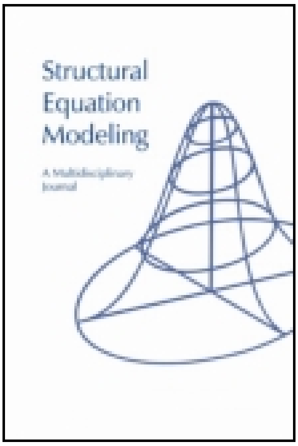

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Latent growth modeling is often conducted using a confirmatory approach whereby specific structures of individual change (e.g., linear, quadratic, exponential, etc.) are fit to the observed data, the best fitting model is chosen based on fit statistics and theoretical considerations, and parameters from this model are interpreted. This confirmatory approach is appropriate when a strong theory guides the model fitting process. However, this approach is often also used when there is not a strong theory to guide the model fitting process, which might lead researchers to misrepresent or miss key change characteristics present in their data. We discuss Tuckerized curves (Tucker, 1958, 1966) as an exploratory way of modeling change processes based on principal components analysis and propose an exploratory approach to latent growth modeling whereby minimal constraints are imposed on the structure of within-person change. These methods are applied to longitudinal data on cortisol response during a controlled experimental manipulation and height changes from early childhood through adulthood collected from 2 different studies. We highlight the additional insights gained, some of the benefits, limitations, and potential extensions of the exploratory growth curve approach and suggest there is much to be gained from using such models to generate new and potentially more precise theories about change and development.

Keywords: change, cortisol, development, exploratory, growth, physical stature

Latent growth modeling is primarily conducted using a confirmatory modeling approach where specific functions of time are tested against longitudinal panel data to determine which of the various shapes of change (e.g., linear, quadratic, exponential, etc.) best represent the within-person change patterns and between-person differences therein. The widespread adoption, application, and promotion of this confirmatory approach to latent growth modeling framework has catapulted the study of change forward (McArdle & Nesselroade, 2003; Preacher, Wichman,

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MacCallum, & Briggs, 2008; Singer & Willett, 2003). Interestingly, looking back at our history, the foundations of latent growth modeling, especially the original work by Tucker (1958, 1966), was much more exploratory. The opportunities for discovery inherent in the original exploratory work might have been lost along the developmental path of growth curve modeling.

We explore the flexibility of the original framework, in terms of the number of components (factors) underlying the change process and their functional form, and suggest that the flexibility of the original framework can be integrated with the contemporary implementation of latent growth curves facilitated by the structural equation modeling (SEM) framework and the recent work on exploratory structural equation modeling (ESEM; Asparouhov & Muthén, 2009). Our contention is that exploratory approaches to latent growth modeling yield additional insights into change processes that might be missed when fitting growth models in the standard confirmatory approach. In this article, we review both how basic latent growth models are used to specify confirmatory tests of specific change functions and how Tucker's (1958, 1966) original framework was used to identify change functions in a data-driven manner. Building from these frameworks we then describe a general SEM-based approach to exploratory latent growth modeling. Finally, using two sets of empirical data characterized by complex within-person change patterns that are nonlinear in time, we demonstrate how additional insights into the underlying change processes can emerge from the exploratory modeling approach.

LATENT GROWTH MODELING

In the structural modeling framework, the latent growth model is fit as a common factor model. This model, with $t = 1$ to T observed scores, $n = 1$ to N individuals, and $r = 1$ to R growth factors, can be written as

$$y_{tn} = \sum_{r=1}^{R} (\lambda_{tr} \eta_{rn}) + u_{tn}, \quad (1)$$

where y_{tn} is the observed score at time t for individual n, λ_{tr} is the factor loading at time t for growth factor r, η_{rn} is the factor score for growth factor r for individual n, and u_{tn} is the unique score at time t for individual n. In matrix notation, the model can be represented as

$$\mathbf{y}_n = \mathbf{\Lambda} \boldsymbol{\eta}_n + \mathbf{u}_n, \quad (2)$$

where $\mathbf{y}_n$ is a $T \times 1$ vector of the repeatedly measured observed scores for individual n, $\mathbf{\Lambda}$ is a $T \times R$ matrix of factor loadings defining the growth factors (e.g., intercept & slope), $\boldsymbol{\eta}_n$ is an $R \times 1$ vector of latent factor scores (e.g., intercept and slope scores) for individual n, and $\mathbf{u}_n$ is a $T \times 1$ vector of unique scores for individual n. The common factor scores are written as deviations from the group mean and specified as

$$\boldsymbol{\eta}_n = \boldsymbol{\alpha} + \boldsymbol{\xi}_n, \quad (3)$$

where $\boldsymbol{\alpha}$ is an $R \times 1$ vector of latent factor means and $\boldsymbol{\xi}_n$ is an $R \times 1$ vector of mean deviations for individual n. The expected mean ($\boldsymbol{\mu}$) and covariance structure ($\mathbf{\Sigma}$) of observed variables are written as

$$\begin{aligned}\boldsymbol{\mu} &= \boldsymbol{\Lambda\alpha} \\ \boldsymbol{\Sigma} &= \boldsymbol{\Lambda\Phi\Lambda}' + \boldsymbol{\Theta}\end{aligned} \tag{4}$$

where $\boldsymbol{\Phi}$ is an $R \times R$ matrix of latent variable covariances and $\boldsymbol{\Theta}$ is a $T \times T$ matrix of uniqueness covariances. Often, an equality constraint is placed on the diagonal elements of $\boldsymbol{\Theta}$ and off-diagonal values are fixed at zero. However, in many situations the equality constraint on the diagonal elements is not needed for identification (see Grimm & Widaman, 2010).

In research applications of the latent growth model, the elements of $\boldsymbol{\Lambda}$ (i.e., λ_{tr}) are typically fixed to specific values to test how a particular functional (e.g., linear, quadratic, exponential) form, or shape, of change represents the data. For example, with five equally spaced measurement occasions and a linear change function $\boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix}$, where the first column defines an intercept factor that captures the predicted score at the first occasion, and the second column defines, when set equal to t, a slope factor that captures changes that progress linearly with respect to time.

The latent growth modeling framework is extremely flexible and able to accommodate many forms of (nonlinear) change (e.g., Browne & du Toit, 1991; Grimm & Ram, 2009; Grimm, Ram, & Hamagami, 2011; Ram & Grimm, 2007). In practice, complexities of change are usually accommodated either by (a) incorporating additional growth factors, or (b) relaxing the constraints that specify a specific functional form or shape of change. We outline these two approaches for accommodating the complexities of (nonlinear) change, and refer back to them later when presenting the exploratory latent growth model.

Additional Growth Factors

Complex functional forms or shapes of within-person change (beyond straight lines) are often accommodated through inclusion of additional growth factors. For example, quadratic and higher order polynomial functions can be specified using additional growth factors with factor loadings equal to t^2, t^3, t^4, and so on. Similarly, spline or multiphase models (Cudeck & Klebe, 2002) can be specified using additional growth factors with nonzero factor loadings within specific time intervals to model change in specific phases of the change process (Ram & Grimm, 2007). Conceptually, additional growth factors suggests that all individuals' observed change trajectories are driven by multiple functions of time (e.g., linear and quadratic), which can be considered multiple developmental processes. That is, each individual's changes are driven by a linear growth factor, a quadratic growth factor, and so on, with person-specific weights (factor scores) indicating the extent to which each factor contributes to an individual's change trajectory. When following typical procedures for fitting of latent growth curves, the number of growth factors and their functional forms are determined a priori (i.e., mathematically, the order and elements of $\boldsymbol{\Lambda}$ are fixed to specific values and tested against the data).

