Mplus 8: Dynamic SEM

Applications

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Intensive longitudinal data

Two approaches we can take when T is large and N>1:

1. **Top-down approach** (i.e., **dynamic multilevel modeling**):
   - use time series models as level 1
   - allow for quantitative individual differences in model dynamics at level 2
   - can be used with relative small T (say 20), but requires at least moderate N (say >30)

2. **Bottom-up approach** (i.e., **replicated time series analysis**)
   - use time series models to model N=1 data
   - allow for quantitative and qualitative differences between persons
   - can be used with small N (say 2), but requires relative large T (say >50)

Alternative approach: **pooled time series analysis** (requires N*T>50).

Outline

1. Top-down approach:
    - **Univariate multilevel AR(1) model**
    - Multiple indicator multilevel AR(1) model
    - Multilevel VAR(1) model
2. Bottom-up approach:
    - Comparison of linear models and regime-switching models
3. Discussion

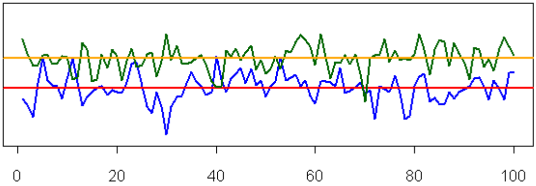
Univariate multilevel AR(1) model: Random mean

Centering part:

$$PA_{it} = \mu_i + PA^*_{it}$$

where

- μ_i is the individual's **mean** (i.e., baseline, trait, equilibrium) of positive affect
- PA^*_{it} is the **within-person centered** (cluster-mean centered) score



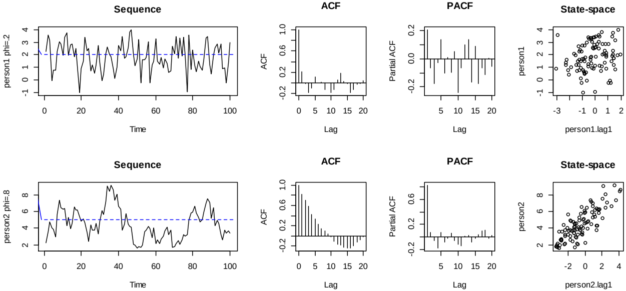
Univariate multilevel AR(1) model: Random inertia

Autoregressive part:

$$PA^*_{it} = \phi_i PA^*_{i,t-1} + \zeta_{it}$$

where

- ϕ_i is the **autoregressive parameter** (i.e., inertia, carry-over, or regulatory weakness)
- ζ_{it} is the **innovation** (residual, disturbance, dynamic error) (with $\zeta_{it} \sim N(0, \sigma^2_\zeta)$)



Univariate multilevel AR(1) model: Level 1

Putting these together we can write:

Level 1: Random mean and inertia

$$PA_{it} = \mu_i + \phi_i PA^*_{i,t-1} + \zeta_{it}$$

where $\zeta_{it} \sim N(0, \sigma^2)$.

Level 2:

$$\mu_i = \mu + v_{0i}$$
$$\phi_i = \phi + v_{1i}$$

where

$$\begin{bmatrix} v_{0i} \\ v_{1i} \end{bmatrix} \sim MN \left[\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{11} & \\ \psi_{21} & \psi_{22} \end{bmatrix} \right]$$

Intermezzo: Centering level 1 predictors?

There are three ways in which we can include level 1 predictors:

- non-centered (**NC**)
- grand mean centered (**GMC**)
- cluster mean centered (**CMC**)

NC and GMC are **equivalent** (i.e., alternative parametrizations).

CMC is **equivalent under some circumstances** (i.e., no random slopes, and predictor means included as level 2 predictor of random intercept), but not always.

Converging consensus: The slope from NC/GMC can be an "**uninterpretable blend**" of the within and between relationship (Raudenbush & Bryck, 2002).

Intermezzo: Centering the lagged predictor?

Hamaker and Grasman (2015) compared four ways of centering the **lagged predictor** in a multilevel AR(1) model:

- NC: no centering
- CMC($\bar{y}_{.i}$): cluster mean centering using the sample mean
- CMC($\hat{\mu}_i$): cluster mean centering using the multilevel estimate
- CMC(μ_i): cluster mean centering using the true mean

Table 4 | Bias and coverage rates for fixed autoregressive parameter ϕ in multilevel autoregressive model under diverse scenarios.

AR parameter	Sample size		Bias				$CR_{0.95}$			
	N	T	NC	C($\bar{y}_{.i}$)	C($\hat{\mu}_i$)	C(μ_i)	NC	C($\bar{y}_{.i}$)	C($\hat{\mu}_i$)	C(μ_i)
$\phi_i \sim N(0.3, 0.1)$	20	20	0.002	−0.072	−0.069	−0.068	0.928	0.762	0.785	0.787
		50	0.000	−0.027	−0.027	−0.026	0.940	0.900	0.901	0.898
		100	0.000	−0.013	−0.013	−0.013	0.932	0.932	0.932	0.932
	50	20	0.005	−0.071	−0.069	−0.067	0.893	0.480	0.512	0.518
		50	0.001	−0.027	−0.026	−0.026	0.936	0.800	0.804	0.805
		100	0.000	−0.013	−0.013	−0.013	0.946	0.902	0.902	0.903
	100	20	0.006	−0.070	−0.068	−0.066	0.892	0.196	0.227	0.242
		50	0.001	−0.027	−0.027	−0.027	0.930	0.623	0.630	0.637
		100	0.000	−0.013	−0.013	−0.013	0.930	0.851	0.854	0.851

Intermezzo: Centering the lagged predictor?

Conclusion (from Hamaker & Grasman, 2015):

- **CMC leads to a downward bias** in the estimation of the AR parameter
- **CMC is better** when interest is in a **level 2 predictor of the AR parameter**

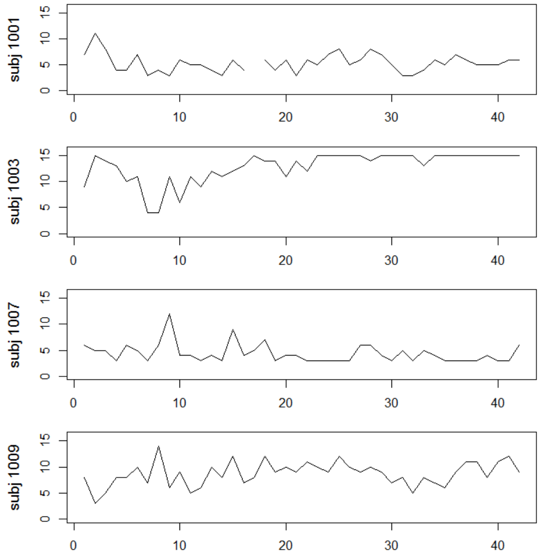
Note that when N=1, the OLS estimate of the AR parameter is known to be biased (e.g., Marriott & Pope, 1954).

