the best overall fit and explanation of the observed response frequencies.

Table 2

<table>
<thead>
<tr>
<th>Mastery</th>
<th>Manifest Markov chain</th>
<th>Latent Markov chain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Below poverty</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>.62</td>
<td>.38</td>
</tr>
<tr>
<td>2</td>
<td>.48</td>
<td>.52</td>
</tr>
<tr>
<td>Above poverty</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>.48</td>
<td>.52</td>
</tr>
<tr>
<td>2</td>
<td>.35</td>
<td>.65</td>
</tr>
</tbody>
</table>

Goodness-of-fit tests

<table>
<thead>
<tr>
<th>Total sample</th>
<th>With poverty covariate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi^2(12, N = 3,575) = 7,073.51, p &lt; .05$</td>
<td>$\chi^2(12, N = 3,575) = 6,974.35, p &lt; .05$</td>
</tr>
<tr>
<td>$\chi_{LR}^2(12, N = 3,575) = 6,155.69, p &lt; .05$</td>
<td>$\chi_{LR}^2(12, N = 3,575) = 6,198.25, p &lt; .05$</td>
</tr>
<tr>
<td>BIC = 19,039.31</td>
<td>BIC = 19,095.91</td>
</tr>
</tbody>
</table>

Note. Ending letter sounds Time 1 (rows) by ending letter sounds Time 2 (columns). For homogenous Markov model, all transition probabilities are the same over time. 1 = nonmastery; 2 = mastery; BIC = Bayesian information criterion.
Table 3
Response Probabilities and Class Proportions for Separate Latent Class Models (Total Sample)

<table>
<thead>
<tr>
<th>Wave</th>
<th>LAK</th>
<th>EWR</th>
<th>ERC</th>
<th>χ² (29)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fall</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LAK*</td>
<td>.47</td>
<td>.02</td>
<td>.01</td>
<td>.00</td>
</tr>
<tr>
<td>EWR</td>
<td>.97</td>
<td>.87</td>
<td>.47</td>
<td>.02</td>
</tr>
<tr>
<td>ERC</td>
<td>1.00</td>
<td>.99</td>
<td>.98</td>
<td>.97</td>
</tr>
<tr>
<td>Spring</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LAK</td>
<td>.56</td>
<td>.06</td>
<td>.00</td>
<td>.00</td>
</tr>
<tr>
<td>EWR</td>
<td>.99</td>
<td>.92</td>
<td>.63</td>
<td>.05</td>
</tr>
<tr>
<td>ERC</td>
<td>.00</td>
<td>.99</td>
<td>.99</td>
<td>.96</td>
</tr>
<tr>
<td>Fall first</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LAK</td>
<td>.52</td>
<td>.08</td>
<td>.01</td>
<td>.00</td>
</tr>
<tr>
<td>EWR</td>
<td>1.00</td>
<td>.92</td>
<td>.71</td>
<td>.05</td>
</tr>
<tr>
<td>ERC</td>
<td>1.00</td>
<td>.99</td>
<td>.98</td>
<td>.98</td>
</tr>
<tr>
<td>Spring first</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LAK</td>
<td>.19</td>
<td>.00</td>
<td>.00</td>
<td>.00</td>
</tr>
<tr>
<td>EWR</td>
<td>.98</td>
<td>.90</td>
<td>.79</td>
<td>.35</td>
</tr>
<tr>
<td>ERC</td>
<td>1.00</td>
<td>.99</td>
<td>.99</td>
<td>.99</td>
</tr>
</tbody>
</table>

Note. Response probabilities are for passed items. Response probabilities for failed items can be computed from 1 – prob(mastery). LR = letter recognition; BS = beginning sounds; ES = ending letter sounds; SW = sight words; WIC = words in context; Fall K = fall kindergarten; Spring K = spring kindergarten; Fall first = fall first grade; Spring first = spring first grade; LAK = low alphabet knowledge; EWR = early word reading; ERC = early reading comprehension.

Table 4
Transition Probabilities From Fall Kindergarten to Spring First Grade (Total Sample)

<table>
<thead>
<tr>
<th>Wave</th>
<th>LAK</th>
<th>EWR</th>
<th>ERC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fall</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LAK</td>
<td>.30</td>
<td>.69</td>
<td>.01</td>
</tr>
<tr>
<td>EWR</td>
<td>.00</td>
<td>.66</td>
<td>.34</td>
</tr>
<tr>
<td>ERC</td>
<td>.00</td>
<td>.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Spring</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LAK</td>
<td>.59</td>
<td>.40</td>
<td>.01</td>
</tr>
<tr>
<td>EWR</td>
<td>.00</td>
<td>.82</td>
<td>.18</td>
</tr>
<tr>
<td>ERC</td>
<td>.00</td>
<td>.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Goodness-of-fit tests

χ²(1,048,528, N = 3,575) = 12,384.21, p = 1.0
χ²(1,048,528, N = 3,575) = 6,732.31, p = 1.0
BIC = 44,590.80

Note. LAK = low alphabet knowledge; EWR = early word reading; ERC = early reading comprehension; Fall K = fall kindergarten; Spring K = spring kindergarten; Fall first = fall first grade; Spring first = spring first grade; BIC = Bayesian information criterion.
all the stayers are in the LAK class, they do not contribute to the transition probabilities for the movers. The slight differences between the mover transition probabilities compared to the transition probabilities in Table 4 are because 3% of the sample is in the stayer class. Finally, it is interesting that based on a comparison of the BICs, the results of the mover–stayer specification provide a better fit to the manifest response frequencies than the LTA model in Table 4. However, the discrepancy between the likelihood-ratio chi-square and Pearson chi-square is, again, indicative of sparse cells and needs to be inspected closely.