Relaxing Constraints on Functional Form

Alternatively, complex forms or shapes of within-person change can be accommodated in the latent growth model by relaxing the constraints on the factor loadings for the slope factor(s). Instead of forcing the factor loadings to change linearly with respect to time, *T-2* factor loadings can be estimated from the data. This model is often referred to as a *latent basis* or unstructured model (McArdle & Epstein, 1987; Meredith & Tisak, 1990). The shape of change (i.e., $\mathbf{\Lambda}$) is latent in that it is estimated from the data. That is, the form of change is not required to follow a prespecified functional form, but rather is an optimal functional form (alternatively conceptualized as an optimal rescaling of time) estimated from the data. Conceptually, as with the additional factor approach, the latent basis growth model suggests that all individuals' observed within-person changes are driven by the same process (i.e., $\mathbf{\Lambda}$ is the same for all persons) and individuals differ only in the magnitude of those changes (person-specific factor scores). Following usual fitting procedures, the latent basis factor captures as much complexity as possible in a single interindividual differences factor.

Although rarely done in practice, these two methods for modeling complexity can be combined within the latent growth model. Specifically, multiple growth factors with relatively few constraints on the patterns of change can be used to capture rather complex forms of change (see the multiphase models in Ram & Grimm, 2007). However, even in such applications, where multiple latent bases are estimated from the data, the number of growth factors is still specified a priori. Later, we push the combined possibilities a bit further within an explicitly exploratory framework where both the functional forms and number of factors are not specified a priori.

Exploratory Forms of Change Modeling

Tucker (1958, 1966) laid the foundations for the latent growth curve model with what he called *generalized learning curves*. Meredith and Tisak (1984) brought Tucker's general approach to modeling longitudinal data to the SEM framework, which had recently gained popularity with the development of LISREL. Meredith and Tisak referred to Tucker's approach as *Tuckerized curves* and discussed more data-driven models (e.g., latent basis model) as well as structured models (e.g., exponential). In this section we review Tucker's work, and then examine how modern SEM-based models might be placed in and make use of the original, explicitly exploratory, framework.

Tuckerized Curves

Following Tucker (1958), the *Tuckerized curve* model can be written as

$$y_{tn} = \sum_{r=1}^{R} (\lambda_{tr} \eta_{rn}), \tag{5}$$

where y_{tn} is the observed score at time t for individual n, λ_{tr} is the component loading at time t for growth factor r, and η_{rn} is the score for component r for individual n. The values of λ_{tr}

capture patterns of change interpreted as *generalized learning curves* with η_{rn} as individual weights (i.e., scores) indicating the contribution of λ_{tr} to individual n's observed curve. For example, if two components are retained and individual n has a large value on η_{1n} and a zero value on η_{2n}, then individual n has an observed change pattern that is proportional to λ_{t1} (by amount η_{1n}).

The Tuckerized curve equation (Equation 5) is similar to the latent growth equation (Equation 1) with one exception—there is no uniqueness score. The Tuckerized curve model is a principal components model (prior to any rotation) and observed scores are completely accounted for by R components when $R = T$. As in other applications of principal components analysis, the solution is often truncated and fewer components ($R < T$) are retained and interpreted. We note explicitly that when working with the truncated solution, the observed data are not completely explained, and we are left with a complex matrix of sums of squares and cross-product residuals. This residual matrix can be decomposed into residual means and correlations as the off-diagonal elements will likely be nonzero, but practically, this matrix is often set aside.

Most relevant to this project is that the number of components retained in a Tuckerized curve analysis and their patterns of change captured in λ_{tr} are not specified a priori. Instead the number of components retained is determined by the relative magnitude of eigenvalues of the sums of squares and cross-products matrix as well as the interpretability of the generalized learning curves. Thus, the approach outlined by Tucker is completely exploratory in that neither the number of change components or generalized learning curves nor their form or function with respect to time are specified a priori—similar to an exploratory factor analysis (EFA; without Procrustean rotation) model where neither the number of factors nor their structure or factor loading pattern are specified a priori.

As in EFA there is the question of rotation in Tuckerized curves and from the outset Tucker (1958, 1966) discussed the possibility of rotating the extracted components to aid interpretation. He pointed out the inappropriateness of simple structure as a criterion for rotation, and proposed three alternative criteria: rotation to all positive loadings, positive slopes over time, and, if possible, rotation to an asymptote. Later, Arbuckle and Friendly (1977) suggested using a rotation criterion where component loadings were rotated to smooth functions—by minimizing successive differences between rows in the component loadings matrix. As in EFA, rotation of Tuckerized curves does not affect model–data fit, only the interpretation of the generalized learning curves and the individual weights.

Tucker's approach, often seen as the foundation for latent growth modeling, was purely exploratory. Interestingly, unlike the historical expansion of ANOVA into the general linear model, the Tuckerized curves model is not a constrained type of latent growth curve model, nor can it be implemented within the modern implementations of latent growth curve modeling, which are often fit in the SEM framework—typically seen as a framework for confirmatory types of analysis. We seek to understand if and how the modern SEM-based latent growth curve approach might be adapted backward toward its more exploratory beginnings.

Exploratory Growth Models

In recent years, a number of exploratory change models have emerged that allow for the discovery of change functions or groups of individuals defined by specific change patterns.

These models rest variously on principal component models (Davison, 2008), profile analysis via multidimensional scaling (Ding, Davison, & Petersen, 2005), k-means cluster analysis (Brossart, Parker, & Willson, 1998), latent profile analysis (Gibson, 1959), growth mixture models (Muthén & Shedden, 1999), and latent class growth analysis (Nagin, 1999).

Here, working directly from the Tuckerized curve model, we build an explicitly exploratory model within the latent growth curve approach (Equations 1–4). Our intent is to see how the basic elements of Tucker's original framework can be incorporated and utilized within the modern, highly accessible and familiar SEM-based modeling framework. The general idea is to allow for exploration by taking the basic structure of the model in Equations 1 through 4, and build it up with minimal constraints. In particular, we concentrate on turning $\mathbf{\Lambda}$ from a completely specified matrix to one with the absolute minimum number of constraints.

Beginning with a one-factor model (see also McArdle & Epstein, 1987), elements of the $\mathbf{\Lambda}$ matrix are freely estimated. The single growth factor has an estimated mean and, for identification, a variance fixed at 1. The pattern of factor loadings from this model often map onto the pattern of observed means across time—similar to a one-component Tuckerized curve. The model can then be expanded iteratively, adding additional factors one at a time. Two-factor, three-factor, and other models can be specified with additional identification constraints (see Howe, 1955; McArdle & Cattell, 1994). For example, in a two-factor model one element of $\mathbf{\Lambda}$ must be fixed at 0. All other factor loadings are freely estimated. Thus, with five measurement occasions $\mathbf{\Lambda} = \begin{bmatrix} \lambda_{1,1} & 0 \\ \lambda_{2,1} & \lambda_{2,2} \\ \lambda_{3,1} & \lambda_{3,2} \\ \lambda_{4,1} & \lambda_{4,2} \\ \lambda_{5,1} & \lambda_{5,2} \end{bmatrix}$. The factors have estimated means, variances fixed at 1, and are uncorrelated with one another (i.e., $\mathbf{\Phi}$ is an identity matrix). Of note, the row of $\mathbf{\Lambda}$ (i.e., time point) with the fixed 0 factor loading is arbitrary and does not affect model fit. However, as is usual in growth modeling, the choice has an effect on the estimated parameters and the interpretation of the factor loading patterns. Thus, this choice is often driven by substantive reasoning so that the time point with a fixed 0 factor loading represents a point in time where the second factor has no influence. In the specification given earlier, the second factor has no influence at the first occasion.