BUT: CMC in Mplus is not associated with this bias (nor is it in WinBUGS, see Jongerling et al., 2015), probably because the **same (individual) parameter** is used as the intercept and for CMC of the lagged predictor.

NOTE: CMC is the default in Mplus when creating lagged variables.

Daily diary data on positive affect (PA)

Data: 89 females measured for 42 days (see Jongerling, Laurenceau & Hamaker, 2015).



Input: Create an observed lagged variable

```
TITLE: Multilevel AR(1) with random mean

DATA: file is fem.dat;

VARIABLE:
names=subj couple day   dhappy
dexcited    denerget    denthusi    PA;
cluster=subj;
useobs are
(subj .ne. 1003) .and.
(subj .ne. 1107) .and.
(subj .ne. 1223) .and.
(subj .ne. 1233) .and.
(subj .ne. 1249) .and.
(subj .ne. 1327) .and.
(subj .ne. 1425);

MISSING = all(999);

USEVAR are PA;

LAGVAR = PA(1);  ! CREATE AN OBSERVED LAGGED VARIABLE
```

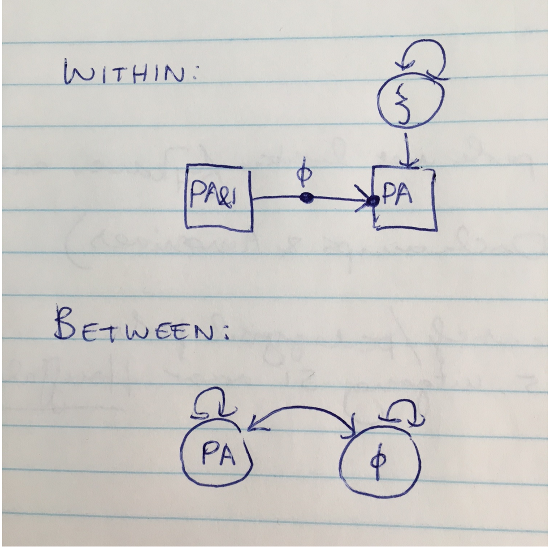
NOTE: Using **LAGVAR = PA(1);** gives a lagged variable based on lagging the observed variable PA by one.

Input: Random AR parameter and random mean

```
ANALYSIS:    TYPE IS TWOLEVEL random;
             estimator=bayes;
             fbiter=10000;
             bseed = 7487;
             proc = 2;

MODEL:

  %WITHIN%
  phi | PA on PA&1; ! AUTOREGRESSION IS RANDOM

  %BETWEEN%
  PA with phi;      ! CORRELATED RANDOM MEAN AND AR
```

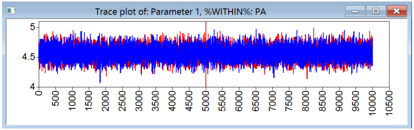
NOTE: The lagged variable (created by **LAGVAR = PA(1);**) is referred to as **PA&1**.

Path diagram of the multilevel AR(1) model

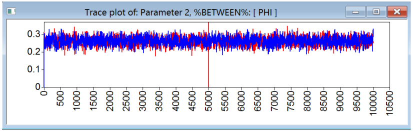


Results: Trace plots (10,000 iterations)

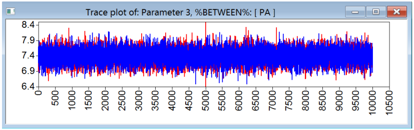
Level 1 residual variance:



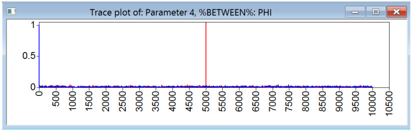
AR parameter:



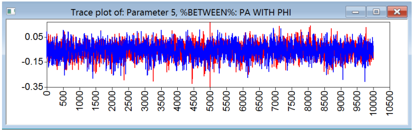
Average mean:



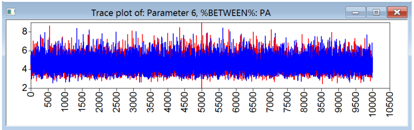
Variance of AR parameter:



Cov. mean and AR parameter:



Variance of mean:



Results: Parameter estimates

MODEL RESULTS

	Estimate	Posterior S.D.	One-Tailed P-Value	95% C.I. Lower 2.5%	95% C.I. Upper 2.5%	Significance
Within Level						
Residual Variances						
PA	4.563	0.109	0.000	4.357	4.784	*
Between Level						
PA WITH						
PHI	-0.053	0.049	0.129	-0.152	0.039	
Means						
PA	7.393	0.231	0.000	6.933	7.842	*
PHI	0.263	0.021	0.000	0.221	0.304	*
Variances						
PA	4.470	0.752	0.000	3.316	6.260	*
PHI	0.010	0.005	0.000	0.002	0.022	*

Testing whether a random effect is significant is problematic; instead we can compare two models (with and without a random effect).

Input: Fixed AR parameter and random mean

```
ANALYSIS:   TYPE IS TWOLEVEL random;
            estimator=bayes;
            fbiter=10000;
            bseed = 6186;

MODEL:

  %WITHIN%
  PA on PA&1 (phi); ! AUTOREGRESSION

  %BETWEEN%
  PA;               ! RANDOM MEAN

OUTPUT: TECH8 TECH1;

PLOT: TYPE = PLOT2;
```

In this model there is no random AR parameter; only a random mean.

Random AR parameter?

Warning: Make sure the DIC is **stable** (this may take *many more iterations* than apparent from trace plots).

To ensure the DIC is stable, run the model at least **twice with a different seed**: This should give the same DIC and pD.

Here we compare the model with a fixed AR parameter (ϕ) to a model with a random AR parameter (ϕ_i).

Model	DIC	pD
ϕ	16501	192
ϕ_i	16498	216

Only slight preference for model with random AR parameter.

Literature on inertia

Affective inertia has been **empirically related to**

- neuroticism (+) and agreeableness (-) (Suls, Green & Hillis, 1998)
- concurrent depression (+) (Kuppens, Allen & Sheeber, 2010, *Psychological Science*)
- future depression (+) (Kuppens, Sheeber, Yap, Whittle, Simmons & Allen, 2012)
- rumination (+) (Koval, Kuppens, Allen & Sheeber, 2012)
- self-esteem (-) (Houben, Van den Noortgate & Kuppens, 20150)
- life-satisfaction (-) (Houben et al., 2015)
- PA (-) and NA (+) (Houben et al., 2015)

Note that inertia in positive affects seems also maladaptive.

