Mplus 8: Dynamic SEM

Time series analysis and state-space modeling

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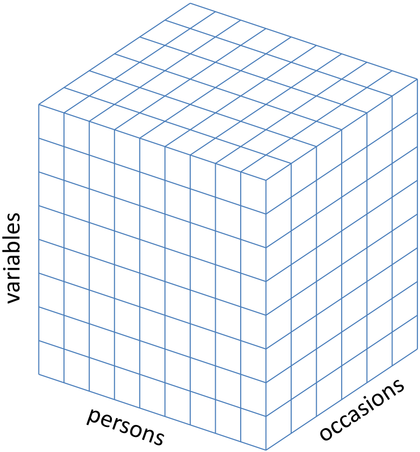
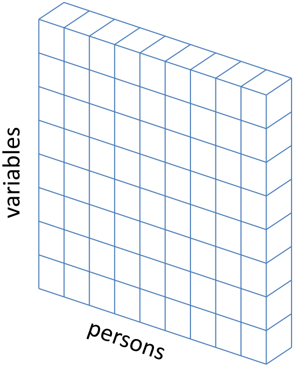
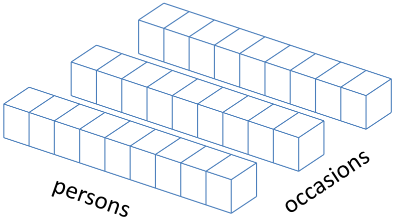
Cattell's data box



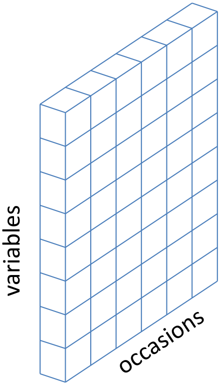
Cross-sectional research: A single snapshot



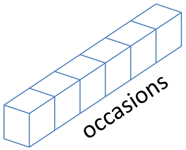
Panel research: A few snapshots



Time series data: Looking at the movie



Time series data: Looking at the movie



Outline

- **Why time series analysis?**
- Autocorrelation
- ARMA models
- Stationarity
- The state-space model
- Kalman filter and parameter estimation
- Diverse models in state-space format
- Miscellaneous

What is time series analysis?

Time series analysis is a class of techniques that is used in econometrics, seismology, meteorology, control engineering, and signal processing.

Main characteristics:

- N=1 technique
- T is large (say >50)
- concerned with *trends*, *cycles* and *autocorrelation structure* (i.e., serial dependency)
- goal: forecasting ($\neq$ prediction)

TSA in the social and medical sciences

In **sociology**:

- quarterly unemployment numbers
- effect of alcohol consumption per capita on criminal violence rates
- effect of suicide news on suicide rates

In **medical research**:

- effect of safety warnings on antidepressants use
- effects of pain control strategies
- effect of 9/11 attacks on weekly psychiatric patient admissions

In **psychology**:

- network of symptoms in depressive patient
- effect of feedback on academic performance
- effect of an intervention on the relationship between stress and affect

Intensive longitudinal data

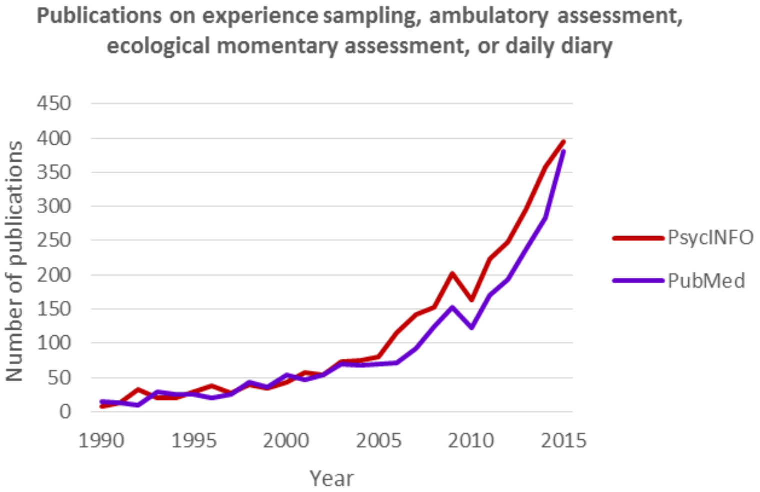
Intensive longitudinal data are gathered using:

- daily diary with end-of-day-measurements (self-report)
- experience sampling method (self-report)
- ecological momentary assessment (self-report)
- ambulatory assessment (including physiological variables)
- event contingency (self-report)
- observational measurements (expert rater)

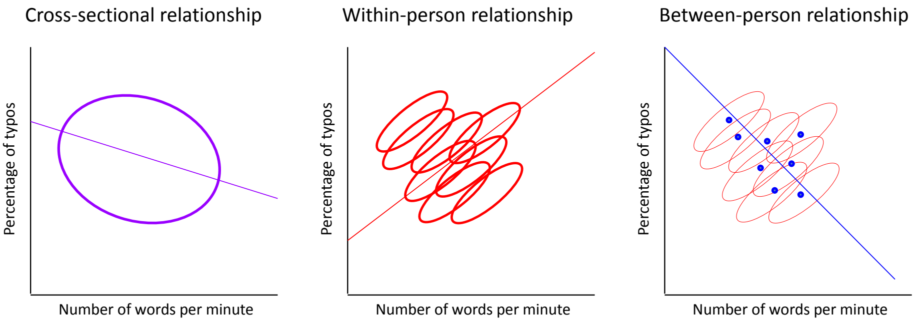
For more info on methodology, check out:

- Tamlin Conner (e.g., her seminar with Joshua Smyth on YouTube)
- Society for Ambulatory Assessment
- Trull and Ebner-Priemer (2013)

It's a revolution!

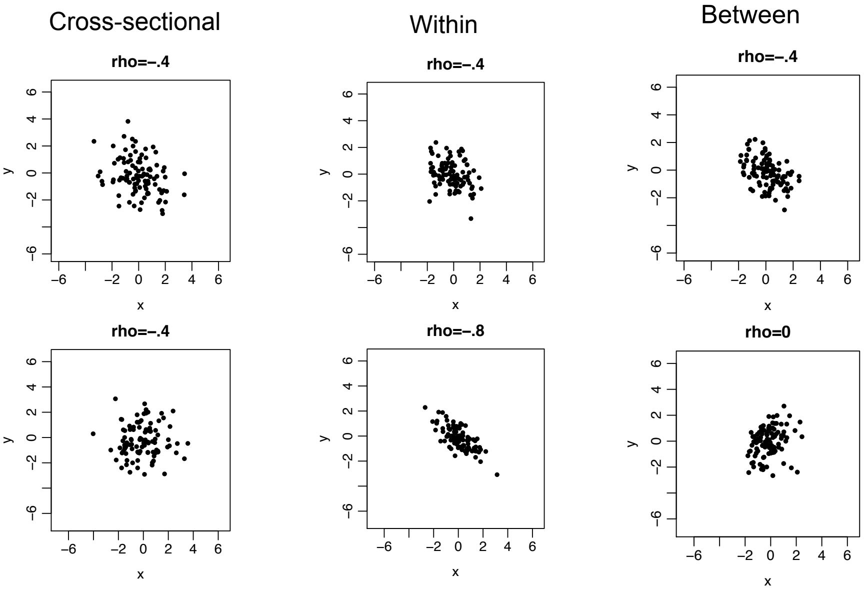


A fundamental problem in a nutshell



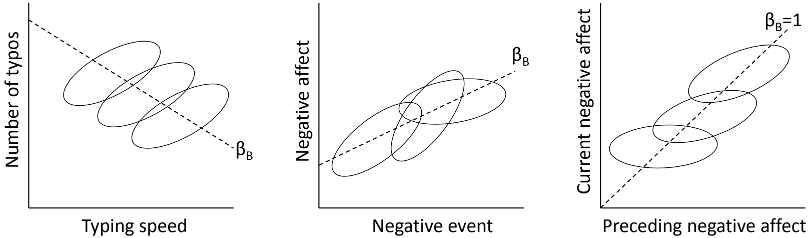
Taken from Hamaker (2012).

Three perspectives on data



Taken from Hamaker (2012).

Interindividual differences in intraindividual variation



Taken from Hamaker and Grasman (2014).