Although the number of factors can be increased in an iterative fashion, the number of fixed 0 loadings needed for identification increases with each additional factor. For example, for a three-factor exploratory model with five measurement occasions, $\mathbf{\Lambda} = \begin{bmatrix} \lambda_{1,1} & 0 & 0 \\ \lambda_{2,1} & \lambda_{2,2} & 0 \\ \lambda_{3,1} & \lambda_{3,2} & \lambda_{3,3} \\ \lambda_{4,1} & \lambda_{4,2} & \lambda_{4,3} \\ \lambda_{5,1} & \lambda_{5,2} & \lambda_{5,3} \end{bmatrix}$. In this specification, the second factor has no influence on the first occasion and the third factor has no influence at the first or second occasion. Following exploratory practice, additional growth factors are added one at a time until (a) degrees of freedom are exhausted, (b) the addition of growth factors does not yield interpretable factor patterns, or (c) convergence issues are encountered. Factor patterns that are not interpretable tend to have values that vary in a random fashion across the rows of $\mathbf{\Lambda}$ (i.e., within a column, across time)—an indication that the factor is capturing random fluctuations.

Finally, we mention the structure of unique factors or residuals. Because we are working in a common factor framework, these are not the complex residuals that are obtained in the Tuckerized curve (truncated principal components) approach. They do have meaningful and interpretable structure. As in current latent growth model specifications, $\mathbf{\Theta}$, the matrix of unique covariances is diagonal and the diagonal elements can be freely estimated or constrained to be equivalent with respect to time (see Grimm & Widaman, 2010).[1]

Exploratory Structural Equation Modeling (ESEM)

Recently, Asparouhov and Muthén (2009) introduced the concept of ESEM and incorporated the EFA model within the general SEM framework. The ESEM framework can be adapted to aid the specification and estimation of the exploratory growth model. Specifically, the ESEM framework begins with an unrotated EFA model with a specific number of factors. The factor loading matrix ($\mathbf{\Lambda}$) of this model is specified in the same way as described previously and then the factors are rotated via various rotation criteria (e.g., Quartimin, Geomin, Target) to aid interpretation.

The ESEM framework can aid the exploratory growth modeling approach through this rotation and because the flexibility of the ESEM framework allows for the estimation of factor means. The available rotations within the ESEM framework, as implemented in M*plus*, were not designed for such models, but allow for the estimation of all factor loadings. Thus, all factors can impact scores at all time points (zero loadings do not need to be specified). Furthermore, oblique rotations can be implemented to allow for factor intercorrelations.

Model Fitting

In practice, the model fitting approach for the exploratory growth model is similar to an approach often taken with growth mixture models. That is, models are fit with an increasing number of factors until one of the stopping rules (described earlier) is encountered. Models are compared using global (Comparative Fit Index [CFI], Tucker–Lewis Index [TLI], and root mean square error of approximation [RMSEA]) and comparative (Bayesian Information Criteria [BIC], Sample Size Adjusted BIC [SSBIC], and chi-square) fit indexes. From the fitted models, a model is selected as the best representation of the individual trajectories. From the selected model, different specifications can be examined by changing the location of fixed zero factor loadings and by fitting the selected model within the ESEM framework using different rotation criteria.

[1]We note that there are at least two alternative ways to specify these same exploratory growth models. One alternative specification involves fixing a factor loading at 1 for each factor and freeing its variance. This specification will have the same model fit as the specification described earlier because this is simply a respecification of identification constraints. And as in the previously described specification, growth factors are uncorrelated. A second alternative specification involves fixing $R - 1$ fixed factor loadings to 0 per growth factor and allowing the growth factors to correlate. Thus, a correlated solution is possible and, in many cases might be more reasonable. However, the trade-off comes in the form of additional fixed factor loadings—which might also constrain emergent interpretation of factor loading patterns. This specification leads to models that account for growth within specific phases of time, similar to spline or multiphase models, because each factor has a large number of 0 factor loadings. Although this is reasonable in some situations, specifying the model with uncorrelated factors allows for the estimation of more factor loadings and is closer to the Tuckerized curve approach.

ILLUSTRATIVE EXAMPLES

To examine and demonstrate additional insights into the underlying change processes that can emerge from the exploratory growth modeling approach, we present two illustrative examples that make use of longitudinal data characterized both by complex patterns of change that are nonlinear with respect to time and by substantial interindividual differences in change.

The *cortisol data*[2] ($T = 9$, $N = 34$) were collected as part of the MacArthur Successful Aging Studies during an investigation of interindividual differences in the time course of cortisol (measured in mmol/1) production and dissipation in response to a controlled intervention (Seeman et al., 1995; Seeman, Singer, & Charpentier, 1995). Previously, we have used a variety of latent growth and growth mixture models to describe and analyze these data (Ram & Grimm, 2007, 2009; Ram, Grimm, Gatzke-Kopp, & Molenaar, 2012; Shiyko, Ram, & Grimm, 2012). A plot of the observed trajectories appears in Figure 1a.

The *Berkeley height data* ($T = 14$, $N = 127$) were collected as part of the Berkeley Growth and Guidance Studies, with participants' height (cm) recorded at annual intervals from age 3 to age 17 (except for age 14 recording). These data represent a subset of the data presented and analyzed using nonlinear growth models in Grimm et al. (2011). A plot of the observed trajectories for both boys and girls is contained in Figure 1b.

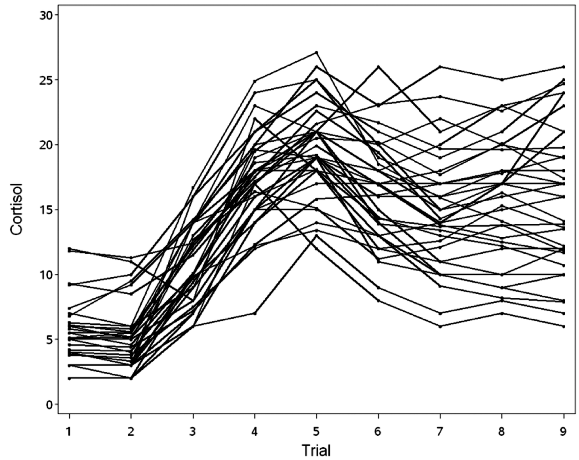
Exploratory Analysis Procedures

Each set of data was examined using both *Tuckerized curve* and *exploratory latent growth* models. Tuckerized curves were fit to the longitudinal data using SAS macros written by Wood (1992). Procedurally, we (a) obtained the principal components solution; (b) examined the eigenvalues, mean square ratios (MSRs), and pattern of loadings to select an appropriate number of components to retain; (c) evaluated different rotated solutions; (d) plotted and examined the rotated component pattern; and (e) calculated principal component scores and estimated trajectories for each individual.

