

Mplus Short Courses  
Topic 9

**Bayesian Analysis Using Mplus**

Bengt Muthén  
Linda K. Muthén

Copyright © 2010 Muthén & Muthén  
[www.statmodel.com](http://www.statmodel.com)  
08/04/2010

1

**Table Of Contents**

Overview of Bayesian Features In Mplus	7
Bayesian Estimation	13
Path Analysis With Indirect Effects	32
Model Fit	45
Factor Analysis	51
Bayesian Analysis When ML Is Slow Or Intractable Due To Many Dimensions Of Numerical Integration	61
Multiple Indicator Growth Modeling With Categorical Variables	74
Mixture Modeling	82
Multilevel Regression With A Continuous Dependent Variable	88
A Simulation Study: Multilevel Regression With A Small Number Of Clusters	99
Multilevel Regression With A Categorical Dependent Variable And Several Random Slopes	109
Multilevel Regression With A Categorical Dependent Variable And Small Random Slope Variance	111

2

## Table Of Contents (Continued)

Multiple Imputation	115
Multiple Imputation With A Categorical Outcome In A Growth Model With MAR Missing Data: Using WLSMV On Imputed Data	126
H <sub>0</sub> Imputation	129
Two-Level Imputation	133
Chi-Square Model Testing Using Multiple Imputation Data: Asparouhov-Muthén (2010): Chi-Square Statistics With Multiple Imputation	141
Overview Of Other New Features In Mplus Version 6	146
References	153

3

## Bayesian Analysis In Mplus

Mplus conceptualization:

- Mplus was envisioned 15 years ago as both a frequentist and a Bayesian program
- Bayesian analysis firmly established and its use growing in mainstream statistics
- Much less use of Bayes outside statistics
- Bayesian analysis not sufficiently accessible in other programs
- Bayes provides a broader platform for further Mplus development

4

## Bayesian Analysis

Why do we have to learn about Bayes?

- More can be learned about parameter estimates and model fit
- Better small-sample performance, large-sample theory not needed
- Analyses can be made less computationally demanding
- New types of models can be analyzed

5

## Writings On The Bayes Implementation In Mplus

- Asparouhov & Muthén (2010a). Bayesian analysis using Mplus. Technical appendix
- Asparouhov & Muthén, B. (2010b). Bayesian analysis of latent variable models using Mplus. Mplus scripts and data.
- Asparouhov, T. & Muthén, B. (2010c). Multiple imputation with Mplus. Mplus scripts and data.
- Muthén (2010). Bayesian analysis in Mplus: A brief introduction. Technical report with Mplus scripts and data. Ongoing writing - comments invited

Posted under Papers, Bayesian Analysis

6

## **Overview Of Bayesian Features In Mplus**

7

## **Bayesian Estimation In Mplus**

- Single-level, multilevel, and mixture models
- Continuous and categorical outcomes (probit link)
- Default non-informative priors or user-specified informative priors (MODEL PRIORS)
- Multiple chains using parallel processing (CHAIN)
- Convergence assessment using Gelman-Rubin potential scale reduction factors
- Posterior parameter distributions with means, medians, modes, and credibility intervals (POINT)
- Posterior parameter trace plots
- Autocorrelation plots
- Posterior predictive checking plots

8

## **Multiple Imputation (DATA IMPUTATION)**

- Carried out using Bayesian estimation to create several data sets where missing values have been imputed
- The multiple imputation data sets can be used for subsequent model estimation using ML or WLSMV
- The imputed data sets can be saved for subsequent analysis or analysis can be carried out in the same run
- Imputation can be done based on an unrestricted H1 model using three different algorithms including sequential regressions
- Imputation can also be done based on an H0 model specified in the MODEL command

9

## **Multiple Imputation (Continued)**

- The set of variables used in the imputation of the data do not need to be the same as the set of variables used in the analysis
- Single-level and multilevel data imputation are available
- Multiple imputation data can be read using TYPE=IMPUTATION in the DATA command

10

## **Plausible Values (PLAUSIBLE)**

- Plausible values used in IRT contexts such as the ETS NAEP, The Nation's Report Card (Mislevy et al., 1992)
- Plausible values are multiple imputations for missing values corresponding to a latent variable
- Available for both continuous and categorical latent variables (factors, random effects, latent classes)
- More informative than only an estimated factor score and its standard error or a class probability
- Plausible values are given for each observation together with a summary over the imputed data sets for each observation and latent variable
- Multiple imputation and plausible values examples are given in the User's Guide, Chapter 11

11

## **Bayesian Analysis Using Mplus: An Ongoing Project**

Features that are not yet implemented include:

- EFA and ESEM
- Logit link
- Censored, count, and nominal variables
- XWITH
- Weights
- Random slopes in single-level models
- Latent variable decomposition of covariates in two-level models
- c ON x in mixtures
- Mixture models with more than one categorical latent variable
- Two-level mixtures
- MODEL INDIRECT
- MODEL CONSTRAINT except for NEW parameters
- MODEL TEST

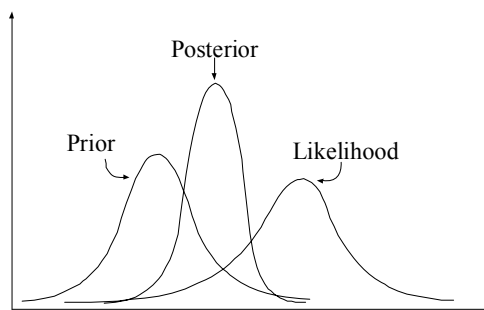
12

## Bayesian Estimation

13

## Prior, Likelihood, And Posterior

- Frequentist view: Parameters are fixed. ML estimates have an asymptotically-normal distribution
- Bayesian view: Parameters are variables that have a prior distribution. Estimates have a possibly non-normal posterior distribution. Does not depend on large-sample theory
  - Diffuse (non-informative) priors vs informative priors



14

## Bayes Theorem

- Probabilities of events A and B:

$$P(A, B) = P(A|B)P(B)$$

$$\text{Bayes theorem: } P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$

- Applied to modeling:

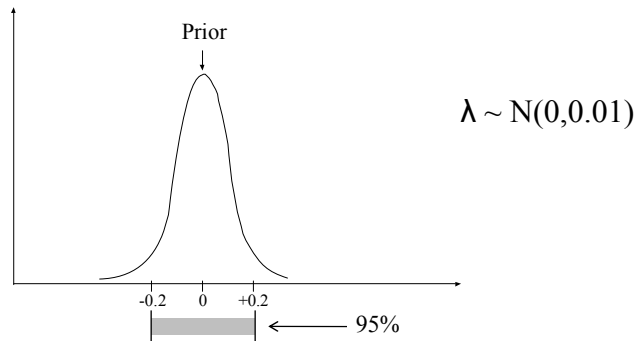
$$\begin{aligned} [parameters|data] &= \frac{[data|parameters][parameters]}{[data]} \\ &= \frac{\text{likelihood} \times \text{prior}}{[data]} \end{aligned}$$

- Posterior  $\propto$  likelihood  $\times$  prior

15

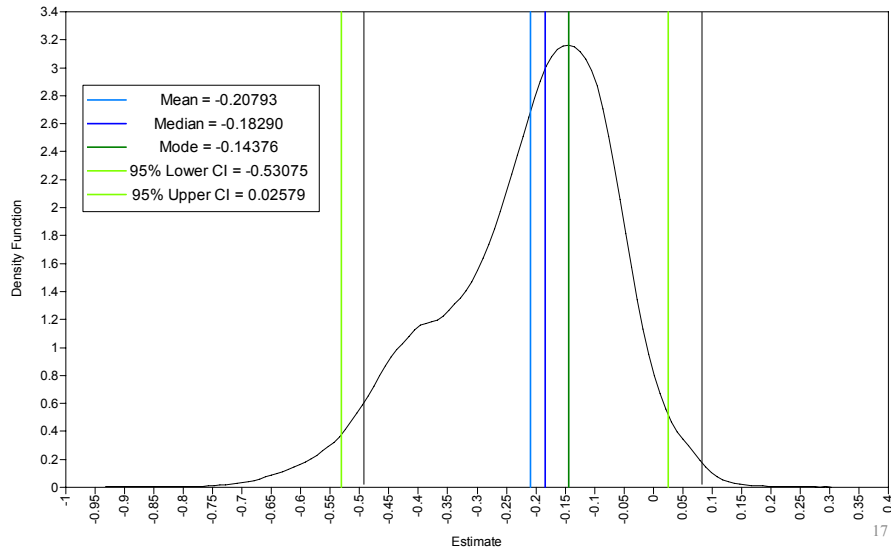
## Where Do Parameter Priors Come From?

- Previous studies
- Hypotheses based on substantive theory
  - Example: Zero cross-loadings in CFA



16

## Non-Normal Posterior Parameter Distribution



## Bayesian Estimation Using the Markov Chain Monte Carlo (MCMC) Algorithm

- $\theta_i$  : vector of parameters, latent variables, and missing observations at iteration  $i$
- $\theta_i$  is divided into  $S$  sets:  

$$\theta_i = (\theta_{1i}, \dots, \theta_{Si})$$
- Update  $\theta$  using Gibbs sampling over  $i = 1, 2, \dots, n$  iterations:  

$$\theta_{1i} \mid \theta_{2i-1}, \dots, \theta_{Si-1}, \text{ data, priors}$$

$$\theta_{2i} \mid \theta_{1i}, \theta_{3i-1}, \dots, \theta_{Si-1}, \text{ data, priors}$$

...

$$\theta_{Si} \mid \theta_{1i}, \dots, \theta_{S-1i-1}, \text{ data, priors}$$

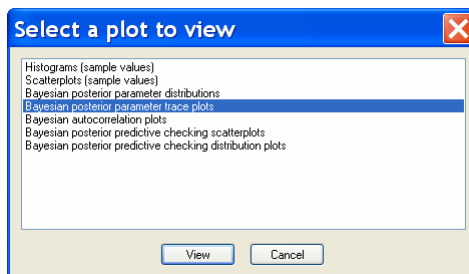
Asparouhov & Muthén (2010a). Bayesian analysis using Mplus.  
 Technical appendix.

## MCMC Iteration Issues

- Trace plot: Graph of the value of a parameter at different iterations
- Burnin phase: Discarding early iterations. Mplus discards first half
- Posterior distribution: Mplus uses the last half as a sample representing the posterior distribution
- Autocorrelation plot: Correlation between different iterations for a parameter. Low correlation desired
- Mixing: The MCMC chain should visit the full range of parameter values, i.e. sample from all areas of the posterior density
- Convergence: Stationary process.