Autoregressive parameter in **daily drinking behavior** has been positively related to being female (Rovine & Walls, 2006); however, the **average** was close to **zero**.

Extension 1: Random innovation variance

Level 1: Random mean, inertia, and innovation variance

$$PA_{ti} = \mu_i + \phi_i PA^*_{t-1,i} + \sigma_i \zeta_{ti}$$

where $\zeta_{ti} \sim N(0,1)$.

Level 2:

$$\mu_i = \mu + v_{0i}$$
$$\phi_i = \phi + v_{1i}$$
$$\sigma_i = \sigma + v_{2i}$$

where

$$\begin{bmatrix} v_{0i} \\ v_{1i} \\ v_{2i} \end{bmatrix} \sim MN \left[\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{11} & & \\ \psi_{21} & \psi_{22} & \\ \psi_{31} & \psi_{32} & \psi_{33} \end{bmatrix} \right]$$

Why random innovation variance? Statistical

For N=1 we have: $y_t = \mu + \phi(y_{t-1} - \mu) + \zeta_t$, such that:

$$\begin{aligned} Var(y_t) = E\Big[\big\{y_t - \mu\big\}^2\Big] &= E\Big[\big\{\mu + \phi(y_{t-1} - \mu) + \zeta_t - \mu\big\}^2\Big] \\ &= E\Big[\big\{\phi(y_{t-1} - \mu) + \zeta_t\big\}^2\Big] \\ &= \phi^2 E\Big[\big\{y_{t-1} - \mu\big\}^2\Big] + \sigma^2 \end{aligned}$$

where $E\Big[\big\{y_t - \mu\big\}^2\Big] = E\Big[\big\{y_{t-1} - \mu\big\}^2\Big] = \sigma_y^2$

$$\begin{aligned} \sigma_y^2 &= \phi^2 \sigma_y^2 + \sigma^2 \\ \sigma_y^2 - \phi^2 \sigma_y^2 &= \sigma^2 \\ (1 - \phi^2)\sigma_y^2 &= \sigma^2 \\ \sigma_y^2 &= \frac{\sigma^2}{1 - \phi^2} \end{aligned}$$

Hence, **individual differences** in σ_y^2 can come from individual differences in ϕ **and/or** σ^2.

Why random innovation variance? Substantive

Level 1: Random mean, inertia, and innovation variance

$$PA_{ti} = \mu_i + \phi_i PA^*_{t-1,i} + \sigma_i \zeta_{ti}$$

where $\zeta_{ti} \sim N(0, 1)$.

Substantive interpretation of random innovation variance:

- individual differences in exposure
- individual differences in reactivity

Level 1: Reactivity to Positive Events (PE)

$$PA_{ti} = \mu_i + \phi_i PA^*_{t-1,i} + \beta_i PE^*_{ti} + \zeta_{ti}$$

Some results for stress sensitivity and reward experience:

- Suls et al. (1998)
- Wichers: relationship with depression and effect of therapy

Extension 2: Measurement error

Level 1: Measurement equation

$$PA_{it} = \mu_i + \eta_{it} + \epsilon_{it}$$

where

- μ_i is the individual's mean
- η_{it} is the individual's true score at occasion t
- ϵ_{it} is the individual's measurement error at occasion t (could also consider individual differences in its variance)

Level 1: Transition equation

$$\eta_{it} = \phi_i \eta_{i,t-1} + \sigma_i \zeta_{it}$$

where $\zeta_{it} \sim N(0, 1)$.

Some thoughts about measurement error in a multilevel AR(1) model:

- advantage: separate signal from noise
- advantage: reliability per person
- disadvantage: AR-effects in error end up in signal
- disadvantage: not identified when $\phi = 0$

Outline

1. Top-down approach:
   - Univariate multilevel AR(1) model
   - **Multiple indicator multilevel AR(1) model**
   - Multilevel VAR(1) model
2. Bottom-up approach:
   - Comparison of linear models and regime-switching models
3. Discussion

Multiple indicator AR(1) model for PA

We have three indicators: excited (EXC), energetic (ENE), and enthusiastic (ENT).

Level 1: Within-person factor model

$$\begin{bmatrix} EXC_{it} \\ ENE_{it} \\ ENT_{it} \end{bmatrix} = \begin{bmatrix} \mu_{EXC,i} \\ \mu_{ENE,i} \\ \mu_{ENT,i} \end{bmatrix} + \begin{bmatrix} 1 \\ \lambda_{2W} \\ \lambda_{3W} \end{bmatrix} PAW_{it} + \begin{bmatrix} \epsilon_{EXC,it} \\ \epsilon_{ENE,it} \\ \epsilon_{ENT,it} \end{bmatrix}$$

where

- μ's are the individual's means
- λ's are the within-person factor loadings
- PAW_{it} is the individual's latent score at occasion t
- ϵ's are the individual's measurement errors at occasion t

Multiple indicator AR(1) model for PA

Note that PAW_{it} has a mean of zero for each person (hence no within-person means here).

Level 1: Within-person latent AR(1)

$$PAW_{it} = \phi_i PAW_{i,t-1} + \sigma_i \zeta_{it}$$

where

- ϕ_i is the individual's autoregressive parameter
- $\sigma_i \zeta_{it}$ is the individual's innovation at occasion t (with var(ζ)=1)

Multiple indicator AR(1) model for PA

Level 2: Between-person factor model

$$\begin{bmatrix} \mu_{EXC,i} \\ \mu_{ENE,i} \\ \mu_{ENT,i} \end{bmatrix} = \begin{bmatrix} \mu_{EXC} \\ \mu_{ENE} \\ \mu_{ENT} \end{bmatrix} + \begin{bmatrix} 1 \\ \lambda_{2B} \\ \lambda_{3B} \end{bmatrix} PAB_i + \begin{bmatrix} \epsilon_{EXC,i} \\ \epsilon_{ENE,i} \\ \epsilon_{ENT,i} \end{bmatrix}$$

Level 2: Fixed and random effects

$$PAB_i = v_{0i}$$
$$\phi_i = \phi + v_{1i}$$
$$\zeta_i = \zeta + v_{2i}$$

where

$$\begin{bmatrix} v_{0i} \\ v_{1i} \\ v_{2i} \end{bmatrix} \sim MN \left[\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{11} & & \\ \psi_{21} & \psi_{22} & \\ \psi_{31} & \psi_{32} & \psi_{33} \end{bmatrix} \right]$$