Conclusions

This article reviewed a selected set of models for stage-sequential developmental processes that represent extensions of the Markov chain model. The models selected for this article were applied to the problem of stage-sequential development of reading proficiency and included the manifest Markov model, the latent Markov model, the latent transition model (with an aside to consider latent class analysis), and the mixture latent transition model, with a special case examining the mover–stayer model.

Not every possible extension of the manifest Markov model was reviewed in this study. There have been recent developments that deserve serious consideration in the context of developmental research. For example, it is also now possible to embed Markov models into the multilevel modeling framework (see Asparouhov & Muthén, 2008). With this extension researchers can now specify the latent status variables to have random intercepts that vary across groups and to model that variability as a function of between-groups covariates. For example, a researcher can specify a multilevel mover–stayer model to examine whether there is between-schools variability in proportion of movers and stayers and how between-schools covariates might explain those differences.

In addition, a very recent extension by Muthén (2008) combines factor mixture analysis with latent transition analysis. In the context of the reading example, this hybrid model would allow researchers to examine if, say, the transition probability from the LAK class to the EWR class is influenced by underlying heterogeneity in the probability of correctly answering the relevant items. That is, those children who are in the LAK class who have higher probabilities of answering the LAK items correctly may have a great probability of transitioning into the EWR class compared to those with a low probability of answering the items correctly. Thus, transition probabilities based on the categorical responses are moderated by continuous underlying heterogeneity in the probability of those responses.

In the interest of providing a uniform computing framework for model estimation and testing, this article used the Mplus software program (Muthén & Muthén, 2006). The Mplus software program is quite comprehensive, incorporating general structural equation modeling for continuous latent and observed variables as well as models for categorical latent and observed variable. Perhaps the single most powerful aspect of the Mplus program is the general underlying model that combines categorical and continuous latent and observed variables into one unifying estimation procedure.

The power and flexibility of Mplus notwithstanding, other software is available that will estimate Markov models and extensions. The Methodology Center at Pennsylvania State University, under the directorship of Linda Collins, freely distributes the software.
program WinLTA (Collins, 2002a), which can estimate most of the models discussed in this article. However, it is not capable of estimating Markov models with a mixture component. The Methodology Center also distributes latent class analysis and latent add-ons for SAS, called PROC LTA and PROC LCA. Another program is Latent Gold (Vermunt & Magidson, 2005), which shares many of the features of the Mplus program in terms of estimating mixture latent transition analysis. As with any other software program, the decision as to which one to use will depend on factors of cost and efficiency.

In summary, the Markov model and its extensions provide only one tool in a large array of quantitative methodologies for the study of change, and as with any advanced statistical methodology, its utility lies in its ability to provide insights into substantive questions. The choice of the proper tool should, of course, be driven by a theory of the kind of developmental change that is taking place. The set of tools outlined in this article are suited to stage-sequential theories of development. They each provide a perspective that can be of great relevance to developmental psychologists and can be further extended to provide insights into the mechanisms that drive stage-sequential development. In the end, however, continued applications of these methodologies will bear out their substantive usefulness.

References

5 The URL for the Methodology Center is http://methodology.psu.edu/


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**Call for Papers:**

**Special Section on Three-Generation Research on Parenting and Its Consequences**

*Developmental Psychology* invites manuscripts for a special section on three-generation research on parenting and its consequences to be compiled by guest editors Rand Conger, Jay Belsky, and Deborah Capaldi working together with Associate Editor Richard Lerner.

The goal of the special section is to highlight recent high quality, prospective, longitudinal research on intergenerational continuities and discontinuities in parenting behavior and their consequences for child and adolescent development. Topics might include, but are not limited to, examination of the following:

- Mediating mechanisms that link quality of parenting in one generation to quality of parenting in the next,
- Social or personal events or conditions that either reduce or increase (i.e., moderate) the degree of intergenerational continuity in the quality of parenting behavior,
- Factors that disrupt intergenerational continuity in abusive parenting,
- The role of continuity in parenting as a nexus for similar developmental trajectories of children or adolescents in one generation and their children in the next generation, and
- Methodological issues related to the study of intergenerational continuity in parenting and its consequences.

Especially welcomed are papers that report the results of research on understudied populations such as ethnic minorities or rural as well as urban parents and children. The submission of recently completed doctoral dissertations is also encouraged.

The submission deadline is **March 15, 2008**. Initial inquiries regarding the special section may be sent to Rand Conger at rdconger@ucdavis.edu, Jay Belsky at j.belsky@bbk.ac.uk, or Deborah Capaldi at deborahc@oslc.org. Manuscripts must be submitted electronically through the Manuscript Submission Portal of *Developmental Psychology* at http://www.apa.org/journals/dev.html. Please be sure to specify in the cover letter that your submission is intended for the special section. For instructions to authors and other detailed submission information, see the journal website at http://www.apa.org/journals/dev.html.