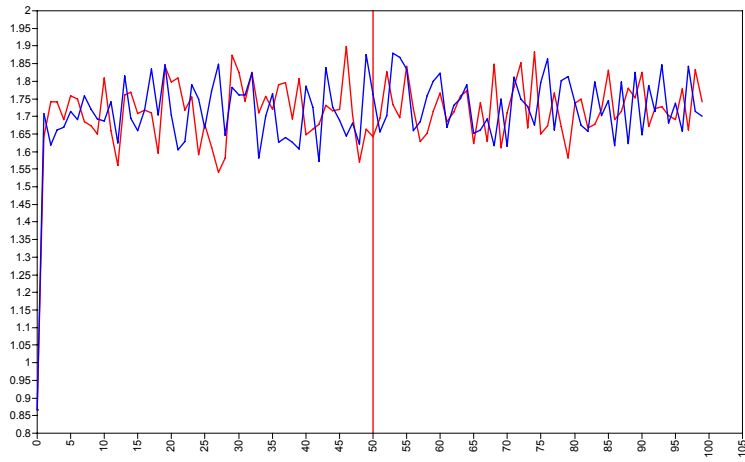
19

## Bayes Graphs



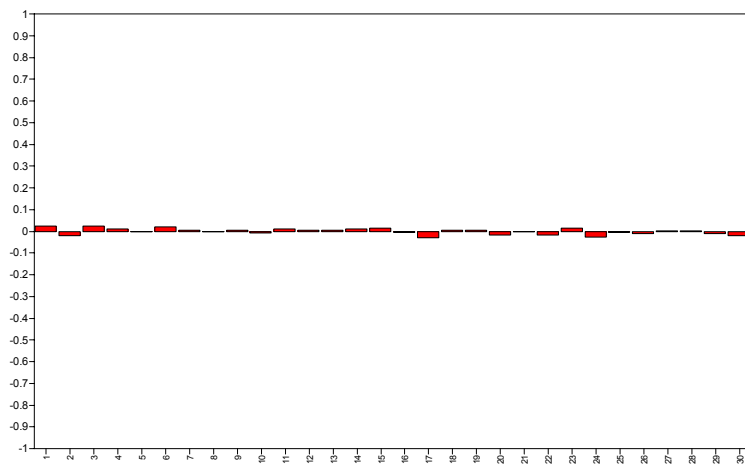
20

## Trace Plot 1



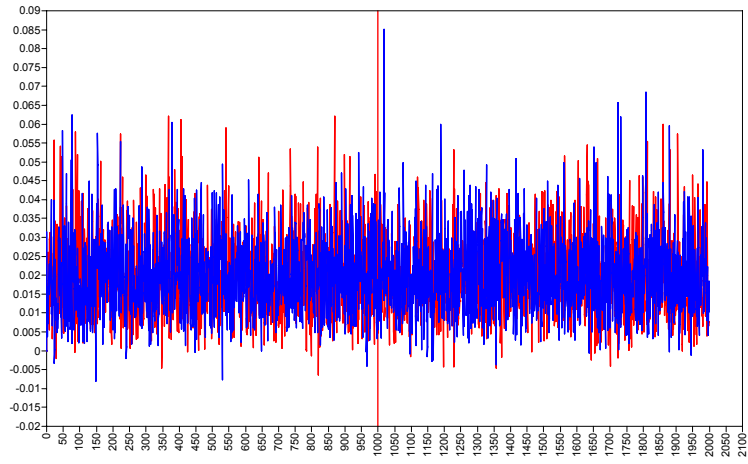
21

## Auto Correlation Plot 1



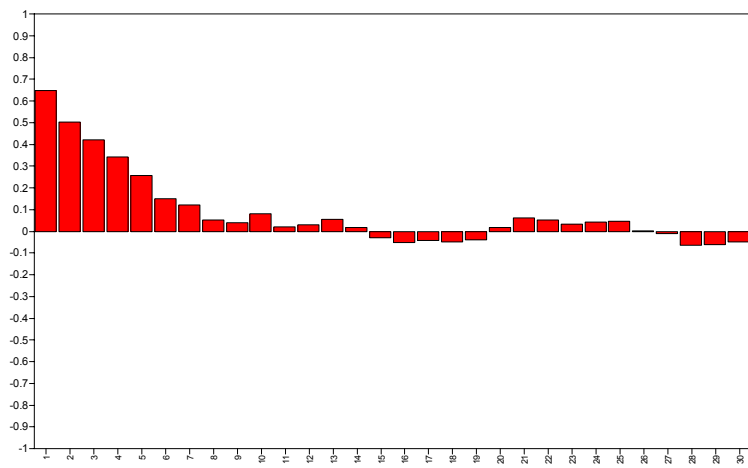
22

## Trace Plot 2



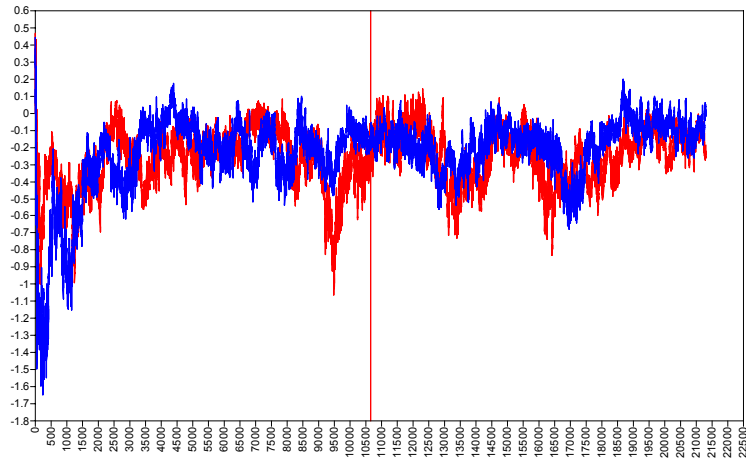
23

## Auto Correlation Plot 2



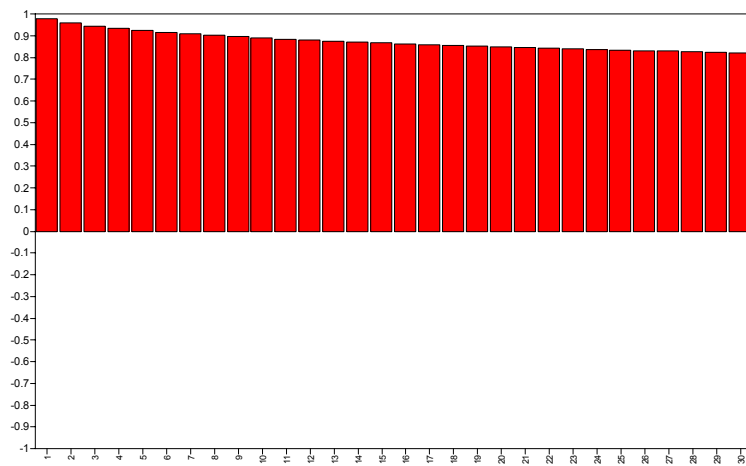
24

### Trace Plot 3



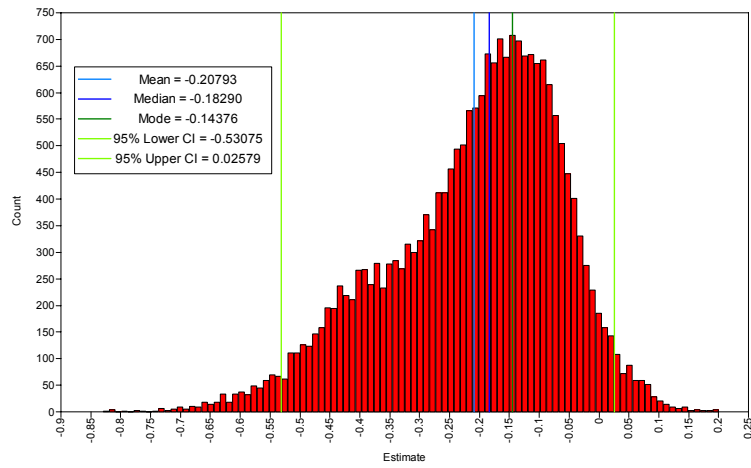
25

### Auto Correlation Plot 3



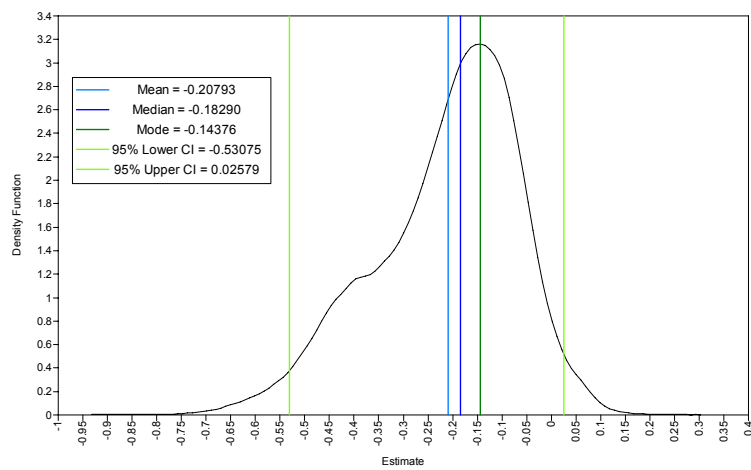
26

## Posterior Using Histogram Option



27

## Posterior Using Kernel Option



28

## Convergence: Potential Scale Reduction Factor (PSR; TECH8)

- Several MCMC iterations carried out in parallel, independent chains. PSR considers  $n$  iterations in  $m$  chains, where  $\theta_{ij}$  is the value of  $\theta$  in iteration  $i$  of chain  $j$ :

$$\bar{\theta}_j = \frac{1}{n} \sum_{i=1}^n \theta_{ij} \qquad \bar{\theta}_{..} = \frac{1}{m} \sum_{j=1}^m \bar{\theta}_j$$

$$B = \frac{1}{m-1} \sum_{j=1}^m (\bar{\theta}_j - \bar{\theta}_{..})^2 \qquad W = \frac{1}{m} \sum_{j=1}^m \frac{1}{n} \sum_{i=1}^n (\theta_{ij} - \bar{\theta}_j)^2$$

$$PSR = \sqrt{\frac{W+B}{W}}$$

- Convergence if PSR is not much larger than 1, e.g. less than 1.05 or 1.1.

29

## TECH8 For Bayes

```

C:\WINDOWS\system32\cmd.exe
Mplus DEVELOPMENT (indev 8/2/2010)
MUTHEN & MUTHEN

Running input file 'c:\share\chapt\explan runs\ba book - topic 1 mplus runs\
ba book\chapt 2 grade 1 - grade 2\CH8_1019.mui' from attempt 1 of 1
ba grade 1 if 4 verbal invariant except talkback race lunch gender.tsp ...

TECHNICAL & OUTPUT FOR BAYES ESTIMATION

      POTENTIAL      PARAMETER WITH      TOTAL
ITERATION  SCALE REDUCTION  HIGHEST PSR      TIME
1000      1.833           5.4           0.31
2000      1.155           5.4           0.28
3000      1.448           5.4           0.28
4000      2.282           5.4           0.28
5000      1.218           5.4           1.4
6000      1.848           5.4           1.7
7000      1.874           5.4           2.0
8000      1.859           4.4           2.3
9000      1.875           4.4           2.6
10000     1.153           4.4           2.8
11000     1.289           4.4           3.1
12000     1.153           4.4           3.4
13000     1.882           11           3.7
14000     1.885           11           4.0
15000     1.822           11           4.2
16000     1.825           11           4.5
17000     1.854           11           4.8
18000     1.875           11           5.1
19000     1.867           11           5.4
20000     1.876           11           5.7
21000     1.146           4.4           6.0
22000     1.179           4.4           6.3
23000     1.163           4.4           6.6
24000     1.153           4.4           6.9
25000     1.232           4.4           7.2
26000     1.231           4.4           7.5
27000     1.279           4.4           7.8
28000     1.288           4.4           8.1
29000     1.243           4.4           8.4
  
```

30

## **Options Related To Bayes Estimation And Multiple Imputation**

See User's Guide pages 559-563:

- POINT (mean, median, mode; default is median)
- CHAINS (default is 2)
- BSEED
- STVALUES (= ML)
- MEDIATOR (observed, latent; default is latent)
- ALGORITHM (GIBBS, MH; default is GIBBS)
- BCONVERGENCE (related to PSR)
- BITERATIONS (to go beyond 50K iterations)
- FBITERATIONS (fixed number of iterations)
- THIN (every kth iteration recorded; default is 1)
- DISTRIBUTION (how many iterations used for MODE)

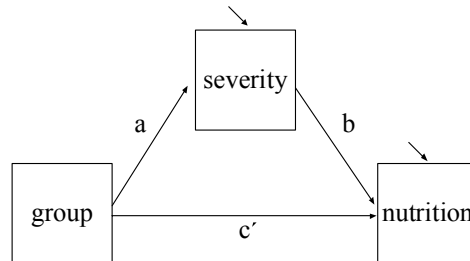
31

## **Path Analysis With Indirect Effects**

32

## Mediation Modeling With Diffuse Priors: ATLAS Example

Source: MacKinnon et al. (2004), MBR.  $n = 861$



- Intervention aimed at increasing perceived severity of using steroids among athletes. Perceived severity of using steroids is in turn hypothesized to increase good nutrition behaviors
- Indirect effect:  $a \times b$

33

## Input For Bayesian Analysis Of ATLAS Example

```
TITLE:          ATLAS
DATA:           FILE = mbr2004atlast.txt;
VARIABLE:       NAMES = obs group severity nutrit;
                USEV = group - nutrit;
ANALYSIS:       ESTIMATOR = BAYES;
                PROCESS = 2;
MODEL:          severity ON group (a);
                nutrit ON severity (b)
                group;
MODEL CONSTRAINT:
                NEW (indirect);
                indirect = a*b;
OUTPUT:         TECH1 TECH8 STANDARDIZED;
PLOT:           TYPE = PLOT2;
```

34

## Output For Bayesian Analysis Of ATLAS Example

Parameter	Estimate	Posterior	One-Tailed	95% C.I.	
		S.D.	P-Value	Lower 2.5%	Upper 2.5%
severity ON					
group	0.282	0.106	0.010	0.095	0.486
nutrit ON					
severity	0.067	0.031	0.000	0.015	0.125
group	-0.011	0.089	0.440	-0.180	0.155
Intercepts					
severity	5.641	0.072	0.000	5.513	5.779
nutrit	3.698	0.191	0.000	3.309	4.018

