# Using Mplus To Do Cross-Lagged Modeling of Panel Data Part 1: Continuous Variables

Bengt Muthén
Professor Emeritus, UCLA
Mplus
bmuthen@statmodel.com

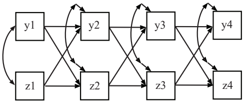
Tihomir Asparouhov
Mplus

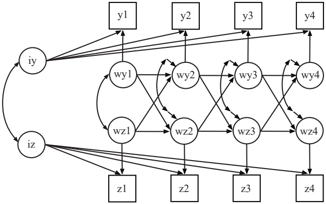
Mplus Web Talks: No. 4
October, 2021

# Overview

- Residual SEM (RSEM) - a new feature in Mplus Version 8.7 with special applications to panel data analysis
    - Asparouhov & Muthén (2021). Residual structural equation models
- Basic multilevel and longitudinal concepts
- Panel data models
- New residual language in Mplus
- Applications to depression and self-esteem, using two different data sets
    - Univariate analysis
    - Bivariate analysis, cross-lagged modeling
- Part 2: Categorical variables

# Cross-Lagged Panel Modeling: CLPM and RI-CLPM




- Direct effects between observed vs latent variables
  - Kenny & Zautra (1995), Cole et al. (2005): TSE
  - Hamaker, Kuiper, Grasman (2015): RI-CLPM
  - Zyphur et al. (2020), Usami (2021): GCLM

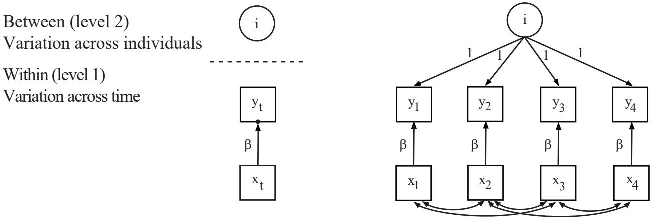
# Multilevel Regression with a Random Intercept

- Individuals $i$ within clusters $j$ (random intercept $\beta_{0j}$):

$$y_{ij} = \beta_{0j} + \beta\, x_{ij} + \varepsilon_{ij}, \tag{1}$$

$$\beta_{0j} = \beta_0 + u_j. \tag{2}$$

- Time points $t$ within individuals $i$ (random intercept $i$; T = 4)
  - Two-level, long format vs Single-level, wide format

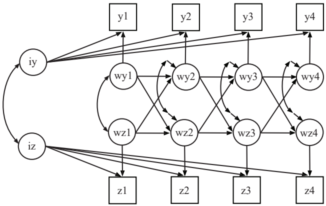


# Observed Variable Centering vs Latent variable Centering

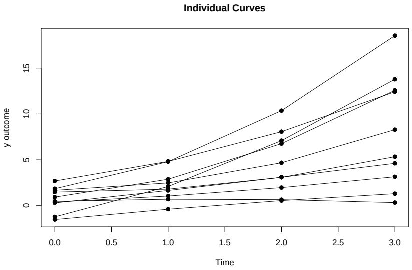
- Observed variable centering - Raudenbush & Bryk (2002). Hierarchical Linear Models. Table 5.11. Time points $t$ within individuals $i$:

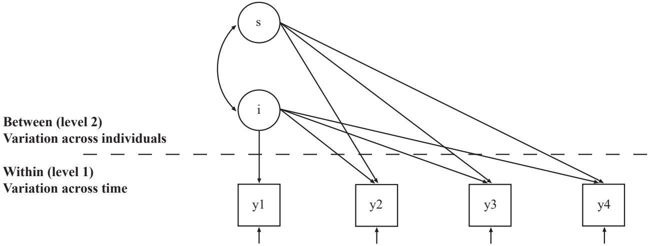
$$y_{it} = \beta_{0i} + \beta_w\,(x_{it} - \bar{x}_{.i}) + \varepsilon_{it},$$
$$\beta_{0i} = \beta_0 + \beta_b\,\bar{x}_{.i} + \delta_i$$