Cross-sectional correlations: A blend

Schmitz (2000):

$$r_{cs} = \eta^2 r_b + (1 - \eta^2) r_w$$

where

- r_{cs} is the cross-sectional correlation
- r_b is the between-person correlation
- r_w is the within-person correlation
- η^2 is the proportion of between-person variance of the total variability

Consequences:

- cross-sectional and panel research may result in an "**uninterpretable blend**" of within-person and between-person relationships (cf. Raudenbush and Bryk, 2002)
- in N=1 time series analysis there is **only within-person** variance

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Lags

Y	Y at lag 1	Y at lag 2
y_1		
y_2	y_1	
y_3	y_2	y_1
y_4	y_3	y_2
y_5	y_4	y_3
y_6	y_5	y_4
y_7	y_6	y_5
y_8	y_7	y_6
$\dots$	$\dots$	$\dots$
y_T	y_{T-1}	y_{T-2}
	y_T	y_{T-1}
		y_T

Autocorrelation function (ACF)

The ACF and the PACF can be used as **diagnostic tools** to determine the nature of the underlying process.

Variance (or: auto-covariance at lag 0):

$$\gamma_0 = \frac{1}{T}\sum_{t=1}^{T}\left(y_t - \bar{y}_t\right)^2$$

Auto-covariance at lag k:

$$\gamma_k = \frac{1}{T-k}\sum_{t=k+1}^{T}\left(y_t - \bar{y}_t\right)\left(y_{t-k} - \bar{y}_t\right)$$

Autocorrelation at lag k:

$$\rho_k = \frac{\gamma_k}{\gamma_0}$$

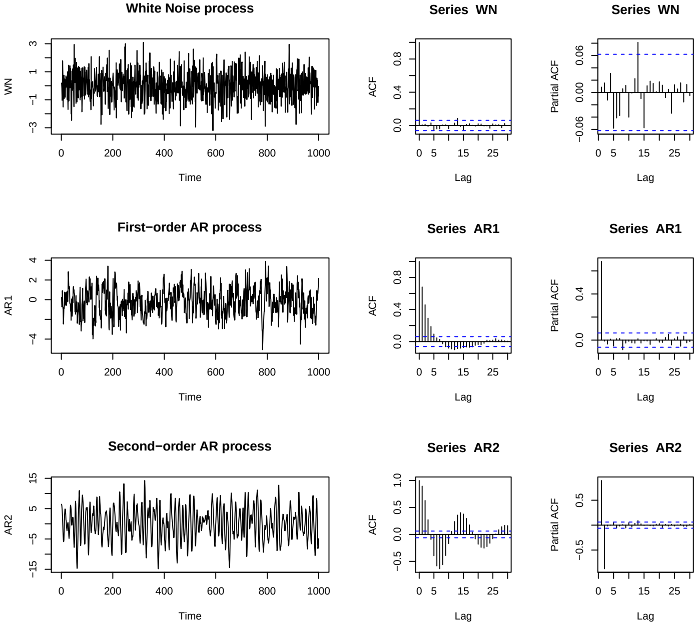
Partial autocorrelation function (PACF)

Partial autocorrelation at lag k is the correlation between y_t and y_{t-k} after **removing the effect of the intermediate observations** (i.e., y_{t-1} to y_{t-k+1}).

Y	Y at lag 1	Y at lag 2
y_1		
y_2	y_1	
y_3	y_2	y_1
y_4	y_3	y_2
y_5	y_4	y_3
$\dots$	$\dots$	$\dots$
y_T	y_{T-1}	y_{T-2}
	y_T	y_{T-1}
		y_T

For instance: Is there a relationship between yesterday's positive affect and tomorrow's positive affect above and beyond their relationship to today's positive affect?