35

## Output For Bayesian Analysis Of ATLAS Example (Continued)

Parameter	Estimate	Posterior	One-Tailed	95% C.I.	
		S.D.	P-Value	Lower 2.5%	Upper 2.5%
Residual variances					
severity	1.722	0.072	0.000	1.614	1.868
group	1.331	0.070	0.000	1.198	1.468
New/Additional parameters					
indirect	0.016	0.013	0.010	0.002	0.052

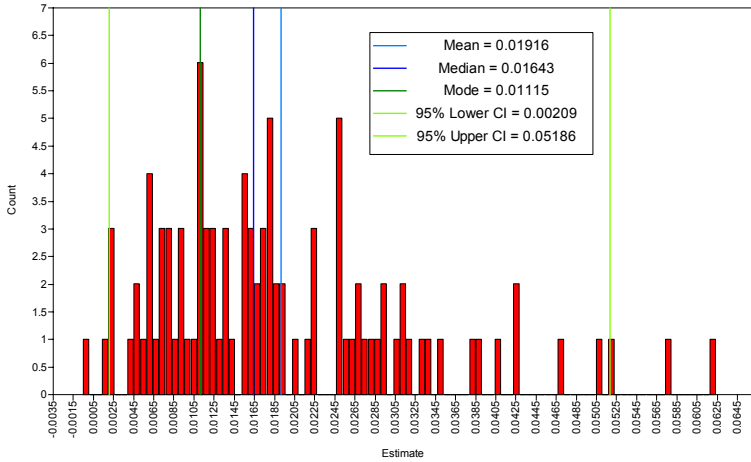
36

# Output For Bayesian Analysis Of ATLAS Example (Continued)

TECHNICAL 8 OUTPUT FOR BAYES ESTIMATION

CHAIN	BSEED	
1	0	
2	285380	
	POTENTIAL	PARAMETER WITH
ITERATION	SCALE REDUCTION	HIGHEST PSR
100	1.037	2

# Posterior Distribution For Indirect Effect: 100 Iterations

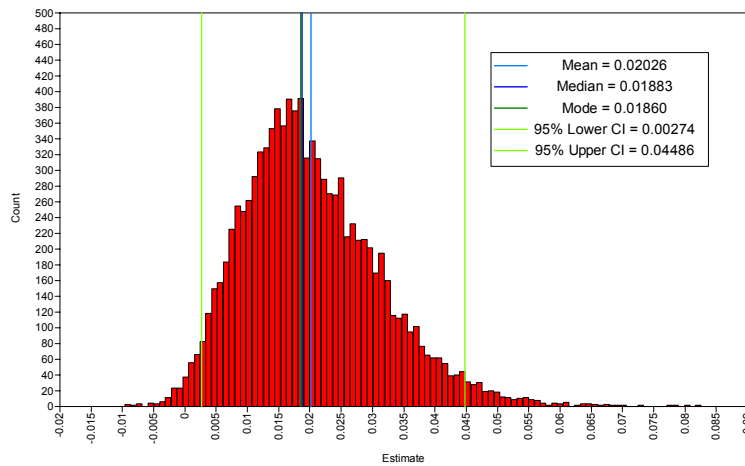


## Mplus Input For Bayesian Analysis Of ATLAS Example (Continued)

```
ANALYSIS: ESTIMATOR = BAYES;  
          PROCESS = 2;  
          FBITER = 10000;
```

39

## Bayesian Posterior Distribution For The Indirect Effect: 10000 Iterations



40

## Bayesian Posterior Distribution For The Indirect Effect

- Bayesian analysis: There is a mediated effect of the intervention
  - The 95% Bayesian credibility interval does not include zero
- ML analysis: There is not a mediated effect of the intervention
  - ML-estimated indirect effect is not significantly different from zero and the symmetric confidence interval includes zero
  - Bootstrap SEs and CIs can be used with ML

41

## Mediation Modeling With Informative Priors: Firefighter Example. $N = 354$

- Source: Yuan & MacKinnon (2009). Bayesian Mediation Analysis. Psychological Methods, 14, 301-322.
  - Nice description of Bayesian analysis
- Informative priors based on previous studies:  $a \sim N(0.35, 0.04)$ ,  $b \sim N(0.1, 0.01)$
- 95% credibility interval for indirect effect shrunken by 16%
- WinBUGS code in Yuan & MacKinnon (2009).  
Mplus code on next slide using MODEL PRIORS. Same results

42

## Mplus Input For Bayesian Analysis With Priors: Firefighters

```
TITLE:      Yuan and MacKinnon firefighters mediation using
            Bayesian analysis
            Elliot DL., Goldberg L., Kuehl KS, et al. The PHLAME
            Study: process and outcomes of 2 models of behavior
            change. J Occup Environ Med. 2007; 49(2): 204-213.

DATA:      FILE = fire.dat;

VARIABLE:  NAMES = y m x;

MODEL:     m ON x (a);
            y ON m (b)
            x;

ANALYSIS:  ESTIMATOR = BAYES;
            PROCESS = 2;
            FBITER = 10000;
```

43

## Mplus Input For Bayesian Analysis With Priors: Firefighters (Continued)

```
MODEL PRIORS:
            a ~ N(0.35, 0.04);
            b ~ N(0.1, 0.01);

MODEL CONSTRAINT:
            NEW (indirect);
            indirect = a*b;

OUTPUT:    TECH1 TECH8;

PLOT:      TYPE = PLOT2;
```

44

## Model Fit

45

## Posterior Predictive Checking

- A posterior predictive p-value (PPP) of model fit can be obtained via a fit statistic  $f$  based on the usual chi-square test of  $H_0$  against  $H_1$ . Low PPP indicates poor fit
- Let  $f(Y, X, \theta_i)$  be computed for the data  $Y, X$  using the parameter values at iteration  $i$
- At iteration  $i$ , generate a new data set  $Y_i^*$  (synthetic/replicated data) of the same size as the original data using the parameter values at iteration  $i$  and compute  $f(Y_i^*, X, \theta_i)$  for these replicated data

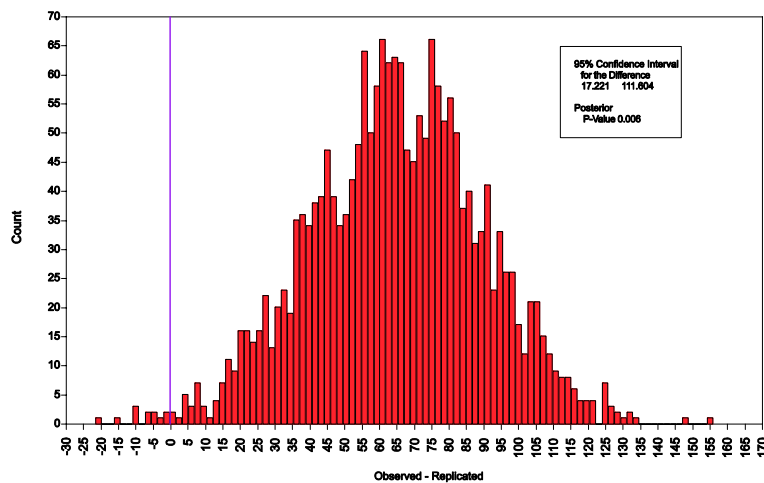
46

## Posterior Predictive Checking (Continued)

- PPP is approximated by the proportion of iterations where  $f(Y, X, \theta_i) < f(Y_1^*, X, \theta_i)$
- Mplus computes PPP using every 10<sup>th</sup> iteration among the iterations used to describe the posterior distribution of parameters
- A 95% confidence interval is produced for the difference in chi-square for the real and replicated data; negative lower limit is good

47

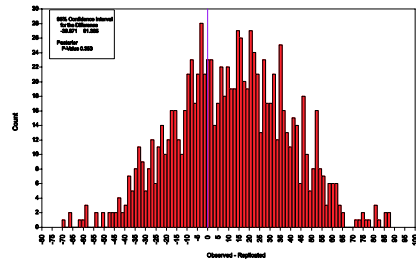
## Posterior Predictive Checking For A Poorly Fitting Model



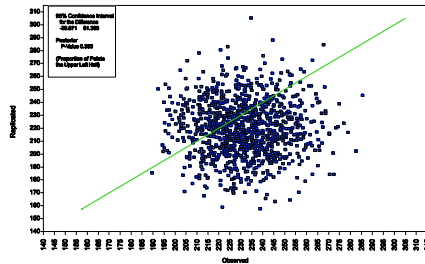
48

## Posterior Predictive Checking For A Well Fitting Model

PPC distribution for Bayes CFA



PPC scatterplot for Bayes CFA



49

## Deviance Information Criterion (DIC)

- DIC is a Bayesian generalization of the ML AIC and BIC (low value is good)
- DIC uses a number of parameters the effective number of parameters referred to as  $p_D$

50

## Factor Analysis

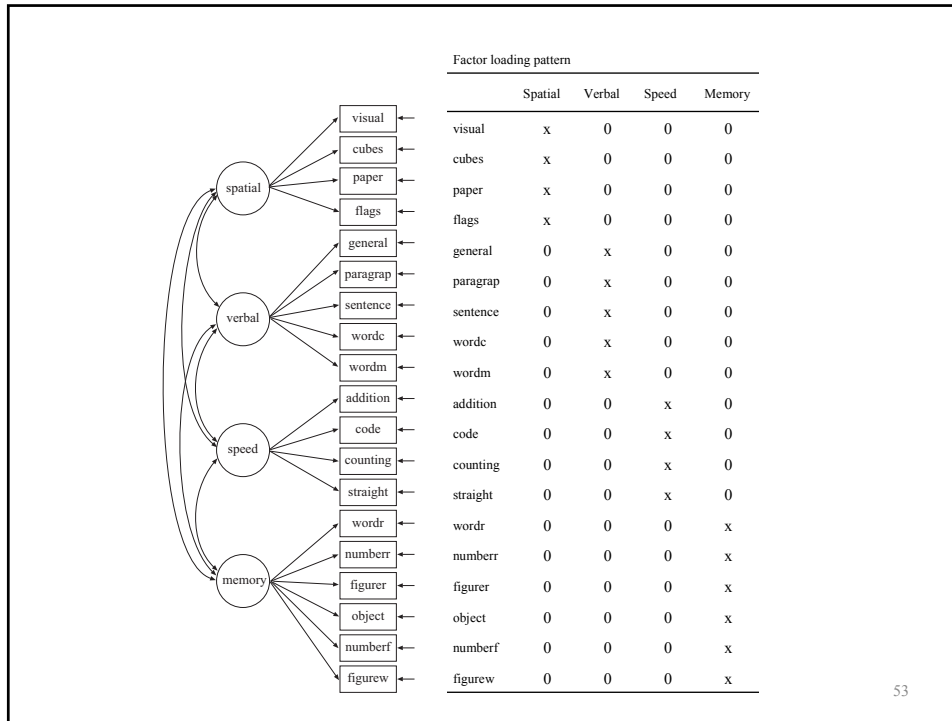
51

## Holzinger-Swineford Data

- 19 tests hypothesized to measure four mental abilities: Spatial, verbal, speed, and memory
- n=145 7<sup>th</sup> and 8<sup>th</sup> grade students from Grant-White elementary school

Source: Muthén (2010). Bayesian analysis in Mplus: A brief introduction. Technical report with Mplus scripts and data.

52



## ML Tests Of Model Fit For Holzinger-Swineford (N = 145)

- Confirmatory factor analysis (CFA) model rejected:
  - ML likelihood-ratio chi-square  $p = 0.0002$   
(CFI = 0.93, RMSEA = 0.057, SRMR = 0.063)
    - Modification indices point to three cross-loadings  
(Modindices by Sörbom, 1989)
    - Model still rejected when those are freed
- Exploratory factor analysis (EFA) with 4 factors using oblique rotation (Geomin):
  - ML likelihood-ratio chi-square  $p = 0.248$   
(CFI = 0.99, RMSEA = 0.025, SRMR = 0.030)
    - The pattern of major loadings is as hypothesized
    - Several significant cross-loadings

## Bayesian CFA Using MCMC For Holzinger-Swineford

- CFA: Cross-loadings fixed at zero - the model is rejected
- A more realistic hypothesis: Small cross-loadings allowed
- Cross-loadings are not all identified in terms of ML
- Different alternative: Bayesian CFA with informative cross-loading priors:  $\lambda \sim N(0, 0.01)$ .