- Latent variable centering - Ludtke et al. (2008), Asparouhov & Muthén (2019), Hamaker & Muthén (2020)



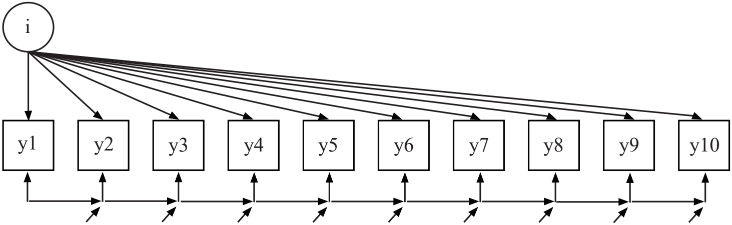
# Basic Longitudinal Modeling Concepts: Growth Modeling with Random Intercept and Slope (T=4)



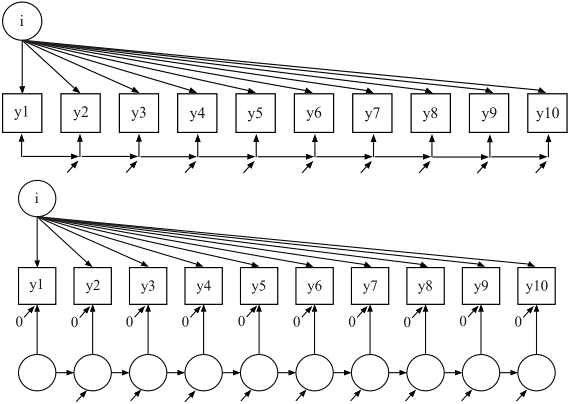



# Basic Longitudinal Modeling Concepts, Continued: Adding Auto-Correlated Residuals (T = 10)

- Statistical theory used in growth modeling:
  - Laird & Ware (1982, Biometrics) random effect model
  - Chi & Reinsel (1989, JASA) added auto-regressions among the residuals, $\varepsilon_t = \beta\ \varepsilon_{t-1} + \delta_t$ (AR-1):
- Special case of no trend: Random intercept plus first-order auto-regressions among the residuals (RI-AR1 modeling):

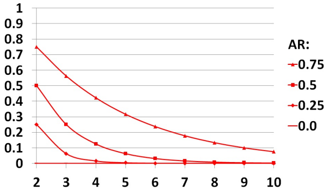


# Random Intercept and Auto-Correlated Residuals RI-AR Modeling Displayed in Two Equivalent Ways

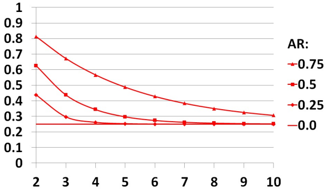



- RI-AR modeling is the univariate part of RI-CLPM
- Time-State-Error (TSE) model allows measurement error but imposes restrictions
- CLPM does not include the random intercept *i*

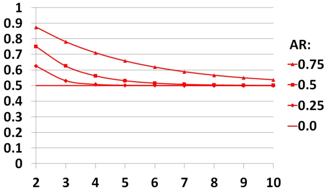
# RI and AR Impact on Correlations Across Time (T = 10)



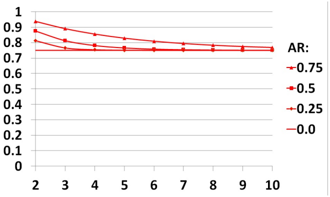

(a) RI variance = 0.00




(b) RI variance = 0.25

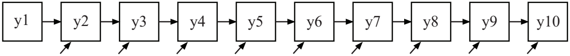



(c) RI variance = 0.50

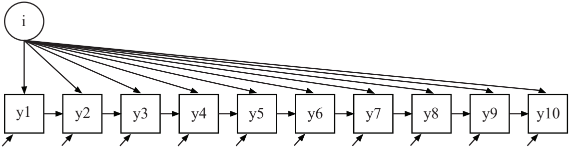



(d) RI variance = 0.75

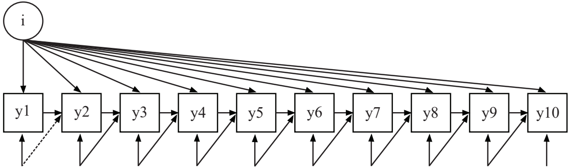
- Auto-Regression of lag 1 (AR1)