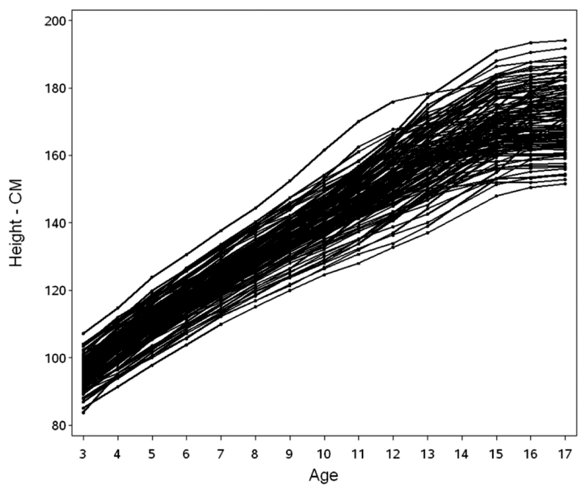
Exploratory latent growth models were fit to the longitudinal data using M*plus* (scripts available at first author's website). Procedurally, we (a) obtained solutions for exploratory growth models with 1, 2, 3, ... up to the maximum number of factors, (b) examined the fit criteria and pattern of loadings to select an appropriate number of factors to retain, (c) fit the selected model within the ESEM framework using different rotation criteria, (d) selected a rotation criteria, (e) plotted and examined the rotated factor loading matrix, and (f) estimated factor scores and examined trajectories for each individual.

We highlight that although conceptualized and presented as a set of step-by-step procedures, the analysis was, as it often is along the way to final solutions, an iterative process with some back and forth and rerunning and rethinking of models, solutions, and plots. Further, because fitting the Tuckerized curves requires complete data, data were limited to participants who provided data at all occasions. This allowed for across-model comparisons, even though fitting exploratory growth models in SEM and ESEM can (with full information maximum likelihood [FIML] estimation) accommodate the incomplete data that were set aside.

[2]We thank Drs. Marilyn Albert and Teresa Seeman for making the data available to us.



(a)



(b)

FIGURE 1 Observed individual trajectories of (a) cortisol over nine trials and (b) height from age 3 to 17.

RESULTS

The results are presented in two sections—first for the cortisol data and then for the Berkeley height data. Within each section we chronicle the fitting of Tuckerized curves and exploratory latent growth models, highlighting what was learned about the underlying change processes that we had not found in our previous applications of the typical, confirmatory latent growth curve procedures applied to these data. Although the details of the results might at times seem rather granular, this level of detail is purposely included to accurately communicate the realities of conducting exploratory work. We hope that the laborious nature of unconstrained explorations is both clear and, as we have found, worthwhile. Our conclusion is that models built on the exploratory underpinnings of the Tuckerized curve have much to offer, and can lead us in new directions—particularly toward more robust examinations of the timing of development processes.

Cortisol Trajectories

Tuckerized curves. The Tuckerized curve solution was first obtained and summary information is provided in Table 1. To determine the number of components to retain, we examined the relative magnitude of associated eigenvalues through a scree plot, evaluated the relative size of the MSRs, and calculated a percentage of variability captured by each component. MSRs are an approximate F-ratio for determining the number of components. Tucker (1966) noted that MSRs were upwardly biased compared to the F distribution with small samples and promoted a relative comparison among MSRs. Lastly, we summed eigenvalues two through nine and calculated the percentage of this total that components two through nine accounted for. The sum of the eigenvalues minus the first eigenvalue generally represents total variability because the first component tracks the mean trajectory; however, the first component does account for some variability as well. This percentage is referred to as the *percentage of variability*.

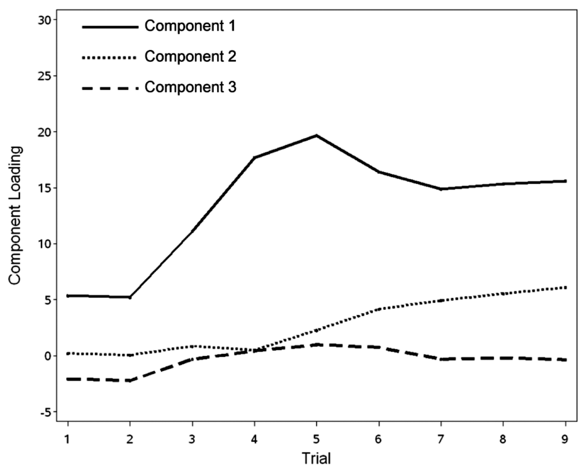
The scree plot for the cortisol data suggested that the fourth component was the beginning of the scree indicating that a three-component solution was appropriate. The MSRs experienced a noticeable jump from the third to the fourth components, after which they generally fluctuated around three. Thus, three components seemed appropriate based on the MSRs, even though subsequent MSRs were significant if compared with the F distribution. In terms of the percentage of variablity, three or five components seemed viable given the relative jumps after these components. Thus, based on all of this information, three components were retained.

TABLE 1
Tuckerized Curve Principal Component Solution Summary Information

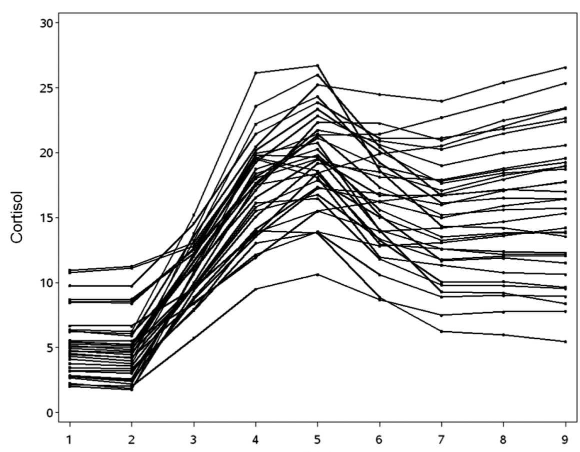
		Degrees of Freedom			
Components	*Eigenvalue*	*Component*	*Error*	*MSR*	*%*
1	65,504	42	264	215.8	—
2	1,138	40	224	8.3	59.7%
3	**383**	**38**	**186**	**4.9**	**20.0%**
4	144	36	150	2.5	7.6%
5	115	34	116	3.1	6.0%
6	54	32	84	1.9	2.8%
7	40	30	54	2.2	2.1%
8	26	28	26	3.4	1.4%
9	7	26	0	—	0.4%

Note. Values in bold indicate chosen model. MSR = mean square ratio.

After trying different rotations, the first two components were rotated to positive values. A plot of the rotated component loading patterns is shown in Figure 2a. The first component mirrors the general change pattern seen in the raw data—a sharp increase in response after the second measurement and a comparatively small decrease in cortisol response after the peak at the fifth measurement. This pattern could be considered the cortisol response function (analogous in concept to a hemodynamic response function) for this sample. The second



(a)



(b)

FIGURE 2 Plot of (a) two rotated component loadings patterns for cortisol based on Tuckerized curve, and (b) predicted individual trajectories for cortisol changes based on two-component Tuckerized curve.

component pattern has near-zero values for the first four measurement occasions followed by a steady increase in values across occasions four through nine. This pattern of loadings suggests that the process captured by the second component was not very active in the early part of individuals' cortisol response, the reactivity phase, but captured additional changes in cortisol during the recovery or dissipation phase. The change pattern displayed by the third component is similar to the first component; however, the changes are not as sharp and dramatic, indicating that this component might account for between-person differences in the shape of the overall cortisol trajectory.