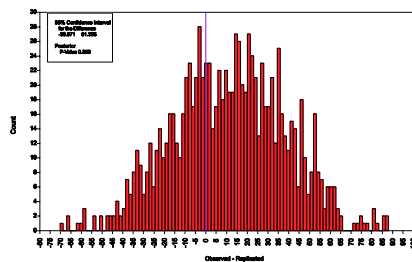
This means that 95% of the prior is in the range -0.2 to 0.2

55

## Bayesian Posterior Predictive Checking For The CFA Model

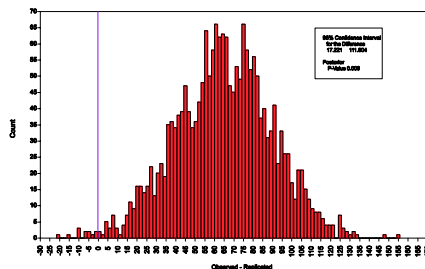
CFA with small cross-loadings  
not rejected by Bayes PPC:

$$p = 0.353$$



Conventional CFA model  
rejected by Bayes PPC:

$$p = 0.006:$$



56

## Summary Of Analyses Of Holzinger-Swineford Data

- Conventional, frequentist, CFA model rejected
- Bayesian CFA not rejected with cross-loadings
- The Bayesian approach uses an intermediate hypothesis:
  - Less strict than conventional CFA
  - Stricter than EFA, where the hypothesis only concerns the number of factors
- Bayes modification indices obtained by estimated cross-loadings

57

## Input Bayes CFA 19 Items 4 Factors Crossloading Priors

```
DEFINE:      visual = visual/sqrt(47.471);
             cubes = cubes /sqrt(19.622);
             paper = paper /sqrt(7.908);
             flags = flags/sqrt(68.695);
             general = general/sqrt(134.970);
             paragraf = paragraf/sqrt(11.315);
             sentence = sentence/sqrt(21.467);
             wordc = wordc/sqrt(28.505);
             wordm = wordm/sqrt(62.727);
             addition = addition/sqrt(561.692);
             code = code/sqrt(275.759);
             counting = counting/sqrt(437.752);
             straight = straight/sqrt(1362.158);
             wordr = wordr/sqrt(116.448);
             numberr = numberr/sqrt(56.496);
             figurer = figurer/sqrt(45.937);
             object = object/sqrt(20.730);
             numberf = numberf/sqrt(20.150);
             figurew = figurew/sqrt(12.845);
```

58

## Input Bayes CFA 19 Items 4 Factors Crossloading Priors (Continued)

```
ANALYSIS: ESTIMATOR = BAYES;
           PROCESS = 2;
           FBITER = 10000;

MODEL:    spatial BY visual* cubes paper flags;
           verbal BY general* paragraf sentence wordc wordm;
           speed BY addition* code counting straight;
           memory BY wordr* numberr figurer object numberf
           figurew;

           spatial-memory@1;

           spatial BY general-figurew*0 (a1-a15);
           verbal BY visual-flags*0 (b1-b4);
           verbal BY addition-figurew*0 (b5-b14);
           speed BY visual-wordm*0 (c1-c9);
           speed BY wordr-figurew*0 (c10-c15);
           memory BY visual-straight*0 (d1-d13);
```

59

## Input Bayes CFA 19 Items 4 Factors Crossloading Priors (Continued)

```
MODEL PRIORS:

           a1-a15 ~ N(0,.01);
           b1-b14 ~ N(0,.01);
           c1-c15 ~ N(0,.01);
           d1-d13 ~ N(0,.01);

OUTPUT:    TECH1 TECH8 STDYX;

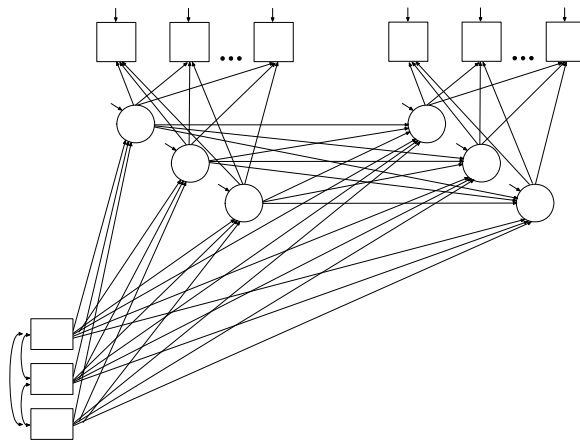
PLOT:      TYPE = PLOT2;
```

60

**Bayesian Analysis When ML Is Slow Or  
Intractable Due To Many Dimensions Of  
Numerical Integration**

61

**Structural Equation Model With  
Categorical Factor Indicators Of  
Three Factors At Two Timepoints**



62

## Johns Hopkins Aggression Study, 13 Items, Cohort 3 (N=678)

Factor loadings

Variables	Grade 1			Grade 3		
	Verbal	Person	Property	Verbal	Person	Property
stubborn	<b>0.892*</b>	0.000	-0.286*	<b>0.913*</b>	0.000	-0.220*
breaks rules	0.413*	0.301*	0.001	0.500*	0.286*	0.001
harms others & property	-0.007	0.508*	0.376*	-0.009	0.526*	0.372*
breaks things	0.016	0.006	<b>0.808*</b>	0.017	0.057	<b>0.669*</b>
yells at others	<b>0.803*</b>	0.019	-0.053	<b>0.821*</b>	0.015	-0.040
takes others' property	0.227	0.026	<b>0.589*</b>	0.277	0.025	0.541*
fight	0.016	<b>0.886*</b>	0.074	0.020	<b>0.838*</b>	0.067
harms property	0.155	0.004	<b>0.819*</b>	0.186	0.004	<b>0.742*</b>
lies	<b>0.793*</b>	-0.254	0.328*	<b>0.759*</b>	-0.191	0.236*
talks back to adults	<b>1.112*</b>	-0.355	0.009	<b>0.949*</b>	-0.238	0.006
teases classmates	0.408*	0.372*	0.006	0.454*	0.326*	0.005
fight with classmates	0.153*	<b>0.792*</b>	0.002	0.183*	<b>0.744*</b>	0.002
loses temper	<b>0.863*</b>	0.043	-0.149	<b>0.826*</b>	0.032	-0.107

63

## Johns Hopkins Aggression Study, 13 Items, Cohort 3 (N=678)

Factor correlations with variances on the diagonal

Variables	Grade 1			Grade 3		
	Verbal	Person	Property	Verbal	Person	Property
Grade 1						
Verbal	1.000					
Person	0.856 (0.037)	1.000				
Property	0.727 (0.082)	0.660 (0.079)	1.000			
Grade 3						
Verbal	0.338 (0.068)	0.253 (0.069)	0.148 (0.078)	1.512 (0.246)		
Person	0.262 (0.077)	0.224 (0.073)	0.109 (0.076)	0.819 (0.068)	0.936 (0.184)	
Property	0.119 (0.078)	0.117 (0.072)	0.075 (0.072)	0.670 (0.105)	0.675 (0.089)	0.855 (0.216)

64

## Computational Issues

- Maximum-likelihood estimation with categorical indicators requires numerical integration with six dimensions which is not feasible (problems of computing time, memory, numerical precision). Computational time grows exponentially with the number of continuous latent variables (factors, random effects).
- Bayes is feasible. Computational time grows linearly with the number of continuous latent variables.
  - Hopkins aggression study: Convergence after 7 minutes using two processors and the default of two MCMC chains, converging in 30K iterations

65

## Statistical Issues

- Measurement invariance
  - (1) None
  - (2) Configural
  - (3) Factor loadings (factors on the same scale)
  - (4) Factor loadings and intercepts (growth studies possible)
- (3) chosen here

66

## Input Excerpts Structural Equation Model With Three Factors At Two Timepoints

```
USEVARIABLES = y1-y13 y301-y313 black lunch312 male;  
CATEGORICAL = y1-y313;  
DEFINE:      CUT y1-y313 (1.5);  
ANALYSIS:    ESTIMATOR = BAYES;  
              PROCESSORS = 2;  
MODEL:       f11 BY y1@1  
              y2*.5 (1)  
              y3@0  
              y4@0  
              y5*1 (2)  
              y6*0 (3)  
              y7*0 (4)  
              y8*0 (5)  
              y9*1 (6)  
              y10*1 (7)  
              y11*.5 (8)  
              y12*0 (9)  
              y13*1 (10);
```

67

## Input Excerpts Structural Equation Model With Three Factors At Two Timepoints (Continued)

```
f12 by y1@0  
y2*.5 (11)  
y3*.5 (12)  
y4*0 (13)  
y5*0 (14)  
y6*0 (15)  
y7@1  
y8@0  
y9*0 (16)  
y10*0 (17)  
y11*0 (18)  
y12*1 (19)  
y13*0 (20);
```

68

**Input Excerpts Structural Equation Model  
With Three Factors At Two Timepoints  
(Continued)**

```
f13 by y1*0 (31)
y2*0 (32)
y3*.5 (33)
y4*1 (34)
y5*0 (35)
y6*.5 (36)
y7*0 (37)
y8@1
y9*.5 (38)
y10@0
y11*8 (39)
y12@0
y13*0 (40);
```

69

**Input Excerpts Structural Equation Model  
With Three Factors At Two Timepoints  
(Continued)**

```
f21 by y301@1
y302*.5 (1)
y303@0
y304@0
y305*1 (2)
y306*0 (3)
y307*0 (4)
y308*0 (5)
y309*1 (6)
y310*1 (7)
y311*.5 (8)
y312*0 (9)
y313*1 (10);
```

70

**Input Excerpts Structural Equation Model  
With Three Factors At Two Timepoints  
(Continued)**

```
f22 by y301@0  
y302*.5 (11)  
y303*.5 (12)  
y304*0 (13)  
y305*0 (14)  
y306*0 (15)  
y307@1  
y308@0  
y309*0 (16)  
y310*0 (17)  
y311*0 (18)  
y312*1 (19)  
y313*0 (20);
```

71

**Input Excerpts Structural Equation Model  
With Three Factors At Two Timepoints  
(Continued)**

```
f23 by y301*0 (31)  
y302*0 (32)  
y303*.5 (33)  
y304*1 (34)  
y305*0 (35)  
y306*.5 (36)  
y307*0 (37)  
y308@1  
y309*.5 (38)  
y310@0  
y311*8 (39)  
y312@0  
y313*0 (40);  
  
f11-f23 ON black lunch312 male;  
OUTPUT: TECH1 STDY TECH8;  
PLOT: TYPE = PLOT3;
```

72

## **Output Excerpts Structural Equation Model With Three Factors At Two Timepoints**

Test of model fit

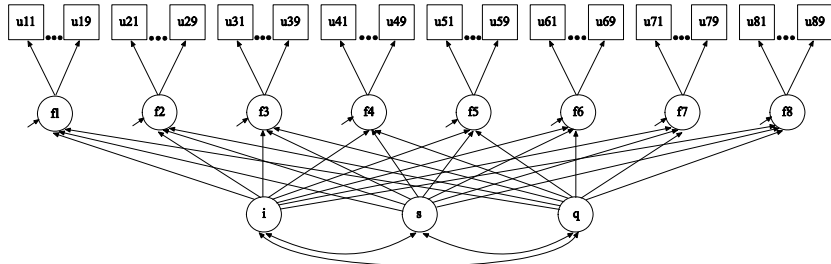
Number of Free Parameters	95
Bayesian Posterior Predictive Checking using Chi-Square	
95% Confidence Interval for the Difference Between the Observed and the Replicated Chi-Square Values	
	-67.231 108.112
Posterior Predictive P-Value	0.324

73

## **Multiple Indicator Growth Modeling With Categorical Variables**

74

## Growth Model With 9 Categorical Indicators Of A Factor Measured At 8 Time Points



75

## Hopkins Aggression Study, Cohort 1, Grade 1 – 7, N = 1174

- Nine binary items
- Measurement invariant loadings and thresholds across time points