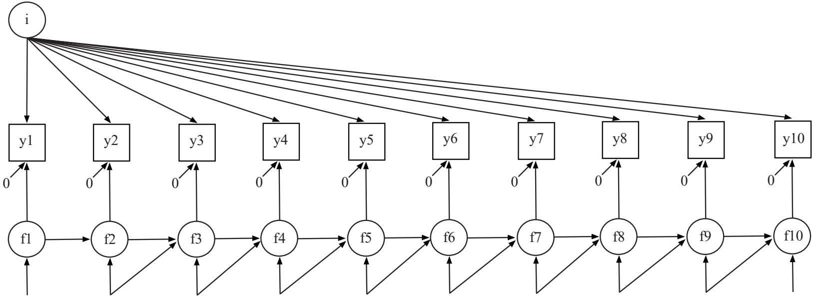

- Dynamic Random Intercept AR1 (D-RI-AR1). Bollen-Brandt (2010)




- Dynamic Random Intercept ARMA (1,1) (D-RI-ARMA11). Zyphur et al. (2020)

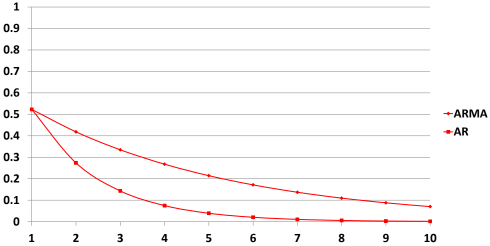


# RI-ARMA Modeling (Asparouhov-Muthén, 2021)



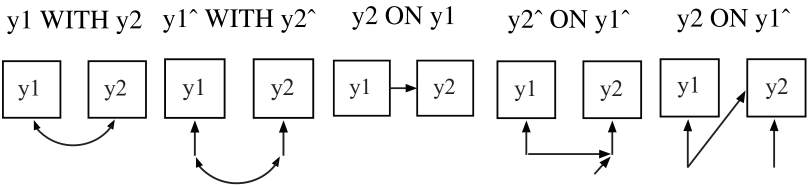

- This model is similar in spirit to RI-AR because of its separation of between- and within-individual variation also referred to as latent centering (centering using the random intercept i), but adds an MA component
- An equivalent measurement error version, RI-MEAR, is available which is more general than TSE but like TSE often presents estimation problems not seen with RI-ARMA

# Correlations Across Time for ARMA and AR Models No Random Intercept, T = 10




- Let $r$ denote the lag1 auto correlation and $b$ denote the MA coefficient:
  - If the second autocorrelation is bigger than $r^2$ then $b$ must be negative (slower decay)
  - If the second autocorrelation is smaller than $r^2$ then $b$ must be positive (faster decay)
  - If the second autocorrelation is exactly $r^2$ then $b$ must be zero (exact exponential decay)
- The MA coefficient $b$ is often negative so that the correlation diminishes slowly over time, slower than for the (RI-)AR model

# Five Ways to Relate Variables




- Residual language for single-level models is new in Version 8.7
- The notation y^ is spoken as y-hat
- Example of ON for residual regression:
  - RI-AR model: Auto-regression among the residuals from the regression of y on the random intercept
- The residual modeling is available for both continuous and categorical outcomes (not for nominal, count, or censored)
- For categorical outcomes, the residual modeling uses a new algorithm for Bayes (Asparouhov & Muthén, 2021)

# Estimators for Five Ways to Relate Variables - Continuous

| Type of relation | ML | WLSMV | Bayes |
|---|---|---|---|
| y1, y2 covariance | y1 WITH y2 | y1 WITH y2 | y1 WITH y2 |
| y1 residual, y2 residual covariance | y1^ WITH y2^ | y1^ WITH y2^ | y1^ WITH y2^ |
| y2 regressed on y1 | y2 ON y1 | y2 ON y1 | y2 ON y1 |
| y2 residual regressed on y1 residual | y2^ ON y1^ | y2^ ON y1^ | y2^ ON y1^ |
| y2 regressed on y1 residual | y2 ON y1^ | y2 ON y1^ | na[1] |

[1] Can be done using the equivalent MEAR formulation in the panel data case.