Sequence, ACF and PACF



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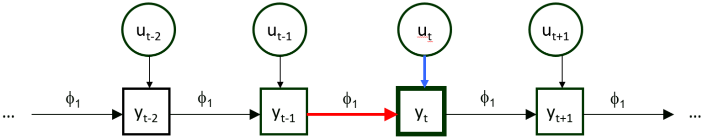
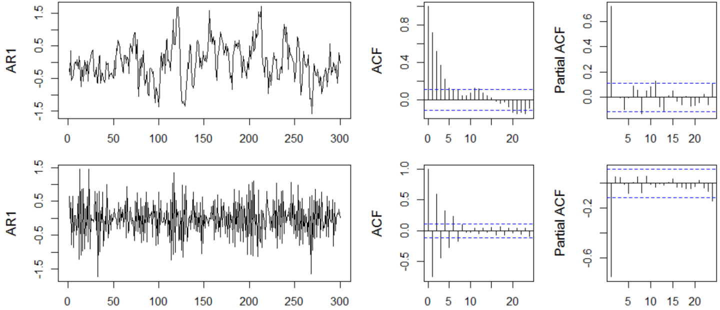
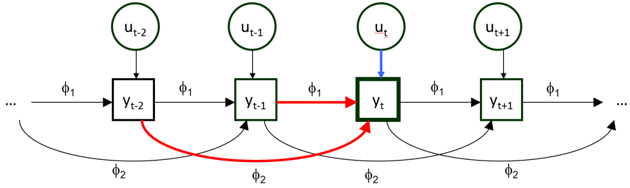
AR(1): $y_t = \phi_1 y_{t-1} + u_t$



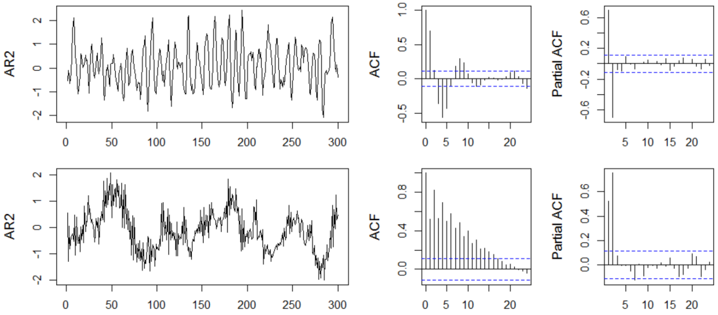
Example with $\phi_1 = 0.7$ and $\phi_1 = -0.7$:



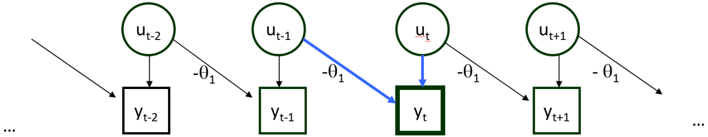
AR(2): $y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + u_t$



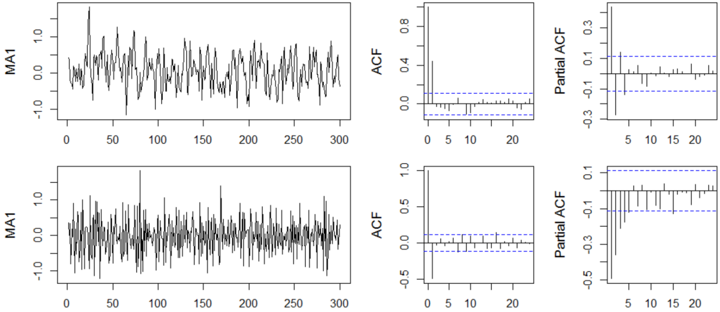
Example with $\phi_1 = 1.2$ and $\phi_2 = -0.7$ and with $\phi_1 = 0.2$ and $\phi_2 = 0.7$:



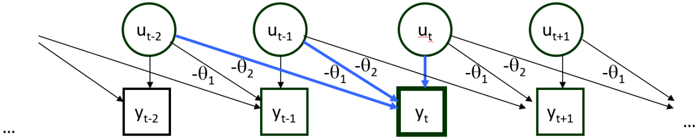
MA(1): $y_t = u_t - \theta_1 u_{t-1}$



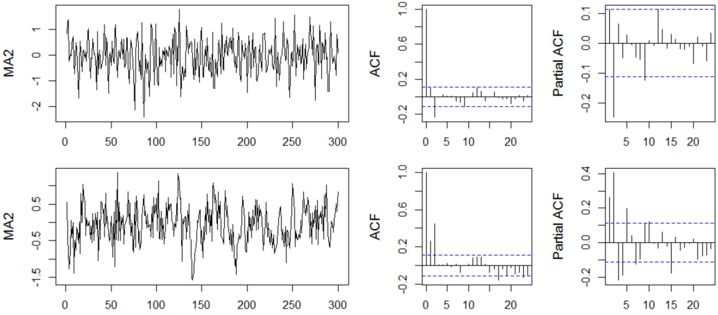
Examples with $\theta_1 = 0.7$ and with $\theta_1 = -0.7$:



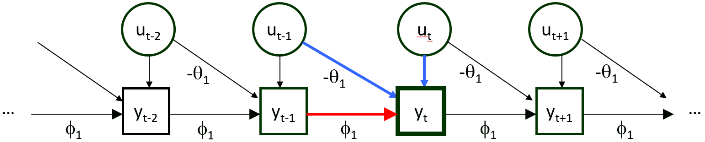
MA(2): $y_t = u_t - \theta_1 u_{t-1} - \theta_2 u_{t-2}$



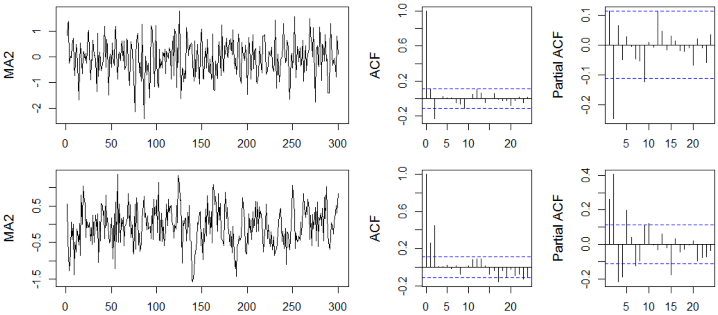
Examples with $\theta_1 = 1.2$ and $\theta_2 = -0.7$, and with $\theta_1 = 0.2$ and $\theta_2 = 0.7$:



ARMA(1,1): $y_t = \phi y_{t-1} + u_t - \theta_1 u_{t-1}$



Example with $\phi_1 = .8$ and $\theta_1 = 0.8$, and with $\phi_1 = -0.8$ and $\theta_1 = -0.8$:



Pure AR, pure MA, or an ARMA(p, q)?

In general:

- an AR(p) can **always** be written as an MA(∞)
- an MA(q) can **always** be written as an AR(∞)

Other (rather unexpected) results found by **Granger and Morris (1976)**:

- AR(1) + WN $\rightarrow$ ARMA(1,1)
- AR(1) + AR(1) $\rightarrow$ ARMA(2,1)
- MA(1) + WN $\rightarrow$ MA(1)

You may consider:

- interpretation (social sciences)
- forecasting (econometrics)
- parsimony

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Stationarity

Stationarity is an important concept in time series analysis:

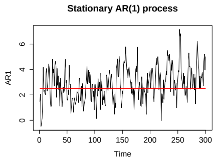
- is based on using **backshift operators** and the **unit root circle** (as all introductory texts on time series analysis do!)
- implies that **all moments** (i.e., means, variances, covariances, lagged covariances, etc.) are **independent of time**

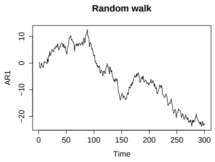
For instance:

- mean is constant over time
- γ_k **depends on the lag** k, **not on** t (i.e., the occasion itself)

Two typical **examples of nonstationary processes**:

- trends over time (including cycles?)
- random walk: $y_t = y_t + e_t$





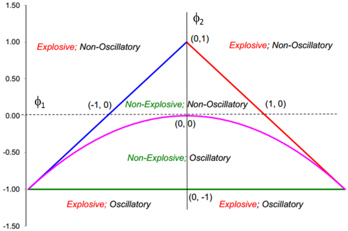
Stationarity of an AR(p)

For an AR(1) to be stationary, $|\phi| < 1$.

For an AR(2) to be stationary we need:

- $\phi_2 - \phi_1 < 1$
- $\phi_2 + \phi_1 < 1$
- $|\phi_2| < 1$

which leads to the following triangle:



(Check out: http://freakonometrics.hypotheses.org/12081)

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Rocket science

State space model with **known parameters**:

- Kalman **filter** predicts the future state (e.g., the location of your space rocket), based on current and previous observations (on-line procedure)
- Kalman **smoother** predicts the state based on previous, current and future observations (off-line procedure)

Often, the **parameter values are NOT known**.

Then, certain **by-products** of the Kalman filter/smoother can be used in a **likelihood function** (see later).