76

## Input For Bayes Multiple-Indicator Growth Modeling

```
TITLE: Hopkins Cohort 1 All time points with Classroom  
Information  
DATA: FILE = Cohort1 classroom ALL.DAT;  
VARIABLE: NAMES = PRCID stub1F bkRule1F harm01F bkThin1F  
yell1F takeP1F ght1F lies1F tease1F stub1S bkRule1S  
harm01S bkThin1S yell1S takeP1S ght1S lies1S tease1S  
stub2S bkRule2S harm02S bkThin2S yell2S takeP2S  
ght2S lies2S tease2S stub3S bkRule3S harm03S bk-  
Thin3S yell3S takeP3S ght3S lies3S tease3S stub4S  
bkRule4S harm04S bkThin4S yell4S takeP4S ght4S  
lies4S tease4S stub5S bkRule5S harm05S bkThin5S  
yell5S takeP5S ght5S lies5S tease5S stub6S bkRule6S  
harm06S bkThin6S yell6S takeP6S ght6S lies6S tease6S  
stub7S bkRule7S harm07S bkThin7S yell7S takeP7S  
ght7S lies7S tease7S gender race des011 sch011  
sec011 juv99 violchld antisocr conductr athort1F  
harmP1S athort1S harmP2S athort2S harmP3S athort3S
```

77

## Input For Bayes Multiple-Indicator Growth Modeling (Continued)

```
harmP4S athort4S harmP5S athort5S harmP6S harmP7S  
athort7S stub2F bkRule2F harm02F bkThin2F yell2F  
takeP2F ght2F harmP2F lies2F athort2F tease2F  
classrm;  
USEVAR = stub1f-tease7s male;  
CATEGORICAL = categorical = stub1f-tease7s;  
MISSING = ALL (999);  
DEFINE: cut stub1f-tease7s(1.5);  
MALE = 2 - gender;  
ANALYSIS: PROCESS = 2;  
ESTIMATOR = BAYES;  
FBITER = 20000;  
MODEL: f1 BY stub1f  
bkrule1f-tease1f (1-8);  
f2 BY stub1s  
bkrule1s-tease1s (1-8);  
f3 BY stub2s  
bkrule2s-tease2s (1-8);
```

78

## Input For Bayes Multiple-Indicator Growth Modeling (Continued)

```

f4 BY stub3s
bkrule3s-tease3s (1-8);
f5 BY stub4s
bkrule4s-tease4s (1-8);
f6 BY stub5s
bkrule5s-tease5s (1-8);
f7 BY stub6s
bkrule6s-tease6s (1-8);
f8 BY stub7s
bkrule7s-tease7s (1-8);
[stub1f$1 stub1s$1 stub2s$1 stub3s$1 stub4s$1] (11);
[stub5s$1 stub6s$1 stub7s$1] (11);
[bkrule1f$1 bkrule1s$1 bkrule2s$1 bkrule3s$1] (12);
[bkrule4s$1 bkrule5s$1 bkrule6s$1 bkrule7s$1] (12);
[harmo1f$1 harmo1s$1 harmo2s$1 harmo3s$1] (13);
[harmo4s$1 harmo5s$1 harmo6s$1 harmo7s$1] (13);
[bkthin1f$1 bkthin1s$1 bkthin2s$1 bkthin3s$1] (14);
[bkthin4s$1 bkthin5s$1 bkthin6s$1 bkthin7s$1] (14);

```

79

## Input For Bayes Multiple-Indicator Growth Modeling (Continued)

```

[yell1f$1 yell1s$1 yell2s$1 yell3s$1 yell4s$1
yell5s$1] (15);
[yell6s$1 yell7s$1] (15);
[takeP1f$1 takeP1s$1 takeP2s$1 takeP3s$1] (16);
[takeP4s$1 takeP5s$1 takeP6s$1 takeP7s$1] (16);
[ght1f$1 ght1s$1 ght2s$1 ght3s$1 ght4s$1] (17);
[ght5s$1 ght6s$1 ght7s$1] (17);
[lies1f$1 lies1s$1 lies2s$1 lies3s$1 lies4s$1
lies5s$1] (18);
[lies6s$1 lies7s$1] (18);
[tease1f$1 tease1s$1 tease2s$1 tease3s$1 tease4s$1]
(19);
[tease5s$1 tease6s$1 tease7s$1] (19);
[f1-f8@0];
i s q | f1@0 f2@.5 f3@1.5 f4@2.5 f5@3.5 f6@4.5 f7@5.5
f8@6.5;
i-q ON male;

```

OUTPUT: TECH1 TECH4 TECH8 TECH10 STANDARDIZED SVALUES;

PLOT: TYPE = PLOT2;

80

## Output For Bayes Multiple-Indicator Growth Modeling

```
Tests of model fit
Number of Free Parameters                36
Bayesian Posterior Predictive Checking using Chi-Square
  95% Confidence Interval for the Difference Between
  the Observed and the Replicated Chi-Square Values
                                         129.488  547.650
Posterior Predictive P-Value            0.002
```

81

## Mixture Modeling

82

## Growth Mixture Modeling

```
TITLE:      this is an example of a GMM for a continuous outcome
            using automatic starting values and random starts

DATA:      FILE = ex8.1.dat;
            NOBS = 200;

VARIABLE:  NAMES = y1-y4 x;
            CLASSES = c(2);

ANALYSIS:  TYPE= MIXTURE;
            ESTIMATOR = BAYES;
            CHAIN = 1;
            FBITER = 20000;

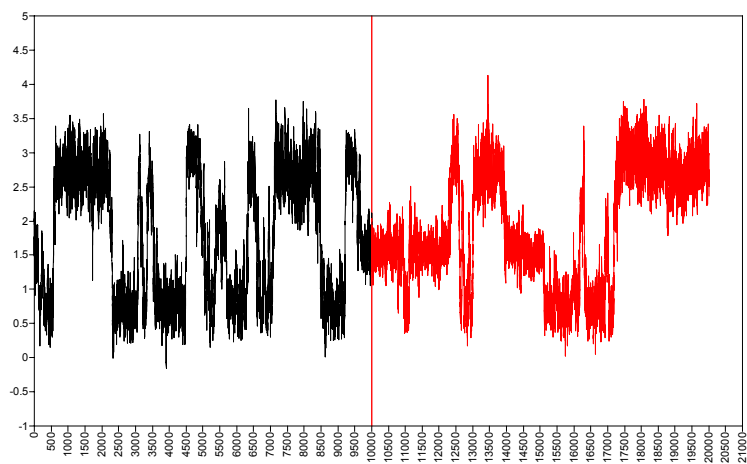
MODEL:     %OVERALL%
            i s | y1@0 y2@1 y3@2 y4@3;
            i s ON x;

OUTPUT:    TECH1 TECH8;

PLOT:     TYPE = PLOT3;
```

83

## Trace Plot For Intercept Mean In Class 1 Mixture Label Switching



84

## Growth Mixture Modeling With Parameter Constraint

```
TITLE:      this is an example of a GMM for a continuous outcome
             using automatic starting values and random starts

DATA:      FILE = ex8.1.dat;
           NOBS = 200;

VARIABLE:  NAMES = y1-y4 x;
           CLASSES = c(2);

ANALYSIS:  TYPE = MIXTURE;
           ESTIMATOR = BAYES;
           FBITER = 20000;
           CHAIN = 1;

MODEL:     %OVERALL%
           i s | y1@0 y2@1 y3@2 y4@3;
           i s ON x;
           %c#1%
           [i*1] (m1);
           %c#2%
           [i*0] (m2);
```

85

## Growth Mixture Modeling With Parameter Constraint (Continued)

```
OUTPUT:    TECH1 TECH8;

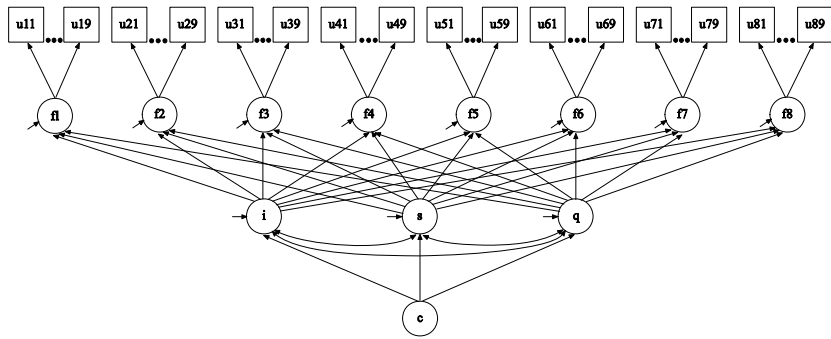
PLOT:      TYPE = PLOT3;

MODEL CONSTRAINT:

           m1 > m2;
```

86

## Growth Mixture Model With Multiple Categorical Indicators



87

## Multilevel Regression With A Continuous Dependent variable

88

## Multilevel Regression With A Random Intercept

Consider a two-level regression model for individuals  $i = 1, 2, \dots, n_j$   
in clusters  $j = 1, 2, \dots, J$ ,

$$y_{ij} = \beta_{0j} + \beta_{1j}x_{ij} + r_{ij}, \quad (1)$$

$$\beta_{0j} = \gamma_{00} + u_{0j}, \quad (2a)$$

$$\beta_{1j} = \gamma_{10} \quad (2b)$$

89

## Multilevel Regression With A Continuous Dependent Variable And A Small Number Of Clusters

- 10 schools, each with 50 students
- Intra-class correlation 0.10

90

## Input For Random Intercept Regression

```
TITLE:
DATA:      FILE = c10n50iccl.dat;
VARIABLE:  NAMES = y x clus;
           WITHIN = x;
           CLUSTER = clus;
ANALYSIS:  TYPE = TWOLEVEL;
           ESTIMATOR = BAYES;
           PROCESS = 2;
           FBITER = 10000;
MODEL:     %WITHIN%
           y ON x;
           y (w);
           %BETWEEN%
           y (b);
```

91

## Input For Random Intercept Regression (Continued)

```
MODEL PRIORS:
           b ~ IG (.001, .001);
MODEL CONSTRAINT:
           NEW(icc);
           icc = b/(b+w);
OUTPUT:    TECH1 TECH8;
PLOT:      TYPE = PLOT2;
```

92

## Output For Random Intercept Regression

Parameter	Estimate	Posterior	One-Tailed	95% C.I.	
		S.D.	P-Value	Lower 2.5%	Upper 2.5%
Within Level					
y ON					
x	0.909	0.069	0.000	0.777	1.042
Residual variances					
y	2.105	0.135	0.000	1.866	2.394

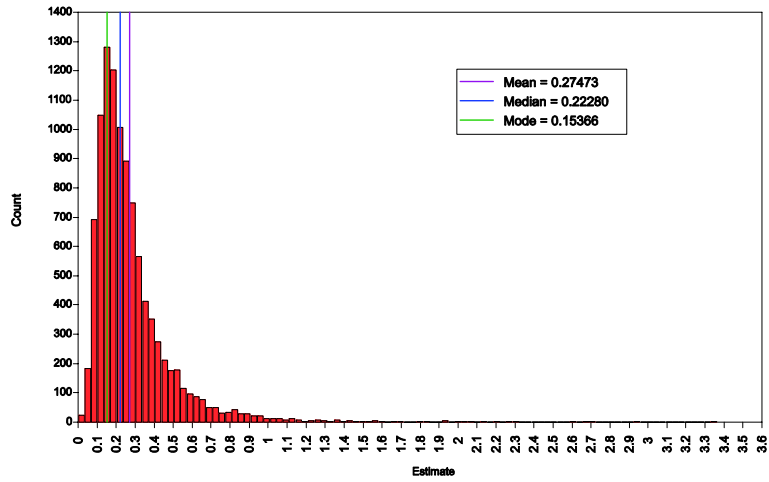
93

## Output For Random Intercept Regression (Continued)

Parameter	Estimate	Posterior	One-Tailed	95% C.I.	
		S.D.	P-Value	Lower 2.5%	Upper 2.5%
Between Level					
Means					
y	0.145	0.178	0.191	-0.209	0.493
Variances					
y	0.223	0.205	0.000	0.073	0.805
New/Additional Parameters					
ICC	0.096	0.063	0.00	0.033	0.276