# Estimators for Five Ways to Relate Variables - Categorical

| Type of relation | ML | WLSMV | Bayes |
|---|---|---|---|
| y1, y2 covariance | na[1] | y1 WITH y2[2] | y1 WITH y2[2] |
| y1 residual, y2 residual covariance | na | y1^ WITH y2^[2] | y1^ WITH y2^[2] |
| y2 regressed on y1 | y2 ON y1[3] | y2 ON y1[2] | y2 ON y1[4] |
| y2 residual regressed on y1 residual | na | y2^ ON y1^ | y2^ ON y1^ |
| y2 regressed on y1 residual | na | y2 ON y1^ | na |

[1] PARAMETERIZATION=RESCOV can be used to allow conditional non-independence
[2] The latent y* variables are used. [3] The observed y variables are used
[4] Observed y1: PREDICTOR=OBSERVED. Latent y1*: PREDICTOR=LATENT (default)

# Mplus Input for Growth with Auto-Correlated Residuals: User's Guide Example 6.17 vs Version 8.7 Using Hats

```
VARIABLE:           NAMES = y1-y4;
MODEL:              i s | y1@0 y2@1 y3@2 y4@3;
                    y1-y4 (resvar);
                    y1-y3 PWITH y2-y4 (p1);
                    y1-y2 PWITH y3-y4 (p2);
                    y1 WITH y4 (p3);
MODEL CONSTRAINT:
                    NEW (corr);
                    p1 = resvar*corr;
                    p2 = resvar*corr**2;
                    p3 = resvar*corr**3;

                    ! Equality of AR
                    ! and residual variances
```

```
VARIABLE:    NAMES = y1-y4;
MODEL:       i s | y1@0 y2@1 y3@2 y4@3;
             y2^-y4^ PON y1^-y3^;

             ! Free AR and residual
             ! variances

             ! Equality of AR and residual
             ! variances needs special
             ! specifications
```

# Panel Data Analysis: Model Notation

- Dynamic models ($y_t$ regressed on $y_{t-1}$):
  - AR: Auto-regressive, classic model which is dynamic by definition
  - ARMA: Auto-regressive, Moving Average, classic model which is dynamic by definition
  - D-RI-AR: AR of the classic, dynamic kind but with a random intercept (RI) added
  - D-RI-ARMA: classic ARMA, that is, dynamic but with RI added
- Non-dynamic models:
  - RI-AR: AR is specified for the residual ("within-level", latent-variable centered) part
  - RI-ARMA: ARMA is specified for the residual ("within-level", latent-variable centered) part

# Mplus Input for RI-AR Modeling: Continuous, Univariate ML using Old Approach vs New Approach in Version 8.7

```
ANALYSIS:
          ESTIMATOR = ML;
          MODEL = NOCOV;
MODEL:
          i BY y1-y10@1;
          f1 BY y1;
          f2 BY y2;
          f3 BY y3;
          f4 BY y4;
          f5 BY y5;
          f6 BY y6;
          f7 BY y7;
          f8 BY y8;
          f9 BY y9;
          f10 BY y10;
          y1-y10@0;
          f2-f10 PON f1-f9;
```

```
ANALYSIS:
          ESTIMATOR = ML;
MODEL:
          i BY y1-y10@1;
          y2^-y10^ PON y1^-y9^;
```

