

How Large Can a Standardized Coefficient be?

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June 22, 1999

A common misunderstanding is that the coefficients in the completely standardized solution must be smaller than one in magnitude and if they are not, something must be wrong. However, this need not be so. In this paper I try to answer the question of how large a standardized coefficient can be.

By a standardized coefficient I mean any estimated coefficient in a measurement or structural relationship in the completely standardized solution. The completely standardized solution in which both observed and latent variables are standardized, is obtained by putting `SC` on the `OU` line in a LISREL syntax file or by putting `SC` on an `Options` line (or on a `LISREL Output` line) in a SIMPLIS syntax file. It can also be obtained as a path diagram by selecting **Standardized Solution** under the **Estimates** menu when the path diagram for the unstandardized solution is visible on the screen.

The misunderstanding probably stems from classical exploratory factor analysis where factor loadings are correlations if a correlation matrix is analyzed and the factors are standardized and *uncorrelated (orthogonal)*. However, if the factors are *correlated (oblique)*, the factor loadings are *regression coefficients and not correlations* and as such they can be larger than one in magnitude. This can indeed happen also for any factor loading or structural coefficient in any LISREL model. Users who are only interested in this issue from a practical point of view can stop reading here. Just remember that a standardized coefficient of 1.04, 1.40, or even 2.80 does not necessarily imply that something is wrong, although, as will be seen, it might suggest that there is a high degree of multicollinearity in the data. The rest of this paper is only of academic interest.

How large can a standardized coefficient be? As most coefficients in LISREL are regression coefficients, I will concentrate on this case. More precisely, elements of Λ_y and Λ_x are regression coefficients, and if $\mathbf{B} = \mathbf{0}$ or if \mathbf{B} is subdiagonal and Ψ is diagonal, then the elements of \mathbf{B} and Γ are also regression coefficients. Otherwise, in the general case, the elements of \mathbf{B} and Γ are structural coefficients, and these can also be larger than one in magnitude in the completely standardized solution.

To look into this problem, let \mathbf{R} be a positive-definite correlation matrix for k variables:

$$\mathbf{R} = \begin{bmatrix} 1 & r_{12} & \cdots & r_{1k} \\ r_{12} & 1 & \cdots & r_{2k} \\ \vdots & \vdots & \vdots & \vdots \\ r_{1k} & r_{2k} & \cdots & 1 \end{bmatrix}$$

and consider the regression of each variable on all the others. The regression coefficient γ_{ij} of variable j in the regression for variable i is

$$\gamma_{ij} = \frac{r^{ij}}{r^{ii}}, j \neq i$$

where r_{ij} are elements of the inverse \mathbf{R}^{-1} of \mathbf{R} . I conjecture that γ_{ij} can be made larger than 1 by suitable choices of the r_{ij} and that when these choices are made such that \mathbf{R} approaches singularity, γ_{ij} can be made arbitrarily large. I don't intend to prove this conjecture, but rather indicate with a simple example that the conjecture might hold.

Consider the case $k = 3$. Then we have three correlations r_{12} , r_{13} , and r_{23} . These cannot be chosen arbitrarily as many combinations of them give a correlation matrix which is not positive definite. If the correlations are strictly between -1 and +1, the condition of positive definiteness can be written explicitly as

$$d = 1 - r_{12}^2 - r_{13}^2 - r_{23}^2 + r_{12}r_{13}r_{23}$$

The regression coefficient γ_{12} of variable 2 in the regression of variable 1 on variables 2 and 3 can be given explicitly as

$$\gamma_{12}(r_{12}, r_{13}, r_{23}) = 1 / (1 - r_{23}^2) / (r_{12} - r_{13}r_{23})$$

Since

$$\gamma_{12}(r_{12}, r_{13}, r_{23}) = \gamma_{12}(r_{12}, -r_{13}, -r_{23}) = \gamma_{12}(-r_{12}, -r_{13}, r_{23}) = \gamma_{12}(-r_{12}, r_{13}, -r_{23})$$

I need only investigate positive values of the correlations. To make γ_{12} large, choose r_{23} as large as possible and then choose values of r_{12} and r_{13} that maximize γ_{12} subject to d being positive. Table 1 gives some values for increasing values of r_{23} .

Table 1
Values of d and γ_{12} for different values of r_{ij}

| Case | r_{12} | r_{13} | r_{23} | d | γ_{12} |
|------|----------|----------|----------|-----------|---------------|
| 1 | 0.800 | 0.300 | 0.800 | 0.0140000 | 1.556 |
| 2 | 0.600 | 0.200 | 0.900 | 0.0060000 | 2.211 |
| 3 | 0.210 | 0.070 | 0.990 | 0.0000060 | 7.070 |
| 4 | 0.190 | 0.091 | 0.995 | 0.0000011 | 9.970 |
| 5 | 0.200 | 0.156 | 0.999 | 0.0000006 | 22.089 |

Table 1 shows that as r_{23} approaches 1, so that \mathbf{R} gets closer to singularity, γ_{12} gets larger and larger. To verify the results using LISREL, run the following SIMPLIS command file corresponding to Case 4:

```
Test
Observed Variables: X1 X2 X3
Covariance Matrix
*1 .19 1 .091 .995 1
Sample Size: 100
X1 = X2 X3
Number of Decimals: 3
End of Problem
```

This gives the following result

```
X1 = 9.970*X2 - 9.830*X3, Errorvar. = 0.000110
      (0.0107)   (0.0107)   (0.000)
      933.934   -920.740   6.964
```

The fact that the error variance is so small shows that there is an almost exact linear relationship between the three variables, and hence that **R** is nearly singular