Input: Multiple indicator AR(1) model

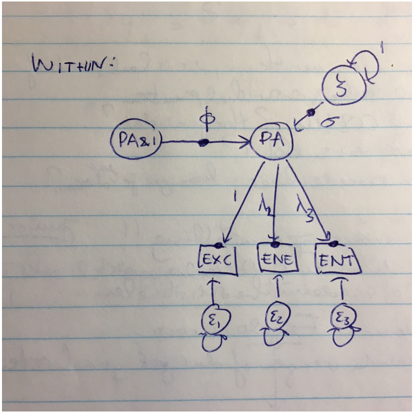
Allowing for:

- random means
- random autoregression
- random innovation SD

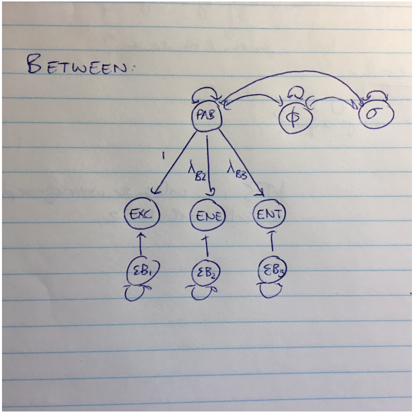
```
MODEL:
  %WITHIN%
  PA BY excited energet enthusi (&1);! FACTOR MODEL AND LAGGED LATENT VARIABLE
  PA@0;                              ! FIX THE RESIDUAL TO ZERO
  zeta BY;                           ! CREATE AN INNOVATION TERM
  PA with zeta@0;                    ! FIX COVARIANCE BETWEEN PA AND ZETA TO ZERO
  zeta@1;                            ! FIX VARIANCE OF THIS TERM TO 1
  sigma | PA on zeta;                ! ALLOW FOR A RANDOM LOADING: INDIVIDUAL SD OF THE INNOVATION
  phi | PA on PA&1;                  ! AUTOREGRESSION IS RANDOM

  %BETWEEN%
  PAB BY excited energet enthusi;    ! FACTOR MODEL
  PAB with sigma;                    ! ALLOW FOR CORRELATED RANDOM EFFECTS
  PAB with phi;                      ! ALLOW FOR CORRELATED RANDOM EFFECTS
  phi with sigma;                    ! ALLOW FOR CORRELATED RANDOM EFFECTS
  [phi*0.2]; phi*0.03;
  [sigma*1.2]; sigma*0.1;
```

Path diagram

Within level:



Between level:



Results: Parameter estimates (within)

MODEL RESULTS

	Estimate	Posterior S.D.	One-Tailed P-Value	95% C.I. Lower 2.5%	95% C.I. Upper 2.5%	Significance
Within Level						
PA BY						
EXCITED	1.000	0.000	0.000	1.000	1.000	
ENERGET	0.953	0.029	0.000	0.898	1.012	*
ENTHUSI	1.049	0.029	0.000	0.993	1.108	*
PA WITH						
ZETA	0.000	0.000	1.000	0.000	0.000	
Variances						
ZETA	1.000	0.000	0.000	1.000	1.000	
Residual Variances						
EXCITED	0.431	0.014	0.000	0.404	0.459	*
ENERGET	0.323	0.012	0.000	0.300	0.346	*
ENTHUSI	0.318	0.012	0.000	0.294	0.343	*
PA	0.001	0.000	0.000	0.001	0.001	

Remember: $Var(PA_i) = \frac{\sigma_i^2}{1-\phi_i^2}$

Results: Parameter estimates (between)

PAB BY						
EXCITED	1.000	0.000	0.000	1.000	1.000	
ENERGET	1.069	0.069	0.000	0.945	1.218	*
ENTHUSI	1.035	0.067	0.000	0.915	1.178	*
PAB WITH						
SIGMA	0.038	0.022	0.032	-0.002	0.085	
PHI	-0.033	0.022	0.056	-0.080	0.008	
PHI WITH						
SIGMA	-0.025	0.009	0.000	-0.046	-0.010	*
Means						
SIGMA	0.562	0.031	0.000	0.502	0.623	*
PHI	0.393	0.029	0.000	0.336	0.450	*
Intercepts						
EXCITED	2.404	0.082	0.000	2.242	2.565	*
ENERGET	2.513	0.083	0.000	2.349	2.676	*
ENTHUSI	2.470	0.081	0.000	2.311	2.629	*
Variances						
PAB	0.470	0.095	0.000	0.321	0.692	*
SIGMA	0.059	0.012	0.000	0.041	0.087	*
PHI	0.025	0.011	0.000	0.009	0.051	*
Residual Variances						
EXCITED	0.086	0.019	0.000	0.056	0.130	*
ENERGET	0.037	0.014	0.000	0.012	0.069	*
ENTHUSI	0.035	0.013	0.000	0.011	0.064	*

NOTE: Means are the fixed effects, variances are the random effects.

Factorial invariance across levels

Are the **factor loadings** for PA **identical across levels**?

```
Within Level

 PA       BY
    EXCITED          1.000       0.000      0.000      1.000      1.000
    ENERGET          0.953       0.029      0.000      0.898      1.012      *
    ENTHUSI          1.049       0.029      0.000      0.993      1.108      *

Between Level

 PAB      BY
    EXCITED          1.000       0.000      0.000      1.000      1.000
    ENERGET          1.069       0.069      0.000      0.945      1.218      *
    ENTHUSI          1.035       0.067      0.000      0.915      1.178      *
```

If $\lambda_w = \lambda_b$, this implies that within-person, state-like fluctuations are **situated on the same underlying dimension** as stable between-person, trait-like differences.

DICs using 500,000 iterations

	$\lambda_w \neq \lambda_b$	$\lambda_w = \lambda_b$
	22355	22364
	22349	22358
	22353	22360
Average:	22352	22361

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Multilevel VAR(1) model

In a vector autoregressive (VAR) model, a vector is regressed on preceding versions of itself.