# Analysis of MWI Data: Depression and Self-Esteem

- Adult sample, N = 663, T = 5, two months apart (Orth et al., 2020)
- Coverage (proportion not missing for each variable and pairs of variables; s = self-esteem, d = depression):

| | S1 | S2 | S3 | S4 | S5 | D1 | D2 | D3 | D4 | D5 |
|---|---|---|---|---|---|---|---|---|---|---|
| S1 | 0.994 | | | | | | | | | |
| S2 | 0.781 | 0.786 | | | | | | | | |
| S3 | 0.692 | 0.643 | 0.697 | | | | | | | |
| S4 | 0.594 | 0.561 | 0.555 | 0.599 | | | | | | |
| S5 | 0.561 | 0.526 | 0.516 | 0.486 | 0.566 | | | | | |
| D1 | 0.988 | 0.783 | 0.695 | 0.597 | 0.564 | 0.994 | | | | |
| D2 | 0.783 | 0.778 | 0.644 | 0.560 | 0.526 | 0.784 | 0.787 | | | |
| D3 | 0.689 | 0.640 | 0.694 | 0.554 | 0.514 | 0.692 | 0.641 | 0.694 | | |
| D4 | 0.596 | 0.563 | 0.555 | 0.597 | 0.486 | 0.599 | 0.561 | 0.554 | 0.600 | |
| D5 | 0.563 | 0.526 | 0.517 | 0.487 | 0.564 | 0.566 | 0.526 | 0.516 | 0.487 | 0.567 |

# Univariate Analysis of MWI Data: Depression (T = 5)

| Model | # par's | LogL | BIC | MLR $\chi^2$ (df) (p-value) | RMSEA (p<0.05) | CFI |
|---|---|---|---|---|---|---|
| 1. AR1 | 14 | -717 | 1525 | $\chi^2(6)$=52 (.0000) | 0.107 (.000) | 0.914 |
| 2. AR2 | 17 | -675 | 1461 | $\chi^2(3)$=13 (.0048) | 0.071 (.156) | 0.987 |
| 3. ARMA11 | 17 | -667 | 1444 | $\chi^2(3)$=2 (.5868) | 0.000 (.925) | 1.000 |
| **4. D-RI-AR1** | 15 | -667 | 1431 | $\chi^2(5)$=2 (.8621) | 0.000 (.994) | 1.000 |
| 5. D-RI-ARMA | 19 | -665 | 1454 | $\chi^2(1)$=0 (.8327) | 0.000 (.926) | 1.000 |
| **6. RI-AR1** | 15 | -671 | 1440 | $\chi^2(5)$=8 (.1480) | 0.031 (.767) | 0.994 |
| 7. RI-ARMA11 | 18 | -666 | 1448 | $\chi^2(2)$=1 (.7062) | 0.000 (.927) | 1.000 |

# Univariate Analysis of MWI Data: Self-Esteem (T = 5)

| Model | # par's | LogL | BIC | MLR $\chi^2$ (df) (p-value) | RMSEA (p<0.05) | CFI |
|---|---|---|---|---|---|---|
| 1. AR1 | 14 | -1293 | 2677 | $\chi^2(6)=104$ (.0000) | 0.157 (.000) | 0.921 |
| 2. AR2 | 17 | -1218 | 2546 | $\chi^2(3)=16$ (.0013) | 0.080 (.082) | 0.990 |
| 3. ARMA11 | 17 | -1208 | 2526 | $\chi^2(3)=3$ (.3990) | 0.000 (.852) | 1.000 |
| **4. D-RI-AR1** | 15 | -1213 | 2523 | $\chi^2(5)=10$ (.0871) | 0.037 (.676) | 0.996 |
| 5. D-RI-ARMA | 19 | -1206 | 2535 | $\chi^2(1)=0$ (.6121) | 0.000 (.819) | 1.000 |
| **6. RI-AR1** | 15 | -1214 | 2526 | $\chi^2(5)=14$ (.0188) | 0.051 (.428) | 0.993 |
| 7. RI-ARMA11 | 18 | No solution | | | | |

- Go to outputs for MWI univariate analysis of self-esteem for Mplus Web Talk No. 4 at `www.statmodel.com`