Component scores were then calculated for the three components for all participants. Scores on Component 1 had a mean of 0.98 and ranged from 0.53 to 1.51, indicating this component of change contributed similarly to the observed trajectories for all participants (with respect to direction of change). Scores on Component 2 had a mean of 0.11 and ranged from −1.59 to 1.99 and scores on Component 3 had a mean of .01 and ranged from −2.30 to 1.81, indicating that participants varied in both the magnitude and direction of the second and third components. Further examination of the distribution of Component 2 scores indicated that these scores were distributed in a relatively uniform manner. Some participants had strong positive weights suggesting a lack of recovery (additional cortisol production), some participants had strong negative weights indicating substantial recovery (dissipation), and some participants had near zero values indicating production and dissipation of cortisol as guided by the generalized learning curve captured by Component 1. Component 3 scores were negatively skewed with most participants having near 0 or slightly positive values and fewer participants with negative and strongly negative values. Participants with positive values on this component had more curved trajectories, whereas participants with negative values had more peaked trajectories. Figure 2b is a plot of the individual-level predictions from the three-component Tuckerized curve solution. The individual predicted trajectories in Figure 2b follow the general pattern of the individual observed trajectories; however the predicted cortisol trajectories appear more linear than the observed trajectories, particularly between the third and fifth occasions—perhaps indicating that an important aspect of the process was not captured and more components are needed.

Exploratory latent growth models. Exploratory latent growth models with one, two, and three growth factors were fit to the longitudinal cortisol data with an equality constraint on the diagonal elements of $\boldsymbol{\Theta}$. Fit indexes for these models are reported in Table 2. Based on absolute fit criteria, none of the exploratory growth models represented the data well, potentially indicating the need for additional growth factors. However, a four-factor solution could not be fit because the number of estimated parameters would be greater than the number of participants. That is, we reached the limits imposed by the available degrees of freedom. Thus, we explored the three-factor model because it showed superior fit over the one- and two-factor models.

The three-factor model was fit within the ESEM framework and two rotations were examined. Both rotations utilized the Geomin criteria—one orthogonal and one oblique. The factor loading patterns were similar for the two rotations. Thus, the orthogonal rotation was retained for simplicity. The factor loading patterns from the three-factor model with orthogonal Geomin rotation are plotted in Figure 3a to display the general change patterns. The pattern displayed

TABLE 2
Fit Indexes for Exploratory Latent Growth Models

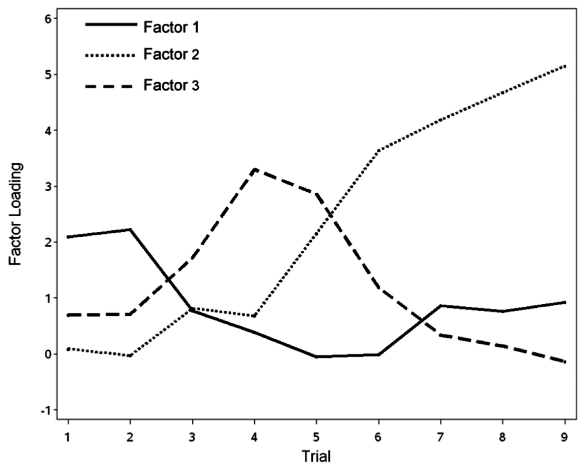
Factors	χ^2 *(parms)*	*CFI*	*TLI*	*RMSEA [CI]*	*AIC*	*BIC*	*ABIC*	*Unique Variance*
1	298(11)	.33	.44	.42 [.37, .46]	1,570	1,587	1,553	7.01
2	158(20)	.67	.65	.33 [.28, .38]	1,448	1,479	1,416	3.23
3	**91(28)**	**.83**	**.76**	**.27 [.21, .33]**	**1,397**	**1,440**	**1,353**	**1.89**

Note. Values in bold indicate chosen model. CFI = Comparative Fit Index; TLI Tucker–Lewis Index; RMSEA = root mean squared error of approximation; AIC = Akaike's Information Criterion; BIC = Bayesian Information Criterion; ABIC = adjusted Bayesian Information Criterion.

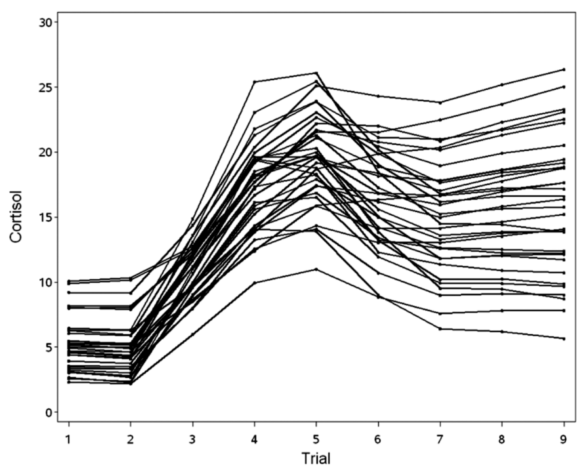
from the growth factor with the greatest mean (Factor 3) mirrored the typical cortisol change pattern and was similar to the pattern of the first component from the Tuckerized curve. The pattern of the second growth factor with the second greatest mean (Factor 2) showed near-zero values for the first four occasions followed by a sharp increase in values—very similar to the second component from the Tuckerized curve. The factor loading pattern for the factor with the smallest mean (Factor 1) was similar to the third component of the Tuckerized curve and was associated with curvature of the cortisol response and diffusion. Means of the three factors were 0.87, 3.05, and 4.53, respectively. This information, in conjunction with the factor variances (fixed at 1) indicate all participants had strong positive factor scores for the second and third factors and most participants had positive factor scores for the first factor. Estimated factor scores ranged from −0.71 to 3.02 for the first factor, 1.17 to 5.05 for the second factor, and 2.67 to 7.06 for the third factor. Estimated factor scores were combined with the factor loading pattern to plot the individual predictions from the three-factor exploratory growth model in Figure 3b. Comparing Figures 2b and 3b, predictions from the Tuckerized curve and the exploratory growth model were highly similar.

Summary. The results from fitting Tuckerized curves and exploratory growth models to the cortisol data suggest that there are at least three important sources of between-person differences in change. First, there was a more or less typical cortisol response curve—participants' cortisol levels increased in response to the intervention and subsequently dissipated and this generalized curve had a strong presence in each participant's observed trajectory. Second, there was substantial between-person variation in the dissipation process. Some participants showed greater dissipation and others showed a lack of dissipation. Third, there were between-person differences in shape of the cortisol response, which is an interesting new result.