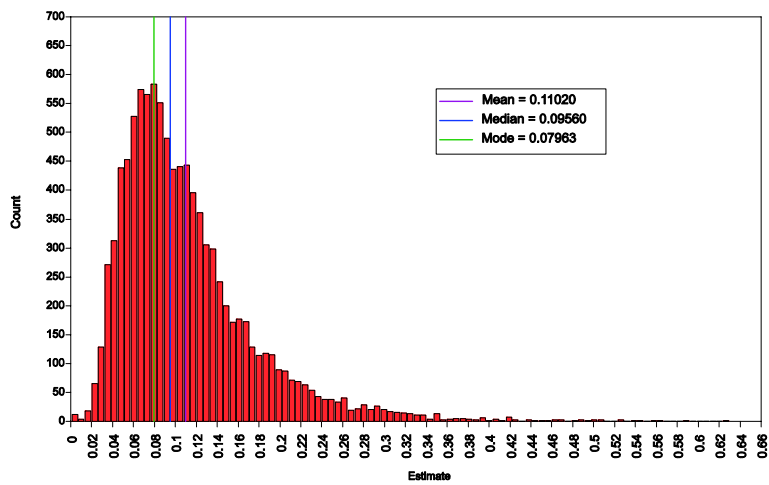
94

## Posterior Distribution Of Between-Level Intercept Variance



95

## Posterior Distribution Of Intra-Class Correlation



96

## Output For ML Twolevel Regression

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Within Level				
y ON				
x	0.909	0.069	13.256	0.000
Residual variances				
y	2.089	0.133	15.653	0.000

97

## Output For ML Twolevel Regression (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Between Level				
Means				
y	0.143	0.152	0.942	0.346
Variances				
y	0.190	0.104	1.828	0.067
New/Additional Parameters				
ICC	0.083	0.042	1.975	0.048

98

**A Simulation Study:  
Multilevel Regression  
With A Small Number Of Clusters**

99

**Output Excerpts For ML In A Monte Carlo  
Study Of Twolevel Regression**

Parameter	Population	Estimates		S.E.	M.S.E.	95%	% Sig
		Average	Std. Dev.	Average		Cover	Coeff
Within Level							
y ON							
x	1.000	0.9957	0.0630	0.0639	0.0040	0.948	1.000
Residual variances							
y	2.000	2.0052	0.1291	0.1281	0.0167	0.946	1.000

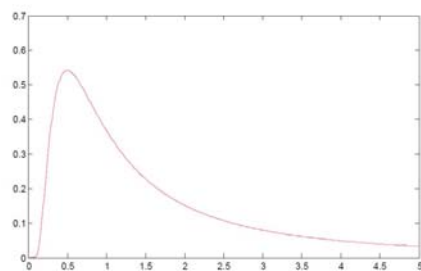
100

## Output Excerpts For ML In A Monte Carlo Study Of Twollevel Regression (Continued)

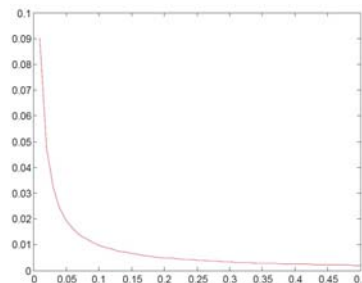
Parameter	Population	Estimates		S.E.	M.S.E.	95%	% Sig
		Average	Std. Dev.	Average		Cover	Coeff
Between Level							
Means							
y	0.000	-0.0035	0.1624	0.1485	0.0263	0.892	0.108
Variances							
y	0.222	0.1932	0.1155	0.1045	0.0141	0.808	0.180
New/Additional Parameters							
ICC	0.100	0.0860	0.0458	0.0422	0.0023	0.812	0.498

101

## Bayes Monte Carlo Study: Different Inverse- Gamma Priors For Variance Parameters



IG(1,1) density



IG(0.001,0.001) density

The mean for  $IG(\alpha, \beta)$  is  $\beta/(\alpha-1)$

The mode is  $\beta/(\alpha+1)$

$\alpha$ : shape parameter

$\beta$ : scale parameter

Overview in Browne & Draper (2006)

102

## Input For Bayes Monte Carlo Twolevel Regression Using An IG (0.001, 0.001) Prior

```
TITLE:          Bayes IG (eps,eps)
MONTECARLO:    NAMES = y x;
               NOBS = 500;
               NREP = 500;
               NCSIZES = 1;
               CSIZES = 10 (50); ! 10 clusters of size 50
               WITHIN = x;

MODEL POPULATION:
               %WITHIN%
               x*1;
               y ON x*1;
               y*2;
               %BETWEEN%
               y*.222; !icc = .222/2.222 = 0.1

ANALYSIS:      TYPE = TWOLEVEL;
               ESTIMATOR = BAYES;
               PROCESS = 2;
               FBITER = 10000;
```

103

## Input For Bayes Monte Carlo Twolevel Regression Using An IG (0.001, 0.001) Prior (Continued)

```
MODEL:         %WITHIN%
               y ON x*1;
               y*2 (w);
               %BETWEEN%
               y*.22(b); !icc = .222/2.222 =0.1

MODEL PRIORS:
               b ~ IG (.001, .001);

MODEL CONSTRAINT:
               NEW(icc*.1);
               icc = b/(w+b);

OUTPUT:        TECH9;
```

104

## Output For Bayes Monte Carlo Twolevel Regression Using An IG (0.001, 0.001) Prior

Parameter	Population	Estimates		S.E.	M.S.E.	95%	% Sig
		Average	Std. Dev.	Average		Cover	Coeff
Between Level							
Means							
y	0.000	0.0128	0.1646	0.1746	0.0272	0.928	0.072
Variances							
y	0.222	0.2322	0.1373	0.2036	0.0189	0.942	1.000
New/Additional Parameters							
ICC	0.100	0.1011	0.0532	0.0618	0.0028	0.934	1.000

105

## Output For Bayes Monte Carlo Twolevel Regression Using A U(0, 1000) Prior

Parameter	Population	Estimates		S.E.	M.S.E.	95%	% Sig
		Average	Std. Dev.	Average		Cover	Coeff
Between Level							
Means							
y	0.000	0.0190	0.1645	0.2106	0.0277	0.966	0.034
Variances							
y	0.222	0.2304	0.1246	0.3255	0.0156	0.930	1.000
New/Additional Parameters							
ICC	0.100	0.1012	0.0484	0.0841	0.0023	0.928	1.000

106

## Output For Bayes Monte Carlo Twolevel Regression Using An IG (-1, 0) Prior

Parameter	Population	Estimates		S.E.	M.S.E.	95%	% Sig
		Average	Std. Dev.	Average		Cover	Coeff
Between Level							
Means							
y	0.000	0.0123	0.1645	0.2122	0.0272	0.966	0.034
Variances							
y	0.222	0.3348	0.1784	0.3767	0.0445	0.928	1.000
New/Additional Parameters							
ICC	0.100	0.1391	0.0626	0.0876	0.0054	0.928	1.000

107

## Monte Carlo Studies Of Twolevel Regression

- Muthén (2010). Bayesian analysis in Mplus: A brief introduction. Mplus scripts and data.
- Asparouhov & Muthén, B. (2010). Bayesian analysis of latent variable models using Mplus. Mplus scripts and data.
- Browne & Draper (2006). A comparison of Bayesian and likelihood-based methods for fitting multilevel models. Bayesian Analysis, 3, 473-514.

108

## Multilevel Regression With A Categorical Dependent Variable And Several Random Slopes

109

## Bias (Percent Coverage) For Random Intercept Variance ( = 0.5 ) In Two-Level Probit Regression With $q$ Random Effects

- 200 clusters of size 20

Prior	$q=1$	$q=2$	$q=3$	$q=4$	$q=5$	$q=6$
$IW(0, -q -1)$	0.03 (90)	0.04 (92)	0.04 (96)	0.08 (81)	0.10 (84)	0.19 (60)
$IW(I, q +1)$	0.03 (89)	0.02 (93)	-0.01 (97)	-0.01 (95)	-0.04 (97)	-0.05 (92)
$IW(2I, q +1)$	0.03 (90)	0.03 (93)	0.01 (96)	0.02 (97)	-0.01 (97)	-0.01 (97)

- $q = 1$ : Random intercept only
- Prior 1 is the default in Mplus 6.0. Prior 2 is the default in Mplus 6.01
- Source: Asparouhov & Muthén, B. (2010). Bayesian analysis of latent variable models using Mplus.

110

## **Multilevel Regression With A Categorical Dependent Variable And Small Random Slope Variance**

111

## **Monte Carlo Simulation With A Small Random Slope Variance**

- Random intercept and two random slopes with the second random slope variance of zero
- 200 clusters of size 20
- Bayes: 5 sec/rep
- ML: 19 minutes/rep due to slow convergence with random slope variance zero

112

## Bias (Percent Coverage) For Small Random Effect Variance Estimation

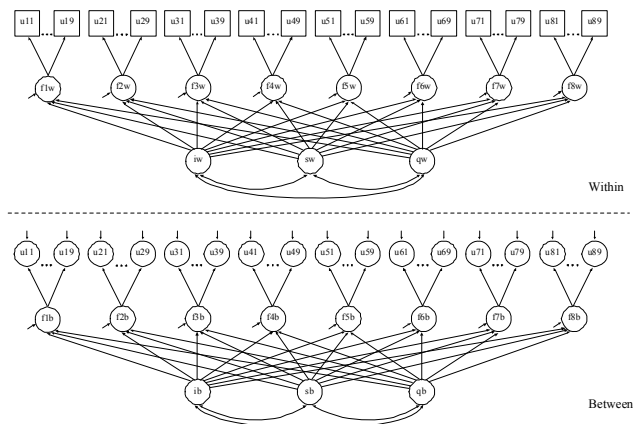
Parameter	ML	Bayes
$\alpha_1$	0.01 (90)	0.01 (93)
$\alpha_2$	0.01 (95)	0.01 (89)
$\alpha_3$	0.00 (96)	0.01 (98)
$\beta_1$	0.01 (96)	0.00 (97)
$\beta_2$	0.00 (98)	0.01 (95)
$\beta_3$	0.00 (95)	0.03 (95)
$\psi_{11}$	0.01 (96)	0.02 (94)
$\psi_{22}$	0.01 (93)	0.03 (94)
$\psi_{33}$	0.01 (99)	0.06 (0)
$\psi_{12}$	0.00 (97)	-0.01 (95)
$\psi_{13}$	0.00 (97)	0.00 (98)
$\psi_{23}$	0.00 (97)	0.01 (97)

113

## Twolevel Growth: 3/4 -Level Analysis

Multiple-indicator growth modeling (T occasions, p items/occ.):

- Number of dimensions:  $2 \times T$ , or  $T + p \times T$  (2-level growth with between-level residuals)



114

## Multiple Imputation

115

## Analysis With Missing Data

Used when individuals are not observed on all outcomes in the analysis to make the best use of all available data and to avoid biases in parameter estimates, standard errors, and tests of model fit.

### Types of Missingness

- MCAR -- missing completely at random
  - Variables missing by chance
  - Missing by randomized design
  - Multiple cohorts assuming a single population

116

## **Analysis With Missing Data (Continued)**

- MAR -- missing at random
  - Missingness related to observed variables
  - Missing by selective design
- Non-Ignorable (NMAR)
  - Missingness related to values that would have been observed
  - Missingness related to latent variables

117

## **Correlates Of Missing Data**

- MAR is more plausible when the model includes covariates influencing missing data
- Correlates of missing data may not have a “causal role” in the model, i.e. not influencing dependent variables, in which case including them as covariates can bias model estimates.

118

## Correlates Of Missing Data (Continued)

- Two solutions:
  - (1) Modeling (ML)
    - Including missing data correlates not as x variables but as “y variables,” freely correlated with all other observed variables
  - (2) Multiple imputation (Bayes; Schafer, 1997) with two different sets of observed variables
    - Imputation model
    - Analysis model

Overview in Enders (2010).

119

## Bayesian Imputation Of Missing Data

- 3 Steps:
  - (1) Estimate the model using Bayes
  - (2) Draw a set of parameter values from the posterior distribution
  - (3) For each set of parameter values, generate missing data according to the model
- Choice of model in step (1):
  - H1: Unrestricted model
  - H0: Restricted model: Factor model, growth model, latent class model, twolevel model, etc

120

## **Three H1 Imputation Approaches In Mplus: DATA IMPUTATION: MODEL =**

- COVARIANCE: Default for continuous variables
  - (1) Bayes estimation of an H1 model, (2) Do multiple draws of (2a) parameters, (2b) missing data generated from those parameters
- SEQUENTIAL: Default for combination of continuous and categorical variables (Ragunathan et al., 2001; chained equations)
- REGRESSION
- Asparouhov, T. & Muthén, B. (2010). Multiple imputation with Mplus.