# NLSY79 Data: Depression and Self-Esteem

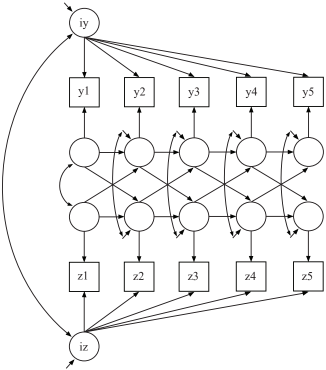
- Adolescents and young adults, N = 8,259, T = 11 (max = 7 - 8 time points observed for any person), 2 years apart 1994 - 2014 (Orth et al., 2020)
- Large amount of missing data: Use ANALYSIS options COVERAGE = 0, STARTS = 50 but be aware that model assumptions are given too much weight relative to data
- Poor coverage, especially for self-esteem (y1 - y11):

| | Y2 | Y3 | Y4 | Y5 | Y6 | Y7 | Y8 | Y9 | Y10 | Y11 | Y1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Y2 | 0.202 | | | | | | | | | | |
| Y3 | 0.148 | 0.259 | | | | | | | | | |
| Y4 | 0.043 | **0.000** | 0.159 | | | | | | | | |
| Y5 | 0.024 | 0.039 | **0.000** | 0.174 | | | | | | | |
| Y6 | 0.171 | 0.225 | 0.138 | 0.158 | 0.608 | | | | | | |
| Y7 | 0.017 | 0.019 | 0.010 | 0.010 | **0.000** | 0.134 | | | | | |
| Y8 | 0.175 | 0.226 | 0.136 | 0.153 | 0.553 | 0.119 | 0.763 | | | | |
| Y9 | 0.007 | 0.010 | 0.008 | 0.009 | 0.023 | 0.005 | **0.000** | 0.096 | | | |
| Y10 | 0.061 | 0.050 | 0.025 | 0.009 | 0.078 | 0.009 | 0.081 | **0.000** | 0.138 | | |
| Y11 | 0.098 | 0.081 | 0.025 | 0.024 | 0.110 | 0.014 | 0.120 | 0.010 | **0.000** | 0.166 | |
| Y1 | 0.104 | 0.065 | 0.042 | 0.007 | 0.096 | 0.012 | 0.100 | 0.002 | 0.062 | 0.028 | 0.118 |

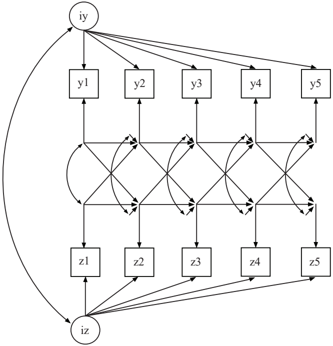
# NLSY79 Data: Depression (T = 11; Tmax = 8)

| Model | # par's | LogL | BIC | MLR $\chi^2$ (df) (p-value) | RMSEA (p<0.05) | CFI |
|---|---|---|---|---|---|---|
| 1. AR1 | 32 | -28035 | 56359 | $\chi^2(43)=1443$ (.0000) | 0.063 (.000) | 0.709 |
| 2. AR2 | 41 | -27371 | 55112 | $\chi^2(34)=456$ (.0000) | 0.039 (1.000) | 0.912 |
| **3. ARMA11** | 41 | -27112 | 54594 | $\chi^2(34)=43$ (.1370) | 0.006 (1.000) | 0.998 |
| 4. D-RI-AR1 | 33 | -27182 | 54662 | $\chi^2(42)=149$ (.0000) | 0.018 (1.000) | 0.978 |
| 5. D-RI-AR2 | 42 | -27121 | 54621 | $\chi^2(33)=57$ (.0054) | 0.009 (1.000) | 0.995 |
| **6. D-RI-ARMA11** | 43 | -27111 | 54610 | $\chi^2(32)=47$ (.0383) | 0.008 (1.000) | 0.997 |
| 7. D-RI-ARMA21 | No solution | | | | | |
| 8. RI-AR1 | 33 | -27185 | 54668 | $\chi^2(42)=153$ (.0000) | 0.018 (1.000) | 0.977 |
| 9. RI-AR2 | No solution | | | | | |
| **10. RI-ARMA11** | 42 | -27110 | 54599 | $\chi^2(33)=40$ (.1924) | 0.005 (1.000) | 0.999 |