VAR(1):

$$\boldsymbol{y}_t = \boldsymbol{c} + \boldsymbol{\Phi}\boldsymbol{y}_{t-1} + \boldsymbol{\zeta}_t \qquad \text{with} \quad \boldsymbol{\mu} = (\boldsymbol{I} - \boldsymbol{\Phi})^{-1}\boldsymbol{c}$$

Alternative expression of a VAR(1):

$$\boldsymbol{y}_t = \boldsymbol{\mu} + \boldsymbol{\Phi}(\boldsymbol{y}_{t-1} - \boldsymbol{\mu}) + \boldsymbol{\zeta}_t$$

When considering a multilevel extension, we want to allow for individual differences in:

- $\boldsymbol{\mu}$: the trait scores of individuals
- $\boldsymbol{\Phi}$: the inertias and cross-lagged relationships

NOTE: We write $\boldsymbol{y}^*_{t-1} = \boldsymbol{y}_{t-1} - \boldsymbol{\mu}$.

Example of a multilevel VAR(1) model

We make use of bivariate data from Emilio Ferrer: Positive Affect and Rumination (see Schuurman, Grasman & Hamaker, 2016).

Six days of ESM data with N=129 and T about 45.

Within level:

$$\begin{bmatrix} PA_{it} \\ RU_{it} \end{bmatrix} = \begin{bmatrix} \mu_{PA,i} \\ \mu_{RU,i} \end{bmatrix} + \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} PA^*_{it-1} \\ RU^*_{it-1} \end{bmatrix} + \begin{bmatrix} \zeta_{PA,it} \\ \zeta_{RU,it} \end{bmatrix}$$

$$= \begin{bmatrix} \mu_{PA,i} + \phi_{11} PA^*_{it-1} + \phi_{12} RU^*_{it-1} + \zeta_{PA,it} \\ \mu_{RU,i} + \phi_{21} PA^*_{it-1} + \phi_{22} RU^*_{it-1} + \zeta_{RU,it} \end{bmatrix}$$

Model specification

```
MODEL:

    %WITHIN%
    E1 BY PA@1 (&1);
    PA@0.01;
    E2 BY pieker@1(&1);
    pieker@0.01;
    E1 with E2;
    E1;
    E2;
    phi11 | E1 on E1&1;
    phi22 | E2 on E2&1;
    phi12 | E1 on E2&1;
    phi21 | E2 on E1&1;
```

At the between level the means and lagged effects are all allowed to correlate.

Results within level

```
Within Level

 E1       BY
    PA                 1.000      0.000      0.000      1.000      1.000

 E2       BY
    PIEKER             1.000      0.000      0.000      1.000      1.000

 E1       WITH
    E2                 0.496      0.047      0.000      0.413      0.593      *

 Residual Variances
    PA                 0.010      0.000      0.000      0.010      0.010
    PIEKER             0.010      0.000      0.000      0.010      0.010
    E1                 1.961      0.046      0.000      1.890      2.063      *
    E2                 2.640      0.062      0.000      2.518      2.759      *
```

Note that the measurement error variances **fixed at 0.01** are **negligibly small** compared to the total variances.

Results between level

```
Between Level

 PA       WITH
    PHI11          0.025      0.007      0.000      0.014      0.040      *
    PHI12          0.015      0.008      0.035      0.000      0.034
    PHI21         -0.033      0.011      0.000     -0.052     -0.010      *
    PHI22         -0.028      0.011      0.000     -0.056     -0.009      *

 PIEKER   WITH
    PHI11         -0.028      0.008      0.000     -0.046     -0.015      *
    PHI12         -0.012      0.010      0.110     -0.034      0.006
    PHI21          0.045      0.013      0.000      0.020      0.074      *
    PHI22          0.067      0.015      0.000      0.041      0.103      *

 PHI11    WITH
    PHI12         -0.002      0.002      0.040     -0.006      0.000
    PHI21         -0.006      0.002      0.000     -0.010     -0.003      *
    PHI22         -0.002      0.002      0.070     -0.005      0.001

 PHI12    WITH
    PHI21          0.000      0.001      0.435     -0.003      0.003
    PHI22         -0.004      0.003      0.055     -0.010      0.001

 PHI21    WITH
    PHI22         -0.002      0.003      0.280     -0.008      0.003

 PA       WITH
    PIEKER        -0.070      0.048      0.085     -0.169      0.018
```

Results between level (continued)

Means						
PA	2.244	0.063	0.000	2.117	2.357	*
PIEKER	1.752	0.069	0.000	1.599	1.872	*
PHI11	0.620	0.008	0.000	0.605	0.635	*
PHI22	0.356	0.017	0.000	0.318	0.392	*
PHI12	0.140	0.011	0.000	0.117	0.160	*
PHI21	0.265	0.014	0.000	0.236	0.292	*
Variances						
PA	0.382	0.055	0.000	0.291	0.496	*
PIEKER	0.624	0.092	0.000	0.446	0.811	*
PHI11	0.006	0.001	0.000	0.003	0.009	*
PHI22	0.020	0.004	0.000	0.013	0.030	*
PHI12	0.006	0.002	0.000	0.003	0.010	*
PHI21	0.014	0.003	0.000	0.008	0.022	*

Means are the fixed effects; **variances** are for the random effects.

Standardizing the cross-lagged parameters

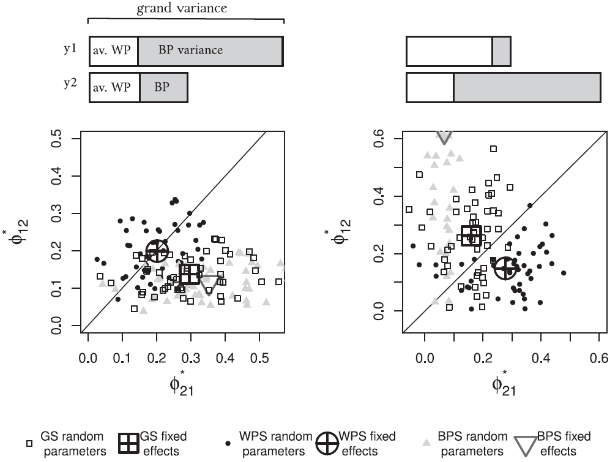
Schuurman et al. (2016) presents three forms of **standardization in multilevel models**:

- total variance (i.e., grand standardization)
- between-person variance (i.e., between standardization)
- average within-person variance
- within-person variance (i.e., within standardization)

Conclusion: last form is most meaningful, as it **parallels standardizing when N=1**.

Standardized fixed effect should be the **average standardized within-person effect**.

Does it make a difference?



From Schuurman et al. (2016)

Networks based on multilevel VAR models

Borsboom has used the idea of **networks as an alternative to latent variables** (in the context of psychopathology).

Dynamical networks are often based on a VAR(1) model.