121

## **Plausible Values**

- Multiple imputations for latent variable values (H0 imputation)
- Available for both continuous and categorical latent variables
- DATA IMPUTATION command saves two data sets:
  - SAVE = imp\*.dat; saves all observations and latent variables for all imputations. Can be used to produce a distribution for each latent variable for each individual, not just a mean and SE
  - PLAUSIBLE = latent.dat; saves for each observation and latent variable a summary over imputed data sets
- Two uses:
  - Interest in individual scores
  - Interest in secondary analysis

122

## Input For Multiple Imputation For A Set Of Variables With Missing Values Followed By The Estimation Of A Growth Model (Ex 11.5)

```
TITLE:      this is an example of multiple imputation for a set of
            variables with missing values

DATA:      FILE = ex11.5.dat;

VARIABLE:  NAMES = x1 x2 y1-y4 z;
            MISSING = ALL (999);

DATA IMPUTATION:

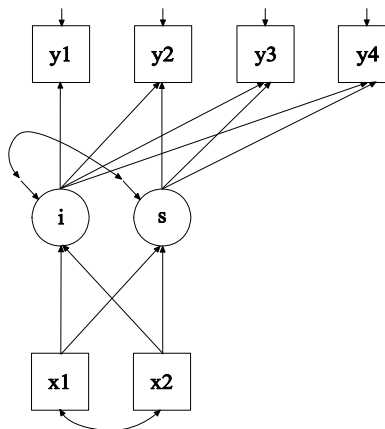
            IMPUTE = y1-y4 x1 (c) x2;
            NDATASETS = 10;
            SAVE = ex11.5imp*.dat;

ANALYSIS:  TYPE = BASIC;

OUTPUT:    TECH8;
```

123

## Linear Growth Model



124

## **Input For Multiple Imputation Followed By The Estimation Of A Growth Model (Ex 11.5), Continued**

```
TITLE:          This is an example of growth modeling using
                 multiple imputation data
DATA:           FILE = ex11.5implist.dat;
                 TYPE = IMPUTATION;
VARIABLES:     NAMES = x1 x2 y1-y4 z;
                 USEVARIABLES = y1-y4 x1 x2;
ANALYSIS:      ESTIMATOR = ML;
MODEL:         i s | y1@0 y2@1 y3@2 y4@3;
                 i s ON x1 x2;
OUTPUT:        TECH1 TECH4;
```

125

## **Multiple Imputation With A Categorical Outcome In A Growth Model With MAR Missing Data: Using WLSMV On Imputed Data**

126

## Choice Of Estimators With Categorical Outcomes

- ML: Intractable with many continuous latent variables (factors, random effects)
- WLSMV: Fast also with many continuous latent variables, but does not handle missing data under MAR
- New alternative: Bayes multiple imputation + WLSMV
- Monte Carlo simulation for 5 time points, binary outcome, linear growth model, n=1000, and MAR missingness as a function of the first outcome, varying the number of imputations as 5 versus 50. Unrestricted (H1) imputation using SEQUENTIAL (Ragunathan et al., 2001)

127

## Bias (Coverage) For MAR Dichotomous Growth Model: WLSMV Versus Imputation+WLSMV

Estimator	WLSMV	WLSMV (5 Imput.)	WLSMV (50 Imput.)
$\mu_1$	-0.03 (.92)	-0.01 (.96)	-0.01 (.93)
$\mu_2$	-0.16 (.02)	0.00 (.92)	0.00 (.93)
$\psi_{11}$	-0.23 (.62)	0.06 (.94)	0.05 (.95)
$\psi_{22}$	0.09 (.96)	0.04 (.91)	0.04 (.91)
$\psi_{12}$	-0.08 (.68)	0.00 (.93)	0.00 (.94)

$\mu$  are means of the two growth factors. True values: 0, 0.20

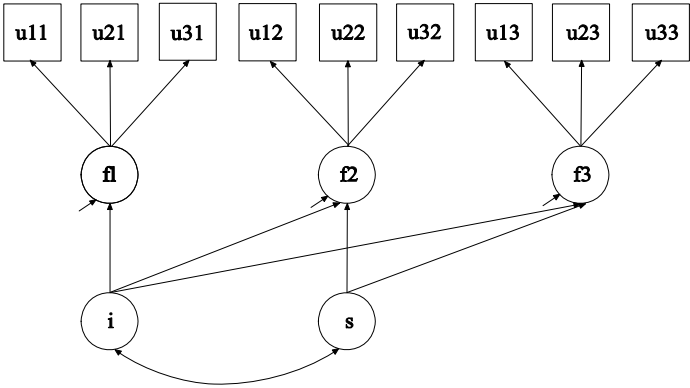
$\psi$  are variances and covariance of the two growth factors. True values: 0.50, 0.50, 0.30

Source: Asparouhov, T. & Muthén, B. (2010). Multiple imputation with Mplus.

128

**H0 Imputation**

**Multiple Indicator Linear Growth Model**



## Input For Multiple Imputation Of Plausible Values Using Bayesian Estimation Of A Growth Model (Ex 11.6). H0 Imputation

```
TITLE:      this is an example of multiple imputation of plausible
            values generated from a multiple indicator linear
            growth model for categorical outcomes using Bayesian
            estimation

DATA:       FILE = ex11.6.dat;

VARIABLE:   NAMES = u11 u21 u31 u12 u22 u32 u13 u23 u33;
            CATEGORICAL = u11-u33;

ANALYSIS:   ESTIMATOR = BAYES;
            PROCESSORS = 2;
```

131

## Input For Multiple Imputation Of Plausible Values Using Bayesian Estimation Of A Growth Model (Continued)

```
MODEL:      f1 BY u11
            u21-u31 (1-2);
            f2 BY u12
            u22-u32 (1-2);
            f3 BY u13
            u23-u33 (1-2);
            [u11$1 u12$1 u13$1] (3);
            [u21$1 u22$1 u23$1] (4);
            [u31$1 u32$1 u33$1] (5);
            i s | f1@0 f2@1 f3@2;

DATA IMPUTATION:

            NDATASETS = 20;
            PLAUSIBLE = ex11.6plaus.dat;
            SAVE = ex11.6imp*.dat;

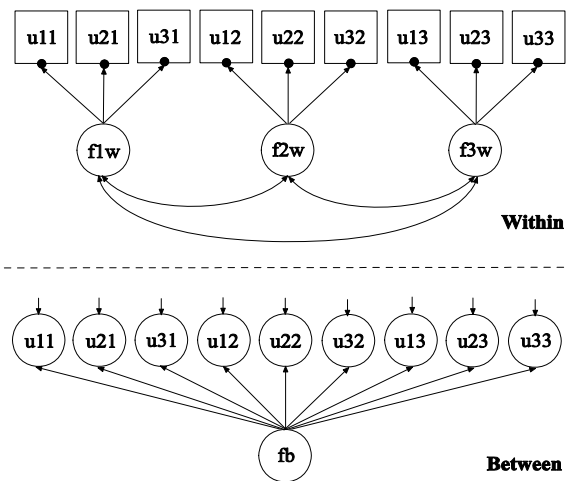
OUTPUT:     TECH1 TECH8;
```

132

## Twolevel Imputation

133

## Twolevel Factor Model



134

## **Input For Multiple Imputation Using A Two-Level Factor Model With Categorical Outcomes Followed By The Estimation Of A Growth Model (Ex 11.7)**

```
TITLE:      this is an example of multiple imputation using a two-
            level factor model with categorical outcomes

DATA:      FILE = ex11.7.dat;

VARIABLE:  NAMES = u11 u21 u31 u12 u22 u32 u13 u23 u33 clus;
            CATEGORICAL = u11-u33;
            CLUSTER = clus;
            MISSING = ALL (999);

ANALYSIS:  TYPE = TWOLEVEL;
            ESTIMATOR = BAYES;
            PROCESSORS = 2;
```

135

## **Multiple Imputation: Two-Level Factor Model With Categorical Outcomes Followed By The Estimation Of A Growth Model**

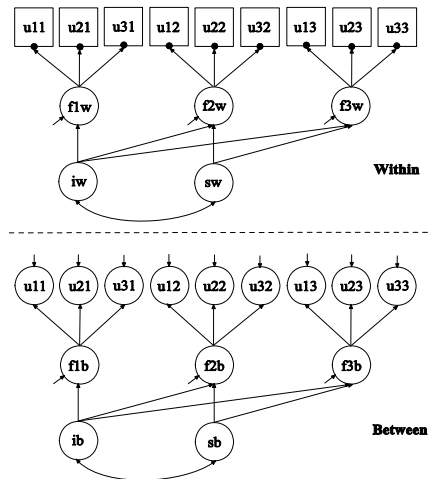
```
MODEL:      %WITHIN%
            f1w BY u11
            u21 (1)
            u31 (2);
            f2w BY u12
            u22 (1)
            u32 (2);
            f3w BY u13
            u23 (1)
            u33 (2);
            %BETWEEN%
            fb BY u11-u33*1;
            fb@1;

DATA IMPUTATION:
            IMPUTE = u11-u33(c);
            SAVE = ex11.7imp*.dat;

OUTPUT:     TECH1 TECH8;
```

136

## Twolevel Growth Model



137

## Input Multiple Imputation Using A Two-Level Factor Model With Categorical Outcomes Followed By The Estimation Of A Growth Model (Second Part)

```

TITLE:      this is an example of a two-level multiple indicator
            growth model with categorical outcomes using multiple
            imputation data

DATA:       FILE = ex11.7implist.dat;
            TYPE = IMPUTATION;

VARIABLE:   NAMES = u11 u21 u31 u12 u22 u32 u13 u23 u33 clus;
            CATEGORICAL = u11-u33;
            CLUSTER = clus;

ANALYSIS:   TYPE = TWOLEVEL;
            ESTIMATOR = WLSMV;
            PROCESSORS = 2;
    
```

138

## Input Multiple Imputation Using A Two-Level Factor Model With Categorical Outcomes Followed By The Estimation Of A Growth Model (Second Part)

```

MODEL:      %WITHIN%
            f1w BY u11
            u21 (1)
            u31 (2);
            f2w BY u12
            u22 (1)
            u32 (2);
            f3w BY u13
            u23 (1)
            u33 (2);
            iw sw | f1w@0 f2w@1 f3w@2;
            %BETWEEN%
            lb BY u11
                u21 (1)
                u31 (2);

```

139

## Input Multiple Imputation Using A Two-Level Factor Model With Categorical Outcomes Followed By The Estimation Of A Growth Model (Second Part)

```

            f2b BY u12
                u22 (1)
                u32 (2);
            f3b BY u13
                u23 (1)
                u33 (2);
            [u11$1 u12$1 u13$1] (3);
            [u21$1 u22$1 u23$1] (4);
            [u31$1 u32$1 u33$1] (5);
            u11-u33;
            ib sb | f1b@0 f2b@1 f3b@2;
            [f1b-f3b@0 ib@0 sb];
            f1b-f3b (6);
OUTPUT:    TECH1 TECH8;
SAVEDATA:  SWMATRIX = ex11.7sw*.dat;

```

140

**Chi-Square Model Testing  
Using Multiple Imputation Data:  
Asparouhov-Muthen (2010): Chi-Square  
Statistics With Multiple Imputation**

141

**Formulas**

$T_m$ : LRT test statistic for the m-th imputed data set

$Q_{0m}$ : estimates for the m-th imputed data set under the  $H_0$  model

$Q_{1m}$ : estimates for the m-th imputed data set under the  $H_1$  model

$$\bar{T} = \frac{1}{M} \sum_{m=1}^M T_m$$

$$\bar{Q}_0 = \frac{1}{M} \sum_{m=1}^M Q_{0m}$$

$$\bar{Q}_1 = \frac{1}{M} \sum_{m=1}^M Q_{1m}$$

142

## Formulas (Continued)

$T_m$ : LRT test statistic with estimates fixed at  $\bar{Q}_0$  for  $H_0$  and  $\bar{Q}_1$  for  $H_1$

$$\bar{T}' = \frac{1}{M} \sum_{m=1}^M T'_m$$

$$T_{imp} = \frac{T'}{1 + r_3}$$

$$r_3 = \frac{M + 1}{(M - 1)(p_1 - p_0)} (T - T')$$

143

## Monte Carlo Study Of Imputation Chi-Square Type I Error: Comparing the Naive And Correct Chi-2

25% missing		
N	$\bar{T}$	$T_{imp}$
100	18.0 (.45)	9.2 (.12)
500	16.2 (.45)	7.8 (.08)
1000	15.7 (.46)	8.1 (.05)
40% missing		
N	$\bar{T}$	$T_{imp}$
100	26.5 (.90)	18.8 (.15)
500	25.9 (.86)	8.7 (.09)
1000	25.5 (.78)	8.3 (.09)

144

## Monte Carlo Study Of Imputation Chi-Square Power: Comparing Imputation And ML

Power study results for 25% missing data case. Percentage rejection rate.