# Bivariate Analysis: RI-CLPM

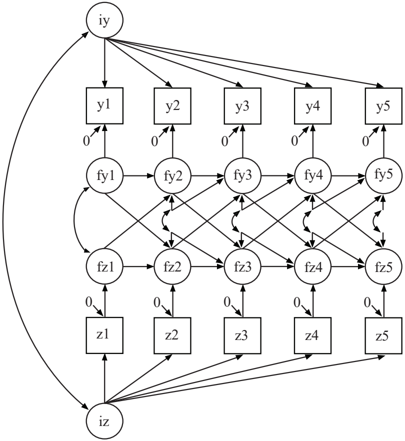



(a) Old approach using factors




(b) New approach using hats

# Bivariate Analysis: RI-ARMA



# Bivariate Analysis of MWI Data: Depression & Self-Esteem (N = 663; MLR, STARTS = 50)

| Model | # par's | LogL | BIC | MLR $\chi^2$(df)<br>$\chi^2$(p value) | RMSEA<br>(p<0.05) | CFI |
|---|---|---|---|---|---|---|
| 0. CLPM1 invariant<br>(Orth's model) | 29 | -1659 | 3507 | $\chi^2(36)=207$<br>(.0000) | 0.085<br>(.000) | 0.932 |
| 1. CLPM1 | 41 | -1651 | 3568 | $\chi^2(24)=189$<br>(.0000) | 0.102<br>(.000) | 0.934 |
| 2. CLPM2 | 47 | -1543 | 3391 | $\chi^2(18)=49$<br>(.0001) | 0.051<br>(.422) | 0.988 |
| **3. ARMA11** | 50 | -1523 | 3371 | $\chi^2(15)=21$<br>(.1387) | 0.024<br>(.970) | 0.998 |
| **4. D-RI-AR1** | 44 | -1537 | 3349 | $\chi^2(21)=33$<br>(.0517) | 0.029<br>(.973) | 0.996 |
| 5. D-RI-ARMA11 | 52 | -1522 | 3382 | $\chi^2(13)=20$<br>(.0862) | 0.029<br>(.926) | 0.997 |
| **6. RI-AR1** | 44 | -1532 | 3349 | $\chi^2(21)=34$<br>(.0323) | 0.031<br>(.958) | 0.995 |
| **7. RI-ARMA** | 50 | -1516 | 3355 | $\chi^2(15)=9$<br>(.8559) | 0.000<br>(1.000) | 1.000 |

# Bivariate Analyses of MWI Depression and Self-Esteem

- Go to outputs for MWI bivariate analysis of depression and self-esteem for Mplus Web Talk No. 4 at `www.statmodel.com`

# Testing of Time-Invariant AR and Cross-Lagged Effects for Depression & Self-Esteem (N = 663)

- Testing time-invariance of auto-regressions and cross-lagged effects using the Wald chi-square test in MODEL TEST with MLR:
    - CLPM1: $\chi^2(12) = 15$ (.2413)
    - CLPM2: $\chi^2(16) = 37$ (.0024)
    - RI-AR1: $\chi^2(12) = 37$ (.0002)
    - RI-ARMA: $\chi^2(16) = 179$ (.0000)
- Time-invariance not rejected for the ill-fitting CLPM1. The time-invariant CLPM1 is used in Orth et al. (2020)
- Time-invariance rejected for the other models

# Wald Chi-Square Testing of Time Invariance: MODEL TEST for CLPM1

MODEL:

```
s2-s5 d2-d5 PON s1-s4 d1-d4 (a1-a8);
s2-s5 d2-d5 PON d1-d4 s1-s4 (c1-c8);
s1-s5 PWITH d1-d5;
```