Taking the more traditional (confirmatory) approach to modeling these data, Ram and Grimm (2007) settled on a three-part multiphase growth model with factors corresponding to baseline (Occasions 1–2), cortisol response (Occasions 2–5), and dissipation (Occasions 5–9) phases. Of note, the confirmatory growth model did not include a growth pattern reflecting the combination of cortisol response and dissipation as was found in the exploratory models (first component and third factor). Furthermore, the exploratory factor (and component) associated with dissipation was active (had nonzero values) beginning at Trial 5 as opposed to Trial 6 in the fitted confirmatory multiphase model. Thus, it appears that the dissipation process began earlier



(a)



(b)

FIGURE 3 Plot of (a) three-factor loading patterns for cortisol based on exploratory growth model, and (b) predicted individual trajectories for cortisol changes based on three factor exploratory growth model.

than previously modeled. Finally, the three-factor exploratory model generally showed superior fit (greater CFI, smaller unique variance) than the three-part multiphase model—highlighting the potential of these exploratory models. Moving from these exploratory models to traditional confirmatory growth models, we would want to test these sources of individual differences in future studies of cortisol response.

Height Trajectories

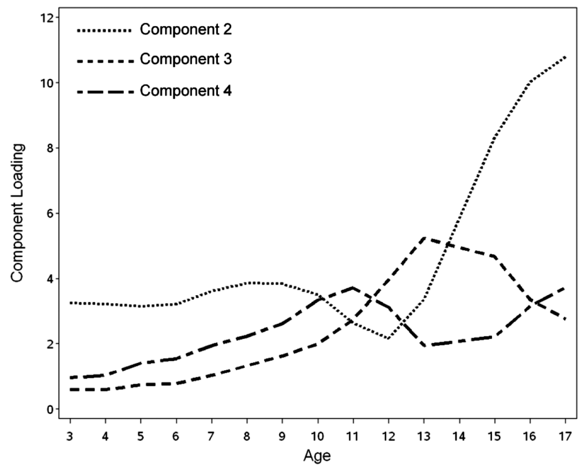
Tuckerized curves. Tuckerized curves were fit to the longitudinal height data and summary information from this model is contained in Table 3. To determine the number of components to retain we examined the scree plot, MSRs, and percentage of variability. The scree plot suggested that the fifth component was the beginning of the scree indicating that a four-component solution was appropriate. The MSRs experienced a sizable jump from the fourth to the fifth components, indicating that four components were appropriate. In terms of the percentage of variability, three or four components seemed viable. Thus, based on all of this information, four components were retained.

The four components were rotated to positive values and a plot of the rotated component loading patterns for the second, third, and fourth components (first component mirrored the general pattern in the data) is contained in Figure 4a. As with the cortisol data, the first component mirrored the general change pattern of the observed data—a more or less gradual increase in height between ages 3 and 15 followed by slower increases between ages 15 and 17. The second through fourth components captured increases during specific phases of development. Processes underlying the second component had a steady influence on height between ages 3 and 10, were less active between 11 and 13 years, and had a strong influence between ages 15 and 17. Thus, it appeared that this component could be characterized as most related to increases in height that occurred between ages 15 and 17—late adolescence. The third component captured changes in height between ages 9 and 13—early adolescence; and the fourth component captured gradual increases between ages 6 and 11 and then sharp increases between ages 12 and 16—possibly associated with a late pubertal growth spurt. The second through fourth components appeared to capture different growth spurts during childhood and adolescence. These components might represent between-person differences in the timing of the pubertal growth spurt whereas the first component captured general increases in height across the entire age range.

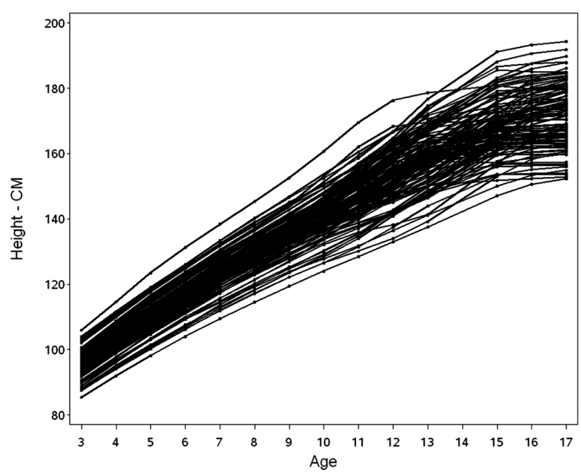
TABLE 3
Height Trajectories Tuckerized Curve Summary Information

		Degrees of Freedom			
Components	*Eigenvalue*	*Component*	*Error*	*MSR*	*%*
1	34,718,515	140	1,638	30,424.59	—
2	9,181	138	1,500	23.93	68.8%
3	2,473	136	1,364	14.61	18.5%
4	**736**	**134**	**1,230**	**7.04**	**5.5%**
5	311	132	1,098	3.98	2.3%
6	223	130	968	3.90	1.7%
7	121	128	840	2.59	0.9%
8	96	126	714	2.59	0.7%
9	59	124	590	1.85	0.4%
10	41	122	468	1.45	0.3%
11	38	120	348	1.56	0.3%
12	30	118	230	1.40	0.2%
13	24	116	114	1.33	0.2%
14	18	114	0	—	0.1%

Note. Values in bold indicate chosen model. MSR = mean square ratio.



(a)



(b)

FIGURE 4 Plot of (a) five rotated component loadings patterns for height based on Tuckerized curve, and (b) predicted individual trajectories for height changes based on five-component Tuckerized curve.

Component scores were calculated for each of the four components. Overall, component scores were more or less normally distributed with few outliers. Individual scores for the first component were all nonzero with a mean of 1.00 and a standard deviation of 0.05. Scores for the remaining components had near-zero means (.03, .00, and .01) and standard deviations near 1.0. Component intercorrelations were near zero, except for the first and second components, which were negatively correlated ($r = -.61$), and the first and fourth components, which were also negatively correlated ($r = -.18$). Figure 4b is a plot of the individual predictions from the four-component Tuckerized curve with rotation to positive values. As can be seen, the predictions appear to closely mirror the observed trajectories.

Exploratory latent growth models. Exploratory latent growth models with 1 through 14 factors were fit to the longitudinal height data (with an equality constraint on the diagonal elements of $\mathbf{\Theta}$). Based on model fit (see Table 4), interpretation of factor loading patterns, and model convergence, models with four or five growth factors appeared to provide an appropriate representation of the data. In examining the factor loading pattern for the five-factor solution, there were two factors with highly similar factor loading patterns and near-zero factor means. Thus, we selected the four-factor exploratory growth model. The four-factor model was then fit within the ESEM framework with orthogonal and oblique rotations. The orthogonal rotations appeared to yield more interpretable factor loading patterns, which are displayed in Figure 5a. Thus, this solution is discussed.