The basic framework

Measurement equation

$$y_t = c_t + Z_t a_t + e_t \qquad \text{with} \qquad e_t \sim MN(0, GG_t)$$

- c_t is the vector with intercepts in the measurement equation
- Z_t is the matrix with factor loadings
- GG_t is the covariance matrix of the measurement errors

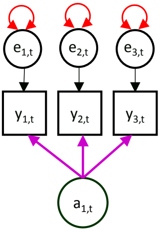
Transition equation

$$a_t = d_t + T_t a_{t-1} + u_t \qquad \text{with} \qquad u_t \sim MN(0, HH_t)$$

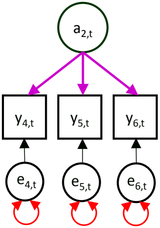
- d_t is the vector with intercepts in the transition equation
- T_t is the matrix with cross- and auto-regressive coefficients
- HH_t is the covariance matrix of the dynamic errors

In a **more basic version** these model matrices are **fixed over time**.

Measurement equation: regressing y_t on a_t

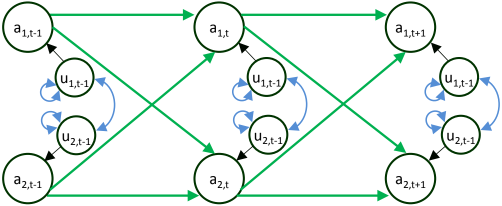


$$y_t = c + \mathbf{Z} a_t + e_t$$
$$e_t \sim MN(0, \mathbf{GG})$$



Transition equation: Regressing a_t on a_{t-1}

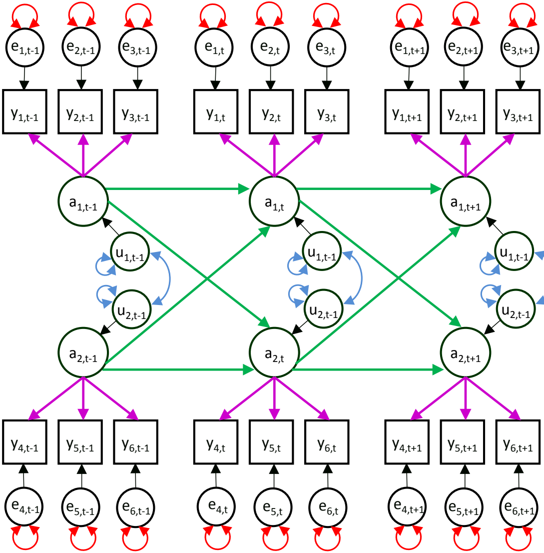
$$a_t = d + \mathbf{T} a_{t-1} + u_t$$
$$u_t \sim MN(0, \mathbf{HH})$$



State-space model = Latent VAR(1) model

$$y_t = c + \mathbf{Z} a_t + e_t$$
$$e_t \sim MN(0, \mathbf{GG})$$

$$a_t = d + \mathbf{T} a_{t-1} + u_t$$
$$u_t \sim MN(0, \mathbf{HH})$$



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State-space model versus SEM

Two ways in which SEM can be use to do TSA:

Toeplitz method, based on making lagged variables

y_1	
y_2	y_1
y_3	y_2
y_4	y_3
$\dots$	
y_T	y_{T-1}

Advantage: easy

Disadvantage: violates assumption of independent cases (=rows); no true ML estimates (and wrong fit measures)

(cf. Hamaker, Dolan & Molenaar, 2002)

Raw maximum likelihood estimation, based on N=1

$$y_1 \quad y_2 \quad y_3 \quad \dots \quad y_T$$

Advantage: gives ML estimates

Disadvantage: requires inversion of (at least) a $T \times T$ matrix (computationally troublesome)

(cf. Hamaker, Dolan and Molenaar, 2003)

See Chow, Ho, Hamaker and Dolan (2010) for further comparison of state-space modeling and SEM.

Kalman filter for parameter estimation

The Kalman filter can be used to **predict future states** when the **parameters are known**.

In practice, the parameter values are often **unknown**.