MODEL TEST:

```
! AR for s:
0 = a2-a1;
0 = a3-a1;
0 = a4-a1;
! AR for d:
0 = a6-a5;
0 = a7-a5;
0 = a8-a5;
! cross-lag for s on d:
0 = c2-c1;
0 = c3-c1;
0 = c4-c1;
! cross-lag for d on s:
0 = c6-c5;
0 = c7-c5;
0 = c8-c5;
```

```
MODEL FIT INFORMATION

Wald Test of Parameter Constraints

  Value: 15.003
  Degrees of Freedom: 12
  P-Value: 0.2413
```

# Conclusions from Data Analyses

- CLPM tends to not fit the data well which is to be expected from statistical considerations
- RI-CLPM needs T $>2$ and tends to fit well to data with $2 < T < 7$
- RI-ARMA tends to be needed for T $>6$ (univariate RI-ARMA needs T $>4$)
- The dynamic alternatives of ARMA and DI-RI-ARMA are also good for T $>6$ and the choice between them and RI-ARMA should be based on substantive considerations

# Further Considerations

- Bootstrapping or Bayes to allow non-symmetric confidence intervals
- Monte Carlo simulations: N, T, and effect sizes needed for good estimates/SEs and power to reject zero cross-lagged effects
- Adding random slopes for linear, quadratic growth: May be needed but can result in no solution or inadmissible solutions unless T is large
- Measurement error parameterization using TSE and MEAR: Can result in no solution or inadmissible solutions
- Causal analysis, unobserved time-varying confounders

# References


- Allison (2009). Fixed effects regression models. Sage
- Antonakis et al. (2019). On ignoring the random effects assumption in multilevel models: Review, critique and recommendations. Organizational Research Methods
- Asparouhov & Muthén (2019). Latent variable centering of predictors and mediators in multilevel and time-series models. Structural Equation Modeling, 26, 119-142
- Asparouhov & Muthén (2021). Residual structural equation models
- Bell & Jones (2015). Explaining fixed effects: Random effect modeling of time-series cross-sectional and panel data. Political Science Research and Methods, 3 , 133-153
- Bollen & Brandt (2010). A general panel model with random and fixed effects: A structural equations approach. Social Forces, 89, 1-34
- Bou & Satorra (2017). Univariate versus multivariate modeling of panel data: Model specification and goodness-of fit testing. Organizational Research Methods
- Cole et al. (2005). Empirical and conceptual problems with longitudinal trait-state models: Introducing a trait-state-occasion model. Psychological Methods
- Curran et al. (2014). The separation of between-person and within-person components of individual change over tinme: A latent curve model with structured residuals. Journal of Consulting and Clinical Psychology, 879-894
- Hamaker, Kuiper, Grasman (2015). A critique of the cross-lagged panel model. Psychological Methods, 1, 102-116

# References


- Hamaker & Mulder (2021). FAQ: Response to Orth et al. (2021) `https://jeroendmulder.github.io/RI-CLPM/faq.html`
- Hamaker & Muthén (2020). The fixed versus random effects debate and how it relates to centering in multilevel modeling. Psychological Methods, 25(3), 365–379.
- Kenny & Zautra (1995). The trait-state-error model for multiwave data. Journal of Consulting and Clinical Psychology
- Ludtke, Marsh, Robitzsch, Trautwein, Asparouhov & Muthén (2008). The multilevel latent covariate model: A new, more reliable approach to group-level effects in contextual studies. Psychological Methods, 13, 203–229.
- Orth et al. (2020). Testing prospective effects in longitudinal research: Comparing seven competing cross-lagged models. Journal of Personality and Social Psychology
- Usami, Murayama, & Hamaker (2019). A unified framework of longitudinal models to examine reciprocal relations. Psychological Methods, 24 (5), 637–657
- Usami (2020). Within-person variability score-based causal inference: A two-step semiparametric estimation for joint effects of time-varying treatments. arXiv Preprint
- Usami (2021). On the differences between general cross-lagged panel model and random-intercept cross-lagged panel model: Interpretation of cross-lagged parameters and model choice. Structural Equation Modeling, 28 (3), 331–344
- Wooldridge (2002). Econometric Analysis of Cross Sectional and Panel Data. The MIT Press
- Zyphur et al. (2020). From data to causes II: Comparing approaches to panel data analysis. Organizational Research Methods