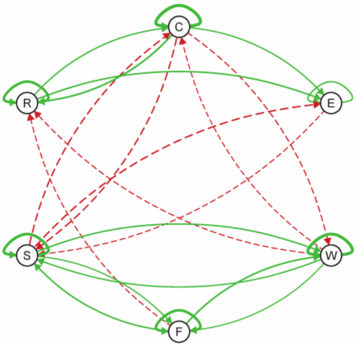
Bringmann et al. (2013) analyzed the lagged relationships between the following variables:

- cheerful (C)
- pleasant event (E)
- worry (W)
- fearful (F)
- sad (S)
- relaxed (R)

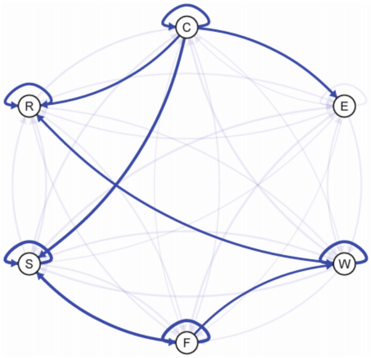
NOTE: They performed **separate multilevel regression analyses** on each of these variables, using all (lagged) variables as predictors.

Results at the population level

Average (fixed effects) network

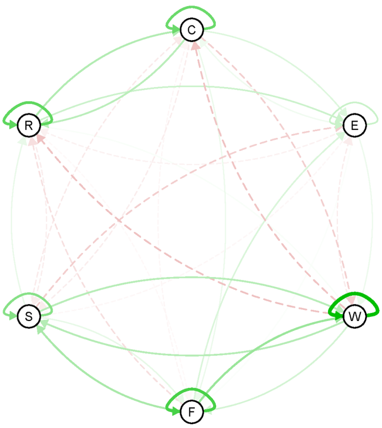


Individual differences network



C=cheerful; E=pleasant event; W=worry; F=fearful; S=sad; and R=relaxed; red solid lines represent positive relationships; green dashed lines represent negative relationship. From Bringmann et al. (2013)

Results at the individual level (2 individuals)



C=cheerful; E=pleasant event; W=worry; F=fearful; S=sad; and R=relaxed
From Bringmann et al. (2013)

Outline

1. Top-down approach:
   - Univariate multilevel AR(1) model
   - Multiple indicator multilevel AR(1) model
   - Multilevel VAR(1) model

2. Bottom-up approach:
   - **Comparison of linear models and regime-switching models**

3. Discussion

Bottom-up: Replicated time series analysis

Characteristics of TSA include:

- N=1
- T is large
- observations are ordered (in time)

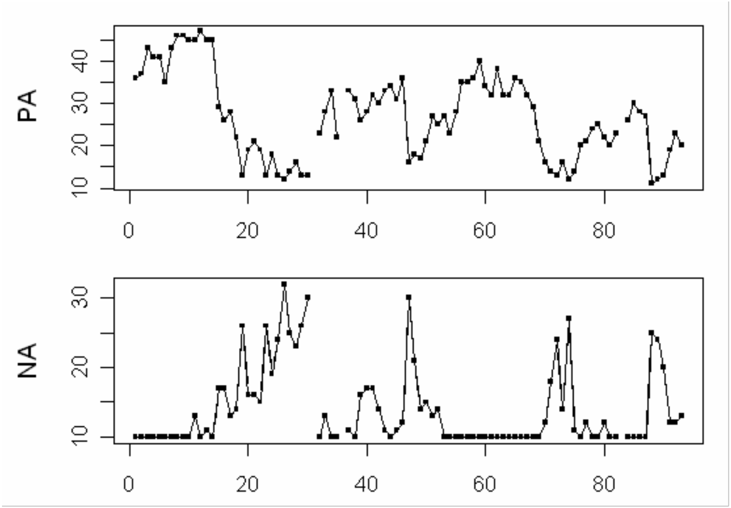
Goals of TSA include:

- prediction and forecasting: weather, currency, earthquakes, epidemic
- signal estimation (Kalman filter): e.g. to control your spacecraft
- identify the nature of the process

Example considered here is based on Hamaker, Grasman and Kamphuis (2016).

Bipolar disorder (BD)

Bipolar disorder is characterized by severe changes in affect and activity: Bipolar patients suffer from **manic** and **depressed episodes**.



BAS dysregulation in BD

BAS may play a crucial role:

- **active BAS**: expecting reward; difficulty inhibiting behavior when approaching a goal; hope
- **inactive BAS**: not expecting reward; difficulty to be motivated; despair

Two forms of BAS dysregulation:

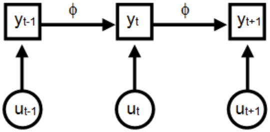
Slow return to baseline

Switches between distinct states

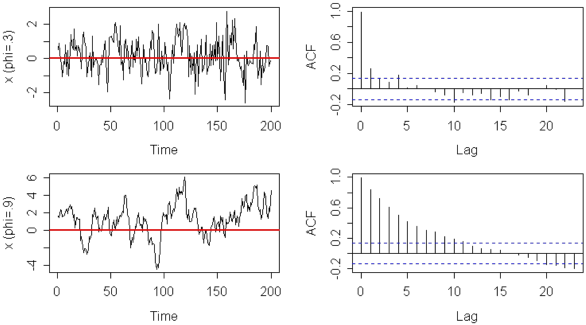
Slow-return-to-baseline model 1: AR(1)

AR(1)

$$y_t = c + \phi y_{t-1} + u_t$$



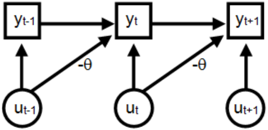
Carry-over: In the AR(1) model today's mood is influenced by yesterday's mood, and the higher ϕ, the more yesterday's mood carries over to today's mood.



Slow-return-to-baseline model 2: ARIMA(0,1,1)

ARIMA(0,1,1)

$$y_t = y_{t-1} - \theta u_{t-1} + u_t$$



Balancing preservation and adaption: The closer θ is to 1, the stronger preservation is; if θ is zero, the system fully adapts to perturbations.

$$\mathrm{E}[y_t | y_{t-1}] = y_{t-1} - \theta e_{t-1}$$

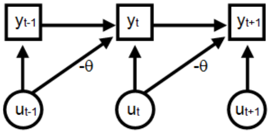
$$= \mathrm{E}[y_{t-1} | y_{t-2}] + e_{t-1} - \theta e_{t-1}$$

The parameter θ is considered to indicate the balance between **preservation** and **adaption**.