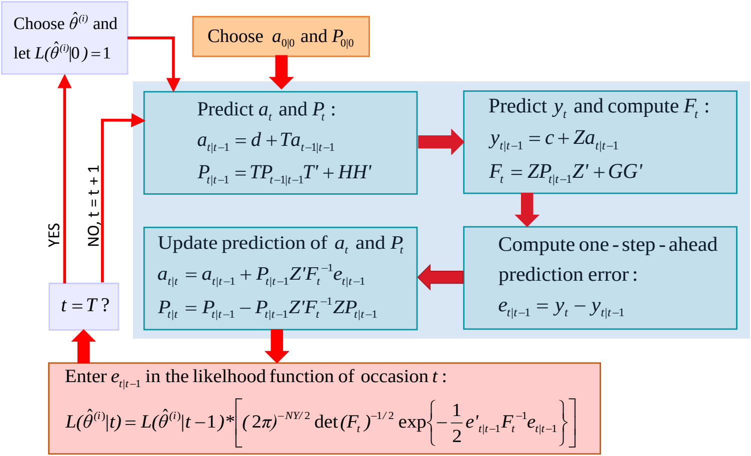
In that case, **by-products** of the Kalman filter can be used to **estimate the parameters**:

- the **one-step-ahead-prediction error** $e_{t|t-1} = y_t - y_{t|t-1}$
- the **covariance matrix** of $e_{t|t-1}$ (i.e., F_t)

These are **plugged into a likelihood function**, which is then **optimized** with respect to the unknown parameters.

Hence, for **each set** of possible parameter values, the **entire Kalman filter** is run from $t = 1$ to $t = T$.

Kalman filter for parameter estimation



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Just a latent vector AR(1) model?

At first sight the state-space model **seems to be** just a latent VAR(1) model.

However, it is actually a **very flexible framework** for all sorts of time series models:

- all ARIMA models
- multivariate extensions
- dynamic factor analysis

Extensions may consist of:

- predictors (e.g., time, intervention, weather conditions) in the measurement and/or transition equation
- time-varying parameters
- regime switches (through combination with a hidden Markov process)

AR(1) in state-space format

Measurement equation:

$$y_t = c + a_t$$

- c is a vector containing the unknown mean
- Z is a 1 by 1 matrix containing 1
- GG is a zero matrix

Transition equation:

$$a_t = Ta_{t-1} + u_t$$

- d is a zero vector
- T is a 1 by 1 matrix containing the autoregressive parameter
- HH is a 1 by 1 covariance matrix containing the variance of the innovations

AR(1) with measurement error

Measurement equation:

$$y_t = c + a_t + e_t$$

- c is a vector containing the unknown mean
- Z is a 1 by 1 matrix containing 1
- GG is a 1 by 1 covariance matrix with the variance of the measurement error

Transition equation:

$$a_t = Ta_{t-1} + u_t$$

- d is a zero vector
- T is a 1 by 1 matrix containing the autoregressive parameter
- HH is a 1 by 1 covariance matrix containing the variance of the innovations

AR(2) in state-space format

Measurement equation:

$$y_t = c + \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} a_t \\ a_{t-1} \end{bmatrix} = c + a_t$$

where GG is a zero matrix.

Transition equation:

$$\begin{bmatrix} a_t \\ a_{t-1} \end{bmatrix} = \begin{bmatrix} \phi_1 & \phi_2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a_{t-1} \\ a_{t-2} \end{bmatrix} + \begin{bmatrix} u_t \\ 0 \end{bmatrix} = \begin{bmatrix} \phi_1 a_{t-1} + \phi_2 a_{t-2} + u_t \\ a_{t-1} \end{bmatrix}$$

- d is a zero vector
- HH is a 2 by 2 covariance matrix containing only the variance of the innovations (element 1,1)

Bivariate VAR(1) in state-space format

The measurement equation:

$$\begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a_{1,t} \\ a_{2,t} \end{bmatrix}$$

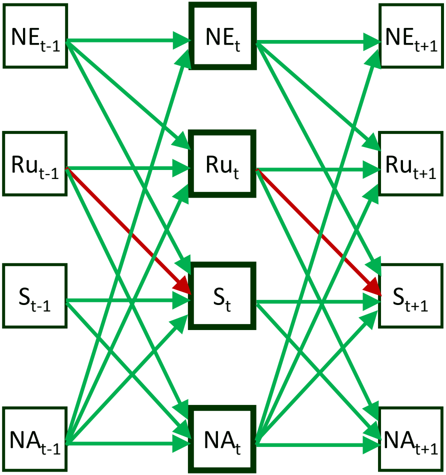
where GG is a zero matrix.