The first growth factor had the strongest mean ($\alpha_1 = 24.64$) and had a prominent role in observed scores over the entire age span and generally tracked the mean trajectory. The factor with the second strongest mean (Factor 3; $\alpha_3 = 7.74$) had positive values from age 3 through age 6, near-zero values from age 7 through 9, negative values from age 10 through 12, and then near-zero values after that. Thus, this factor appears to be linked to early positive changes in height and slowed height changes from ages 10 to 12. The factor with the third strongest mean (Factor 4; $\alpha_4 = 3.15$) had near-zero values until age 12 and after age 14. Thus, this factor appears to be inversely associated with a strong midadolescence growth spurt. The final factor (Factor 2; $\alpha_2 = 0.69$) had near-zero values from ages 3 through 13, after which

TABLE 4
Fit Indexes for Exploratory Latent Growth Models

Factors	χ^2 *(parms)*	*CFI*	*TLI*	*RMSEA [CI]*	*AIC*	*BIC*	*ABIC*	*Unique Variance*
1	3,232(16)	.409	.478	.489 [.475, .504]	9,316	9,361	9,311	8.087
2	1,857(30)	.666	.658	.396 [.380, .411]	7,970	8,055	7,960	2.736
3	956(43)	.834	.801	.302 [.285, .319]	7,094	7,217	7,081	1.215
4	**547(55)**	**.909**	**.870**	**.244 [.225, .263]**	**6,709**	**6,865**	**6,691**	**.756**
5	368(66)	.940	.898	.216 [.196, .237]	6,552	6,740	6,531	.569
6	203(76)	.970	.936	.171 [.148, .195]	6,407	6,624	6,383	.420
7	130(85)	.982	.951	.149 [.122, .177]	6,352	6,594	6,325	.344
8	60(93)	.994	.977	.102 [.068, .136]	6,298	6,563	6,269	.275
9	33(100)	.997	.987	.077 [.028, .119]	6,285	6,570	6,254	.238
10	24(106)	.998	.986	.081 [.023, .132]	6,288	6,590	6,254	.216
11	12(111)	.999	.992	.060 [.000, .129]	6,286	6,602	6,251	.187
12	5(115)	1.000	.994	.052 [.000, .152]	6,288	6,615	6,251	.163
13	0(118)	1.000	1.000	.000 [.000, .171]	6,288	6,624	6,251	.139
14*	0(119)	1.000	1.000	.000 [.000, .000]	6,290	6,629	6,252	.000

Note. Values in bold indicate chosen model. CFI = Comparative Fit Index; TLI Tucker–Lewis Index; RMSEA = root mean squared error of approximation; AIC = Akaike's Information Criterion; BIC = Bayesian Information Criterion; ABIC = adjusted Bayesian Information Criterion.

*Indicates unique variance fixed at 0.000 in this model.

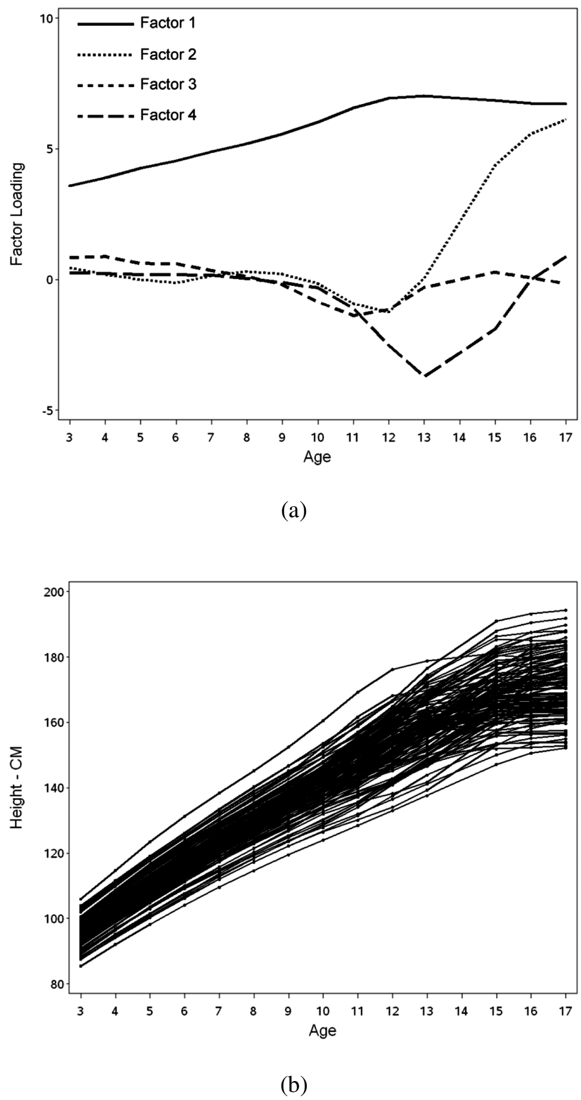


FIGURE 5 Plot of (a) four-factor loadings patterns for height based on exploratory growth model, and (b) predicted individual trajectories for height changes based on the four-factor exploratory growth model.

the factor loadings strongly increased through age 17. Thus, this factor captured additional growth during late adolescence for some participants and lack of growth for other participants.

Factor scores were estimated from the four-factor model. As expected, distributions of the estimated factor scores were more or less normal and scores had near zero correlations ($r < |.03|$). Estimated factor scores were combined with the factor loading pattern to plot the individual predictions from the four-factor exploratory growth model in Figure 5b. Comparing

Figures 4b and 5b, predictions from the Tuckerized curve and the exploratory growth model appear highly similar.

Summary. Changes in height during adolescence and into adulthood were characterized by complex patterns of change and appear to require at least four growth factors (or components) to capture that complexity. Both the Tuckerized curve and exploratory growth approaches yielded a dominant growth factor as well as a small set of growth factors and components that captured changes during specific periods of development. The location (with respect to time) of these minor factors varied depending on the approach, but nevertheless, phase-specific factors appear to capture important variability of physical growth.

In previous analyses, Grimm et al. (2011) fit the Preece–Baines model (Preece & Baines, 1978; PB1), a highly structured nonlinear model with five growth factors (two of which were highly correlated, $r = .986$) specifically designed to account for human growth. Parameters of the model map onto specific features of the developmental process, including the timing of puberty, adult height, height at peak growth velocity, and rates of change before and during puberty. The exploratory approach taken here yielded a model with fewer growth factors that fit the data similarly (residual variance for Preece–Baines model fit to the subset of data analyzed here was .752, whereas the exploratory model was .756). Additionally, the appearance of a factor associated with growth during early childhood appeared in both the exploratory growth and Tuckerized curve approaches. The factors might be associated with a prepubertal growth spurt that is now recognized (e.g., Gasser et al., 1985).

DISCUSSION

The purpose of this article was to examine how the exploratory frameworks that spawned latent growth curve analysis can be integrated into the contemporary implementation within the SEM framework. We reviewed how basic latent growth models are used to specify confirmatory tests of specific change functions and how Tucker's (1958, 1966) original framework was used to identify change functions in a more data-driven approach. Building from these frameworks, we proposed a general SEM-based approach to exploratory latent growth modeling and demonstrated how the model could be applied to two sets of longitudinal data. Additional insights into underlying changes were found to emerge from the exploratory approach. Our contention is that exploratory approaches to latent growth modeling might yield additional insights that would be missed when fitting standard confirmatory growth.

Exploratory Approaches Facilitating Greater Understanding

In recent years there has been a resurgence of exploratory methods for examining change, most prominently those making use of finite mixture models. Growth mixture, latent class, and latent profile models identify *subgroups* (persons) within the overall sample of observations. In complement, the exploratory growth modeling procedures reviewed and proposed here distinguish weighted *subtrajectories* (occasions) within the overall sample of observations. Given that both sets of techniques identify components, factors, or latent groupings that facilitate interpretation and understanding of the observed data and its relation to theory, the SEM-based exploratory growth modeling and the mixture and latent class procedures fall within the same zeitgeist.