N	100	150	200	250	300
$T_{imp}$	34	53	68	75	85
$T_{FIML}$	50	60	76	86	92

Power study results for 40% missing data case. Percentage rejection rate.

N	100	150	200	250	300
$T_{imp}$	30	32	44	51	69
$T_{FIML}$	40	52	55	75	84

145

## Overview Of Other New Features In Mplus Version 6

146

## General Features

- Input statements that contain parameter estimates from the analysis as starting values (SVALUES)
- Merging of data sets (SAVEDATA)
- 90% confidence intervals (CINTERVALS)
- Bivariate frequency tables for pairs of binary, ordered categorical (ordinal), and/or unordered categorical (nominal) variables (CROSSTABS)
- Saving of graph settings (Axis Properties)

147

## General Features (Continued)

- Standard errors for factor scores
- New method for second-order chi-square adjustment for WLSMV, ULSMV, and MLMV resulting in the usual degrees of freedom
  - Asparouhov & Muthén, 2010: Simple second order chi-square correction. Technical appendix, see [www.statmodel.com/techappen.shtml](http://www.statmodel.com/techappen.shtml)

148

## Survival Analysis

- Muthén, Asparouhov, Boye, Hackshaw & Naegeli (2009). Applications of continuous-time survival in latent variable models for the analysis of oncology randomized clinical trial data using Mplus. Technical Report. Mplus scripts posted.
  - Collaboration with Eli Lilly on continuous-time survival analysis of cancer trials including quality of life growth curves:
- Survival intercept capturing treatment-control differences

149

## Survival Analysis Plots

- Survival plots (for discrete-time survival specify the event history variables using the DSURVIVAL option of the VARIABLE command):
  - Kaplan-Meier curve (log rank test)
  - Sample log cumulative hazard curve
  - Estimated baseline hazard curve
  - Estimated baseline survival curve
  - Estimated log cumulative baseline curve

150

## Missing Data Convenience Features For Longitudinal Data

- Research on modeling with missing data that violates MAR:
  - Muthén, Asparouhov, Hunter & Leuchter (2010). Growth modeling with non-ignorable dropout: Alternative analyses of the STAR\*D antidepressant trial. Submitted for publication. Mplus scripts posted.
- Creation of missing data dropout indicators for non-ignorable missing data (NMAR) modeling of longitudinal data using DATA MISSING:
  - TYPE=SDROPOUT for (Diggle-Kenward) selection modeling
  - TYPE=DDROPOUT for pattern-mixture modeling
- Descriptive statistics for dropout (DESCRIPTIVE)
- Plots of sample means before dropout

151

## Complex Survey Data Replicate Weights And Other Features

- Using and generating replicate weights to obtain correct standard errors (REPWEIGHTS)
  - Asparouhov & Muthén (2009). Resampling methods in Mplus for complex survey data.
  - Finite population correction factor for TYPE=COMPLEX (FINITE)
- Pearson and loglikelihood frequency table chi-square adjusted for TYPE=COMPLEX for models with weights
  - Asparouhov & Muthén (2008). Pearson and log-likelihood chi-square test of fit for latent class analysis estimated with complex samples. Technical appendix.
- Standardized values in TECH10 adjusted for TYPE=COMPLEX for models with weights

152

## References

- Asparouhov T. & Muthen B. (2010). Multiple Imputation with Mplus. Technical Report. [www.statmodel.com](http://www.statmodel.com)
- Asparouhov, T. & Muthen, B. (2010a). Bayesian analysis using Mplus. Technical appendix. Los Angeles: Muthen & Muthen. [www.statmodel.com](http://www.statmodel.com)
- Asparouhov, T. & Muthen, B. (2010b). Bayesian analysis of latent variable models using Mplus. Technical report in preparation. Los Angeles: Muthen & Muthen. [www.statmodel.com](http://www.statmodel.com)
- Asparouhov, T. & Muthen, B. (2009). Exploratory structural equation modeling. *Structural Equation Modeling*, 16, 397-438.

153

## References (Continued)

- Barnard, J., McCulloch, R. E., and Meng, X. L. (2000). Modeling covariance matrices in terms of standard deviations and correlations, with application to shrinkage. *Statistica Sinica* 10, 1281-1311.
- Botev, Z.I., Grotowski, J.F. & Kroese, D.P. (2010). Kernel density estimation via diffusion. Forthcoming in *Annals of Statistics*.
- Browne, W.J. & Draper, D. (2006). A comparison of Bayesian and likelihood-based methods for fitting multilevel models. *Bayesian Analysis*, 3, 473-514.
- Casella, G. & George, E.I. (1992). Explaining the Gibbs sampler. *The American Statistician*, 46, 167-174.
- Dominicus, A., Ripatti, S., Pedersen, N.L., & Palmgren, J. (2008). A random change point model for assessing the variability in repeated measures of cognitive function. *Statistics in Medicine*, 27, 5786-5798.

154

## References (Continued)

- Dunson D., Palomo J., and Bollen K. (2005) Bayesian structural equation modeling. SAMSI TR2005-5.  
<http://www.samsi.info/TRntr2005-05.pdf>
- Enders, C.K. (2010). Applied missing data analysis. New York: Guilford.
- Fox, J.P., and Glas, C. A.W. (2001). Bayesian estimation of a multilevel IRT model using Gibbs sampling. *Psychometrika*, 66, 271-288.
- Gelfand, A.E., Hills, S.E., Racine-Poon, A., & Smith, A.F.M. (1990). Illustration of Bayesian inference in normal data models using Gibbs sampling. *Journal of the American Statistical Association*, 85, 972-985.
- Gelman A., Jakulin A., Pittau M. G., Su Y. S. (2008a). A weakly informative default prior distribution for logistic and other regression models. *The Annals of Applied Statistics*, 2, 1360-1383.<sup>155</sup>

## References (Continued)

- Gelman A., van Dyk, D. A., Huang Z., Boscardin J. (2008b) Using Redundant Parameterizations to Fit Hierarchical Models. *Journal Of Computational And Graphical Statistics*, 17, 95-122.
- Gelman A. (2006) Prior distributions for variance parameters in hierarchical models. *Bayesian Analysis* 1, 515-533.
- Gelman, A., Carlin, J.B., Stern, H.S. & Rubin, D.B. (2004). Bayesian data analysis. Second edition. Boca Raton: Chapman & Hall.
- Gelman, A., Meng, X.L., Stern, H.S. & Rubin, D.B. (1996). Posterior predictive assessment of model fitness via realized discrepancies (with discussion). *Statistica Sinica*, 6, 733-807.
- Gelman, A. & Rubin, D.B. (1992). Inference from iterative simulation using multiple sequences. *Statistical Science*, 7, 457-511.
- Green, P. (1996). MCMC in image analysis. In Gilks, W.R., Richardson, S., & Spiegelhalter, D.J. (eds.), *Markov chain Monte Carlo in Practice*. London: Chapman & Hall.

## References (Continued)

- Harman, H.H. (1976). Modern factor analysis. Third edition. Chicago: The University of Chicago Press.
- Hjort, N. L., Dahl, F. A. and Steinbakk, G. H. (2006). Post-processing posterior predictive p-values. *J. Amer. Statist. Assoc.* 101, 1157-1174.
- Holzinger, K.J. & Swineford, F. (1939). A study in factor analysis: The stability of a bi-factor solution. Supplementary Educational Monographs. Chicago.: The University of Chicago Press.
- Lee S. Y., Song X. Y., Cai J. H. (2010) A Bayesian Approach for Non-linear Structural Equation Models With Dichotomous Variables Using Logit and Probit Links. *Structural Equation Modeling*, 17, 280-302.
- Lee, S.Y. (2007). Structural equation modeling. A Bayesian approach. Chichester: John Wiley & Sons.

157

## References (Continued)

- Little, R. J. & Rubin, D. B. (2002). Statistical analysis with missing data. Second edition. New York: John Wiley and Sons.
- Lynch, S.M. (2010). Introduction to applied Bayesian statistics and estimation for social scientists. New York: Springer.
- MacKinnon, D.P. (2008). Introduction to statistical mediation analysis. New York: Erlbaum.
- MacKinnon, D.P., Lockwood, C.M., & Williams, J. (2004). Condence limits for the indirect effect: Distribution of the product and resampling methods. *Multivariate Behavioral Research*, 39, 99-128.
- McLachlan, G. J. & Peel, D. (2000). Finite mixture models. New York: Wiley and Sons.
- Meng, X.L. & Rubin, D.B. (1992). Performing likelihood ratio tests with multiply-imputed data sets. *Biometrika*, 79, 103-111.

158

## References (Continued)

- Muthen, B. (2010) Bayesian Analysis In Mplus: A Brief Introduction. Mplus Technical Report. <http://www.statmodel.com>
- Muthen, B., Asparouhov, T., Hunter, A. & Leuchter, A. (2010). Growth modeling with non-ignorable dropout: Alternative analyses of the STAR\*D antidepressant trial. Submitted for publication.
- Muthen B. & Asparouhov, T. (2009). Growth mixture modeling: Analysis with non-Gaussian random effects. In Fitzmaurice, G., Davidian, M., Verbeke, G. & Molenberghs, G. (eds.), *Longitudinal Data Analysis*, pp. 143-165. Boca Raton: Chapman & Hall/CRC Press.
- Muthen, B. & Muthen, L. (1998-2010). *Mplus User's Guide*. Sixth Edition. Los Angeles, CA: Muthen & Muthen.

159

## References (Continued)

- Nylund, K. L., Asparouhov, T., & Muthén, B. O. (2007). Deciding on the number of classes in latent class analysis and growth mixture modeling: A Monte Carlo simulation. *Structural Equation Modeling*, 14, 535-569.
- Olsen, M.K. & Schafer, J.L. (2001). A two-part random effects model for semicontinuous longitudinal data. *Journal of the American Statistical Association*, 96, 730-745.
- Patz R. and Junker B. (1999) A Straightforward Approach to Markov Chain Monte Carlo Methods for Item Response Models *Journal of Educational and Behavioral Statistics* Summer, 24, 146-178.
- Raghunathan, T. E., Lepkowski, J. M., Van Hoewyk, J., and Solenberger, P. (2001) A Multivariate Technique for Multiply Imputing Missing Values Using a Sequence of Regression Models. *Survey Methodology*, Vol. 27, No 1. 85-95.

160

## References (Continued)

- Rubin, D.B. (1987) Multiple Imputation for Nonresponse in Surveys. J. Wiley & Sons, New York.
- Rupp, A.A., Dey, D.K., & Zumbo, B.D. (2004). To Bayes or not to Bayes, from whether to when: Applications of Bayesian methodology to modeling. *Structural Equation Modeling*, 11, 424-451.
- Savalei, V. (2010) Small Sample Statistics for Incomplete Nonnormal Data: Extensions of Complete Data Formulae and a Monte Carlo Comparison. *Structural Equation Modeling*, 17, 241-264.
- Schafer, J.L. (1997). Analysis of incomplete multivariate data. London: Chapman & Hall.
- Scheines, R., Hoijtink, H., & Boomsma, A. (1999). Bayesia estimation and testing of structural equation models. *Psychometrika*, 64, 37-52.

161

## References (Continued)

- Segawa E., Emery S., and Curry S. (2008) Extended Generalized Linear Latent and Mixed Model, *Journal of Educational and Behavioral Statistics*, 33, 464-484.
- Shrout, P.E. & Bolger, N. (2002). Mediation in experimental and nonexperimental studies: New procedures and recommendations. *Psychological Methods*, 7, 422-445.
- Spiegelhalter, D.J., Thomas, A., Best, N. & Gilks, W.R.(1997). BUGS: Bayesian Inference Using Gibbs Sampling, Version 0.60. Cambridge: Medical Research Council Biostatistics Unit.
- Spiegelhalter, D.J., Best, N. G., Carlin, B.P., & van der Linde, A. (2002). Bayesian measures of model complexity and t (with discussion). *Journal of the Royal Statistical Society, Series B (Statistical Methodology)* 64, 583-639.

162

## References (Continued)

- Song X.Y., Xia Y. M., Lee S. Y. (2009) Bayesian semiparametric analysis of structural equation models with mixed continuous and unordered categorical variables. *Statistics in Medicine*, 28, 2253-2276.
- van Dyk, D. A., and Meng, X. L. (2001), The Art of Data Augmentation. *Journal of Computational and Graphical Statistics*, 10, 1-111.
- Yuan, Y. & MacKinnon, D.P. (2009). Bayesian mediation analysis. *Psychological Methods*, 14, 301-322.