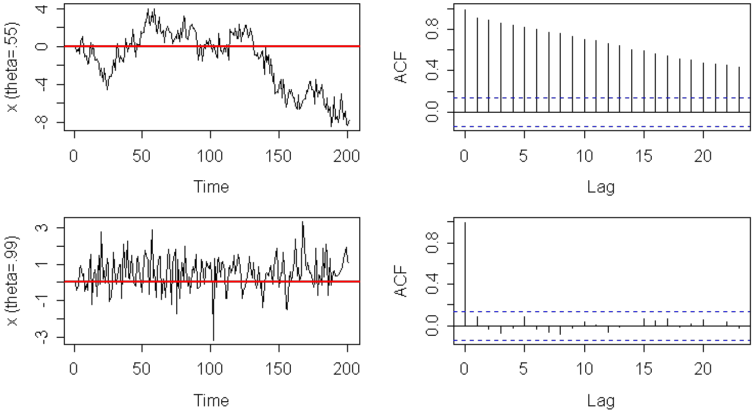
Slow-return-to-baseline model 2: ARIMA(0,1,1)

ARIMA(0,1,1)

$$y_t = y_{t-1} - \theta u_{t-1} + u_t$$



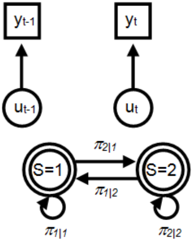
Balancing preservation and adaption: The closer θ is to 1, the stronger preservation is; if θ is zero, the system fully adapts to perturbations.



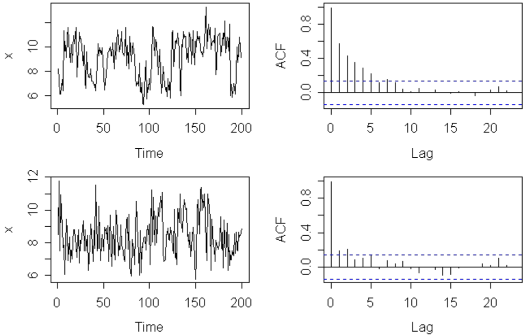
Regime-switching model 1: HM model

HMM

$$y_t = \mu_{S_t} + \sigma_{S_t} u_t$$



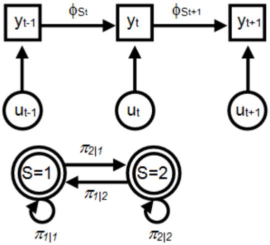
Switching: In the HMM model the system switches between two different WN processes (different means and variances). For each state, there is a probability to stay in it ($\pi_{1|1}$ and $\pi_{2|2}$) and a probabilities to switch ($\pi_{1|2}$ and $\pi_{2|1}$) .



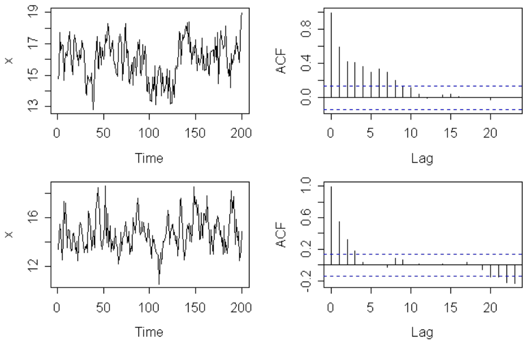
Regime-switching model 2: MSAR(1) model

MSAR(1)

$$y_t = c_{S_t} + \phi_{S_t} y_{t-1} + \sigma_{S_t} u_t$$



Switching with carry-over: The MSAR model is characterized by switches between two different AR(1) processes (different constant c, AR parameter ϕ and variance). Switches are smoother than in the HMM, due to the carry-over.



VAR(1) model and results

```
model:
    y1 with y2;
    y1 y2 on y1&1 y2&1;
```

Note we make use of observed lagged variables y1&1 and y2&1.

MODEL RESULTS

	Estimate	Posterior S.D.	One-Tailed P-Value	95% C.I. Lower 2.5%	95% C.I. Upper 2.5%	Significance
Y1 ON						
Y1&1	0.881	0.079	0.000	0.717	1.042	*
Y2&1	0.041	0.140	0.379	-0.234	0.312	
Y2 ON						
Y1&1	-0.101	0.072	0.066	-0.246	0.037	
Y2&1	0.476	0.124	0.000	0.236	0.709	*
Y1 WITH						
Y2	-15.438	3.366	0.000	-23.886	-10.165	*
Intercepts						
Y1	2.439	3.873	0.242	-4.875	10.487	
Y2	9.931	3.443	0.004	3.565	17.121	*
Residual Variances						
Y1	26.481	4.307	0.000	19.982	36.577	*
Y2	22.405	3.674	0.000	17.120	31.582	*

VARIMA(0,1,1) model

```
model:
    e1 with e2;
    y1-y2@0.5; [y1-y2@0];
    e1 by y1@1 (&1);
    e2 by y2@1 (&1);
    y1 on y1&1@1 e1&1;
    y2 on y2&1@1 e2&1;
```

where:

- e1 by y1@1; defines e1 as the innovation of the process y1
- e1 by (&1); defines a lagged version of e1 (i.e., innovation at previous time point)
- y1 on y1&1@1; defines the I(1) part (random walk)
- y1 on e1&1; defines the MA(1) part (moving average process)

and:

- y1@0.5; sets the measurement error variance to a negligible small number
- and [y1@0]; sets the mean of the process to zero (because it is a unit root process; mean is not identified)

VARIMA(0,1,1) results

MODEL RESULTS

	Estimate	Posterior S.D.	One-Tailed P-Value	95% C.I. Lower 2.5%	95% C.I. Upper 2.5%	Significance
E1 BY						
Y1	1.000	0.000	0.000	1.000	1.000	
E2 BY						
Y2	1.000	0.000	0.000	1.000	1.000	
Y1 ON						
E1&1	-0.200	0.098	0.025	-0.384	-0.000	*
Y2 ON						
E2&1	-0.483	0.098	0.000	-0.658	-0.271	*
Y1 ON						
Y1&1	1.000	0.000	0.000	1.000	1.000	
Y2 ON						
Y2&1	1.000	0.000	0.000	1.000	1.000	
E1 WITH						
E2	-17.078	3.639	0.000	-25.295	-11.432	*
Intercepts						
Y1	0.000	0.000	1.000	0.000	0.000	
Y2	0.000	0.000	1.000	0.000	0.000	
Variances						
E1	27.380	4.583	0.000	20.380	37.988	*
E2	24.409	3.974	0.000	18.196	33.781	*
Residual Variances						
Y1	0.500	0.000	0.000	0.500	0.500	
Y2	0.500	0.000	0.000	0.500	0.500	