Applying the SEM-based exploratory growth curve modeling procedures to the cortisol data, we identified a potentially important set of between-person differences in the transition from cortisol response to dissipation (around Occasions 4–6). Specifically, results indicated that we should revisit when the dissipation process began and develop specific hypotheses about what processes govern the transition between phases and why some individuals transition earlier than others. Next steps would include formulating a confirmatory model that explicitly models between-person differences in timing and examine how those differences are related to other between-person measures. For example, the inclusion of time-invariant predictors (e.g., gender, depression; see Seeman et al., 1995) for the resultant components of the exploratory models would be a potentially fruitful way to square these factors with extant theories regarding the time course of cortisol response among individuals. Here again, the emergent benefit of the exploratory approach is in the lack of a priori constraints regarding the shape of trajectories over time.

Similarly, in applying the exploratory growth curve modeling procedures to the height data, we identified systematic patterns indicative of an early-childhood growth spurt. Our results suggest expanding the Preece–Baines model to reexamine the need for an early childhood growth spurt in addition to the midadolescence growth spurt. More generally, both of our exploratory analyses highlighted some features of the change process that were overlooked with the traditional confirmatory approach. Although we do not know how these findings might generalize across populations and data sets, the results suggest further consideration of how transient processes might systematically contribute to within-person changes and how between-person differences in timing of spurts or transitions might be modeled explicitly (see Grimm et al., 2011; Marceau, Ram, Houts, Grimm, & Susman, 2011). In sum, we conclude that exploratory models derived from the origins of growth modeling still hold promise for identifying specific ways in which to expand or build theory—and should be (re) incorporated into our regular procedures.

Benefits and Limitations

Within the exploratory zeitgeist, the SEM-based exploratory growth model proposed here complements models based on principal components and profile analysis via multidimensional scaling (Davison, 2008; Ding et al., 2005). Benefits of these other models include that (a) the component and scaling scores can be directly computed in an assumption-free manner (i.e., no need for normality assumption of between-person differences), (b) the changes need not follow any prespecified change pattern (see Davison, 2008), (c) substantial flexibility provided by setups with very few constraints, and (d) the inherent ordering of the relative contribution of each component or generalized learning curve to the observed change patterns provided by the natural ordering of principal components (by eigenvalue).

Benefits of the SEM-based exploratory growth model demonstrated here include (a) implementation within a widely used framework that links directly to typical confirmatory growth modeling procedures, (b) straightforward and commonly implemented evaluation of model fit (e.g., RMSEA, CFI), (c) handling of incomplete data using FIML, (d) incorporation of multivariate measurement models, and (e) modeling of multiple observed and latent groups.

The major limitation of the exploratory growth approach is the need to constrain a subset of factor loadings to equal 0 (or other value) for model identification. Moving to the ESEM framework allows for rotation, which enables the estimation of all factor loadings at each measurement occasion and for correlated factors. However, a question remains regarding the most appropriate types of rotation for these models. Our work suggested orthogonal rotations provided more interpretable solutions.

Extensions

Overall, we were pleasantly surprised by the findings that emerged from our examination of how Tuckerized curve and exploratory growth modeling approaches might be applied to longitudinal data with which we were familiar. We learned something new about each data set and can search out and develop new theoretical perspectives to subsequent modeling efforts. As well, we look forward to expanding the reach of the exploratory approach to obtain new insights from other data. In addition to the substantive explorations, there are a number of areas where the method itself might be extended and examined further.

Combined exploratory and confirmatory model specifications. Typically growth curve models include an intercept factor (indicated by all occasions with factor loadings equal to 1). As well, Davison (2008) and colleagues (Ding et al., 2005) included an intercept term in their exploratory principal components and profile analysis via multidimensional scaling models. Intercept terms were included because they map directly onto hypothesized between-person differences in overall levels or initial levels of ability or response. An intercept term can be easily included within the exploratory growth framework. For example, a one-factor exploratory growth model with an intercept factor is equivalent to the latent basis growth model (Meredith & Tisak, 1990). Fitting exploratory growth models with a latent intercept factor to the longitudinal cortisol and height data resulted in similar outcomes as those presented here. In particular, we found that a model with two exploratory growth factors and a latent intercept factor provided a reasonable representation of the cortisol data. The fit of this model, $\chi^2 = 99$ with 22 estimated parameters, was superior to the two-factor model without the intercept term and captured the same change patterns—one factor described the general trend in the data (i.e., rise and subsequent fall) and the second factor described additional changes during the dissipation phase. In sum, additional growth factors with specific properties (i.e., substantive interpretations) can be included into the exploratory growth curve model in a straightforward manner—and thus simultaneously provide for confirmation and extension of existing descriptions of change processes.

Rotation. Tucker (1958, 1966) and later Arbuckle and Friendly (1977) discussed rotation of components in Tuckerized curves. Constraints on specific factor loadings to specific values (and maintenance of orthogonal factors) might facilitate interpretation of growth factors by effectively locking down the location of one or more dimensions of change—a Procrustes-like rotation. Use of nonequality or other pattern-oriented constraints are now easily invoked in most SEM programs. For example, factor loadings could be constrained to have only positive values or to change monotonically across occasions. We see that it would be very useful to systematically outline the sets of constraints that are most appropriate for interpreting exploratory solutions in relation to a specific family of change functions (e.g., sigmoids or exponentials). This would facilitate interpretation within one or more broad theoretical frameworks of change, still allowing for a wide variety of growth factors to emerge. Furthermore, the rotation criteria with the ESEM framework should be evaluated to determine which rotations are appropriate for modeling longitudinal data in this manner.

Subprocesses and subgroups. As noted earlier, there are some parallels between growth mixture models and the exploratory growth models described here—the former primarily being used to identify subgroups of persons, and the latter to identify subprocesses that manifest across occasions. Within a generalized SEM framework, these two lines can be integrated into explicitly exploratory growth mixture models. Such models would be particularly useful in developmental contexts where it is expected that there are groups of individuals with different patterns of change that might result from different subprocesses. Typically, growth models invoke an approach where every individual follows a pattern of change similar in shape to the average trend. In the exploratory growth model presented in this article, each growth factor contributes to each individual's observed change pattern with different weights (factor or component scores). Additionally allowing for the presence of latent classes (through a mixture model), the exploratory models can be used to identify subgroups of individuals that have categorically different sets of growth factors. In principle, there might be several components or factors that are simply not relevant for particular subgroups (i.e., they would have zero or very near-zero scores on those factors). Adding an additional layer of categorical between-person differences in the manifestation of multiple subprocesses would allow for a substantially broad set of change patterns that might map more closely onto developmentally oriented theoretical frameworks (e.g., Baltes, Lindenberger, & Staudinger, 2006; Ford & Lerner, 1992).

Concluding Remarks

The scientific enterprise cycles through formulation of theory-driven hypotheses, empirical tests with observed data, theory rejection, and subsequent revision of theory (Cattell's [1966] inductive-hypothetico-deductive spiral). The precursors of contemporary growth curve modeling used a primarily exploratory approach to identify patterns of change that might inform the development of theory-driven hypotheses about the processes that were driving the observed changes (Tucker, 1958, 1966). The advent and adoption of confirmatory SEM has generated a plethora of empirical results. Our contention is that we should again seriously consider how exploratory models for change can be used for hypothesis generation (Davison, 2008) as opposed to fitting a series of confirmatory models (e.g., linear, quadratic, latent basis) and using those models to reject a very select set of (usually very simple) hypotheses about change.

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