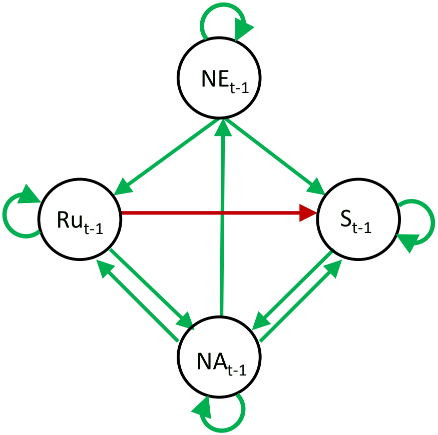
The transition equation:

$$\begin{bmatrix} a_{1,t} \\ a_{2,t} \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{21} \\ \phi_{12} & \phi_{22} \end{bmatrix} \begin{bmatrix} a_{1,t-1} \\ a_{2,t-1} \end{bmatrix} + \begin{bmatrix} u_{1,t} \\ u_{2,t} \end{bmatrix} = \begin{bmatrix} \phi_{11} a_{1,t-1} + \phi_{21} a_{2,t-1} + u_{1,t} \\ \phi_{22} a_{2,t-1} + \phi_{12} a_{1,t-1} + u_{2,t} \end{bmatrix}$$

- d is a zero vector
- HH is a 2 by 2 covariance matrix of $u_{1,t}$ and $u_{2,t}$

Graphical representations





Applications of VAR models

VAR models are of interest, because

- they allow you to study **Granger causality**: Can you predict Y from X, after controlling for previous levels of Y?
- they allow you to determine which variable is "**causally dominant**" when there are reciprocal effects
- they can be interpreted as **networks** (alternative to latent variable approach)

Some interesting **replicated VAR applications**

- Schmitz and Skinner (1994): Perceived control, effort and academic performance
- Rosmalen et al. (2012): Depression and physical activity
- Snippe et al. (2014): Mindfulness, repetitive thinking and depressive symptoms
- Van Gils et al. (2014): Stress and functional somatic symptoms

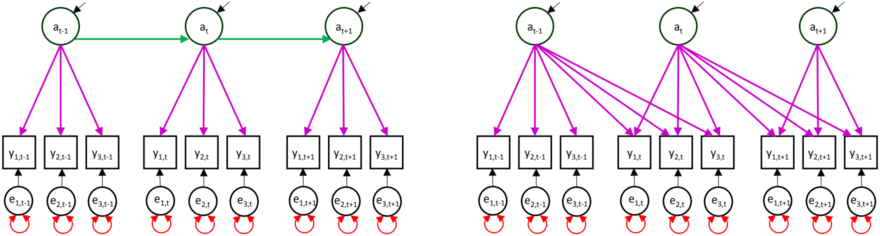
In all these studies they find **important differences across individuals**.

Dynamic factor model

Dynamic factor analysis is used for time series data consisting of **multiple indicators** of an underlying construct.

There are **two popular versions**:

- at the latent level there is a **VARMA model**; the factor loadings only appear **at lag 0**
- at the latent level there is **white noise**; the factor loadings appear at **different lags** (e.g., EEG data)



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Covariance matrix of the series

For a **univariate AR(1)**, we have: $\sigma_y^2 = \frac{\sigma_u^2}{1-\phi^2}$.

Similarly, for a **(latent) VAR model** we can express the **covariance matrix** of y_t in terms of

- **lagged regression parameters** Φ
- **covariance matrix of the innovations** Γ (i.e., HH in the state-space model)

Specifically (from Kim and Nelson, 1999):

$$\Sigma_y = mat\Big[\big(I - \Phi \otimes \Phi\big)^{-1} vec(\Gamma)\Big]$$

where

- $vec()$ implies you put all the **matrix elements in a vector**
- $mat()$ implies you place all the **vector elements in a square matrix**

Model fit

Despite the **similar appearance**, state-space modeling and SEM are **not the same**: For a time series there is **no saturated model** against which we can test other models.

We can **compare our model to other models**, including the white noise model (independence model), using

- log likelihood ratio test (for nested models)
- AIC, BIC, DIC, etc. (for all models)

Fit may be **less interesting** to econometricians and meteorologists: Their primary interest is **forecasting**.

To conclude

- time series analysis is a large class of diverse techniques to analyze N=1 data
- ARMA models are only a small (but basic) part of this
- time series models may be extended with cycles or trends over time
- in psychology we typically have N>1; there are different ways of handling this

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