HMM model

```
model:
    %overall%
    c on c&1;
    y1 with y2; y1-y2; [y1-y2];

model c:

     %C#1%

     y1 WITH y2*-0.12152 (v3);

     [ y1*2.02322 ];
     [ y2*1.66623 ];

     y1*0.40301 (v1);
     y2*0.27785 (v2);

     %C#2%

     y1 WITH y2*-0.12661 (w3);

     [ y1*2.05252 ];
     [ y2*1.61515 ];

     y1*0.40550 (w1);
     y2*0.20074 (w2);

model prior:
v1~IW(2,2);
v2~IW(2,2);
v3~IW(0,2);

w1~IW(2,2);
w2~IW(2,2);
w3~IW(0,2);
```

The overall model part:

- C ON C&1; specifies hidden Markov model
- y1 with y2; ensures the variables are allowed to correlate

Rest is used for specifying starting values and priors

HMM results

MODEL RESULTS

	Estimate	Posterior S.D.	One-Tailed P-Value	95% C.I. Lower 2.5%	95% C.I. Upper 2.5%	Significance
Latent Class Pattern 1 1						
Y1 WITH						
Y2	-29.667	8.942	0.000	-52.482	-16.570	*
Means						
Y1	20.767	1.241	0.000	18.314	23.326	*
Y2	17.659	0.962	0.000	15.725	19.515	*
Variances						
Y1	59.325	14.126	0.000	39.518	94.774	*
Y2	37.273	8.495	0.000	25.278	58.170	*
Latent Class Pattern 1 2						
Y1 WITH						
Y2	0.176	0.394	0.317	-0.618	0.936	
Means						
Y1	33.508	1.283	0.000	30.991	35.930	*
Y2	10.044	0.055	0.000	9.949	10.157	*
Variances						
Y1	57.985	14.079	0.000	39.033	93.425	*
Y2	0.092	0.032	0.000	0.044	0.167	*
Categorical Latent Variables						
C#1 ON						
C&1#1	0.819	0.061	0.000	0.682	0.921	*
C&1#2	0.192	0.061	0.000	0.087	0.327	*
Class Proportions						
Class 1	0.409	0.031	0.000	0.341	0.460	
Class 2	0.091	0.031	0.000	0.039	0.158	
Class 3	0.096	0.031	0.000	0.044	0.164	
Class 4	0.404	0.031	0.000	0.336	0.456	

MSVAR(1) model

```
model:
    %overall%
    C on C&1;
    y1 with y2; y1-y2; [y1-y2];
     y1 y2 on y1&1 y2&1;

MODEL C:
     %C#1%
     y1 y2 on y1&1 y2&1;

     [ y1*20.76743 ] (1);
     [ y2*17.65870 ] (2);

     y1*59.32514 (v1);
     y2*37.27272 (v2);
     y1 WITH y2 (v3);

     %C#2%
     y1 y2 on y1&1 y2&1;

     [ y1*33.50785 ] (6);
     [ y2*10.04370 ] (7);

     y1*57.98539 (w1);
     y2*0.09211 (w2);
     y1 WITH y2*0 (w3);


model prior:
v1~IW(2,2);
v2~IW(2,2);
v3~IW(0,2);

w1~IW(2,2);
w2~IW(2,2);
w3~IW(0,2);
```

The overall model part:

- C ON C&1; specifies hidden Markov model
- y1 y2 on y1&1 y2&1; specifies a VAR(1) model
- y1 with y2; ensures the innovations are allowed to correlate

Rest is used for starting values and priors

MSVAR(1) results

MODEL RESULTS

	Estimate	Posterior S.D.	One-Tailed P-Value	95% C.I. Lower 2.5%	95% C.I. Upper 2.5%	Significance
Latent Class Pattern 1 1						
Y1 ON						
Y1&1	0.814	0.131	0.000	0.543	1.053	*
Y2&1	0.133	0.182	0.219	-0.220	0.494	
Y2 ON						
Y1&1	-0.096	0.126	0.224	-0.338	0.159	
Y2&1	0.370	0.184	0.026	-0.001	0.732	
Y1 WITH						
Y2	-21.215	5.869	0.000	-35.638	-12.993	*
Intercepts						
Y1	1.008	5.500	0.428	-9.502	11.979	
Y2	13.713	5.439	0.007	2.799	24.266	*
Residual Variances						
Y1	27.773	6.527	0.000	18.603	43.894	*
Y2	29.673	6.904	0.000	20.094	46.646	*
Latent Class Pattern 1 2						
Y1 ON						
Y1&1	0.836	0.091	0.000	0.649	1.009	*
Y2&1	0.063	0.276	0.404	-0.477	0.611	
Y2 ON						
Y1&1	0.001	0.006	0.394	-0.010	0.013	
Y2&1	0.054	0.020	0.011	0.014	0.091	*
Y1 WITH						
Y2	-0.001	0.192	0.499	-0.368	0.407	
Intercepts						
Y1	5.076	5.182	0.155	-5.268	15.452	
Y2	9.401	0.341	0.000	8.728	10.097	*
Residual Variances						
Y1	17.086	4.395	0.000	11.082	27.990	*
Y2	0.063	0.024	0.000	0.038	0.130	*

MSVAR(1) results

Categorical Latent Variables

C#1 ON						
C&1#1	0.807	0.064	0.000	0.663	0.914	*
C&1#2	0.215	0.064	0.000	0.107	0.355	*

Class Proportions

Class 1	0.404	0.032	0.000	0.332	0.457
Class 2	0.096	0.032	0.000	0.043	0.168
Class 3	0.107	0.032	0.000	0.054	0.177
Class 4	0.393	0.032	0.000	0.322	0.446

Outline

1. Top-down approach:
    - Univariate multilevel AR(1) model
    - Multiple indicator multilevel AR(1) model
    - Multilevel VAR(1) model

2. Bottom-up approach:
    - Comparison of linear models and regime-switching models

3. **Discussion**

Some other issues to consider

- data may be irregularly spaced (e.g., ESM data), which should be taken into account when estimating lagged effects
- time is treated as discrete here, but it might be more appropriate to consider it as continuous (Deboeck & Preacher, 2015; Voelkle et al., 2012)
- there may be trends and cycles present which should (or not?) be accounted for (Liu & West, 2015; Wang & Maxwell, 2015)
- random factor loadings (allowing for idiographic loadings)
- level 2 predictors for the individual differences in dynamics
- time-varying parameters
- multilevel extension of the regime-switching models
- fit measure that allows for all models to be compared...

References and suggested readings

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Thank you

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