New Developments in Mplus Version 7: Part 1

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Mplus
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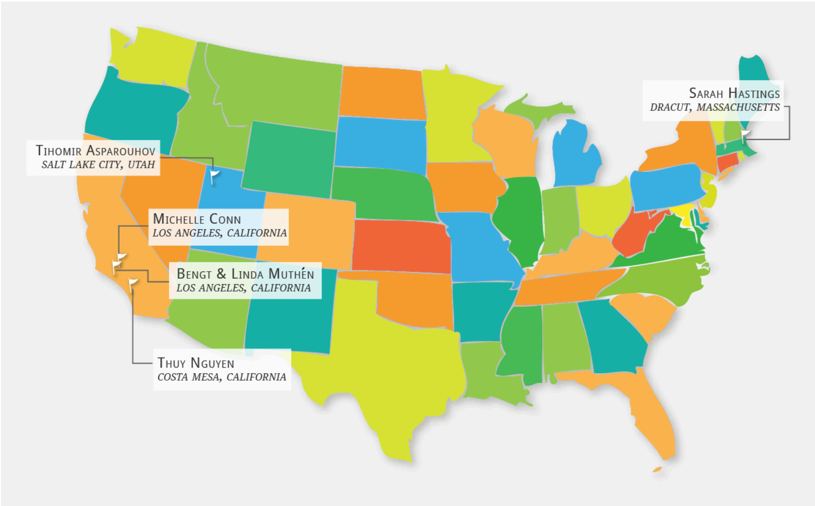
The Map of the Mplus Team



Mplus Background

- Inefficient dissemination of statistical methods:
  - Many good methods contributions from biostatistics, psychometrics, etc are underutilized in practice
- Fragmented presentation of methods:
  - Technical descriptions in many different journals
  - Many different pieces of limited software
- Mplus: Integration of methods in one framework
  - Easy to use: Simple, non-technical language, graphics
  - Powerful: General modeling capabilities

Mplus

Several programs in one

- Path analysis
- Exploratory factor analysis
- Structural equation modeling
- Item response theory analysis
- Growth modeling
- Mixture modeling (latent class analysis)
- Longitudinal mixture modeling (Markov, LTA, LCGA, GMM)
- Survival analysis (continuous- and discrete-time)
- Multilevel analysis
- Complex survey data analysis
- Bayesian analysis
- Monte Carlo simulation

Fully integrated in a general latent variable framework

What's New in Mplus Version 7?

5 big new features:

1. Surprise
2. Factor analysis
   - Bi-factor EFA rotations, bi-factor ESEM, two-tier modeling
   - Bayesian EFA and CFA (BSEM), bi-factor BSEM
3. Analysis of several groups with approx. measurement invariance
   - using a Bayes approach (multiple-group BSEM)
   - using a two-level analysis with random intercepts and loadings
4. Analysis of individual differences SEM using measurement parameters that vary across subjects
5. Mixture analysis
   - Using a proper 3-step analyze-classify-analyze approach to investigate covariates and distal outcomes
   - Latent transition analysis with new output, covariates influencing transition probabilities, and probability parameterization
   - Exploratory LCA using Bayesian analysis

What's New in Mplus Version 7, Continued

5 more big features:

1. 3-level SEM analysis, complex survey data handling, and multiple imputation
2. Cross-classified SEM analysis including random subjects and contexts (2 random modes)
3. IRT analysis with random items
4. Longitudinal analysis with approx. measurement invariance
   - using a Bayes approach (multiple-time point BSEM)
   - using cross-classified analysis of time and subjects
5. Analysis of changing membership over time

What's New in Mplus Version 7, Continued

and 5 other new features:

1. Parallel analysis
2. LOOP plots (moderated mediation, cross-level interactions, etc)
3. Bayes plausible value factor score distribution plots for each subject
4. Two-tier algorithm
5. New convenience options: LOOP, DO, COV, DIFF, DO DIFF, MODEL=ALLFREE, auto-labeling, BY with random loadings, BITER = (minimum), TECH15, TECH16

What's New in Mplus Version 7, Continued

- and if you don't see what you had on your wish list, stay tuned for

Version 7.1

Version 7.2

...

Version 8

Schedule

- Part 1:
  - Hardware and Timings
  - New Options
  - Recap of Bayesian Analysis in Mplus
  - Advances in Factor Analysis
  - Advances in Multiple-Group Analysis: Invariance Across Groups
- Part 2:
  - Two-Level Random Loadings in IRT
  - Advances in Individual Differences Modeling: Invariance Across Subjects
  - Advances in Mixture Modeling
  - 3-Level Analysis, Complex Survey Data, and Multiple Imputation
  - Introductory Cross-Classified Analysis
- Part 3:
  - Advanced Cross-Classified Analysis: Two Random Modes
  - Random Items in IRT
  - Advances in Longitudinal Analysis: Invariance Across Time
  - Advances in Longitudinal Analysis: Growth Modeling with Many Time Points

Hardware

Don't settle for using an outdated computer. Use 64-bit instead of 32-bit, use several processors instead of only 1 or 2, use fast CPUs.

- PC:
  - IntelR CoreT i7-3770K 3.5GHz/3.9GHz Turbo 8MB L3 Cache HD 4000 (Intel's i7 processor released in April is also available for laptops)
  - over-clocked to 4.2 to 4.5GHz
  - 8 procs
  - 32GB RAM
  - 64-bit
- Mac Pro:
  - Intel Xeon 3.33GHz, 6-core
  - 12 procs
  - 24GB RAM
  - 64-bit

Bengt's PC as of June 2012: $1,500 Dell XPS 8500, i7-3770 with 8 processors, CPU of 3.40 GHz, 12 GB RAM, 64-bit.

Software and Hardware

New option: PROCESSORS = a b; ! a = # processors, (b = # threads)
Can be used with: STARTS = c d; ! New default: c = 20, d = 4.

Multiple processors:

- Programming parallelized code, executable distributes the computing over different processors
- Parallelized code implemented for numerical integration (sample split into parts), missing data patterns, but not for Bayes
- Speed increases as a increases in PROCESSORS = a;

Multiple threads:

- Used with STARTS = , typically for TYPE = MIXTURE
- Different starting value sets are analyzed in different threads using different processors, possibly using more than one processor per thread
- Using as many threads as processors is fastest, but choosing fewer threads than processors is less memory demanding

Software and Hardware, Continued

PROCESSORS = a b; ! a = # processors, (b = # threads)
Replaces PROCESSORS = a (STARTS);

- Analyses with STARTS= (typically mixtures): Using many threads saves time. Mplus makes best decision when saying PROCESSORS = a; although memory demand can be reduced by choosing b < a, for instance PROCESSORS = 8 2;
- Non-mixture analyses with numerical integration: Using many processors saves time, while number of threads has no effect
- Bayesian analysis: Time not dependent on number of threads or number of processors (PROCESSORS = 2 suitable for the default of 2 chains)

Processors/Threads Available When?

- Multiple processors and multiple threads with random starts and multiple processors without random starts available when
  - TYPE=MIXTURE
  - Bayesian analysis with more than one chain if STVALUES=ML
  - Models that require numerical integration
- Multiple processors and multiple threads with random starts (w/o random starts, one processor is used) available when
  - TYPE=RANDOM
  - TYPE=TWOLEVEL and TYPE=THREELEVEL, continuous outcomes, ESTIMATOR= ML, MLR, and MLF without numerical integration
- Multiple processors but not multiple threads available when
  - Models with all continuous variables, missing data, and maximum likelihood estimation
  - Bayesian analysis with more than one chain
  - TYPE=TWOLEVEL, categorical outcomes, and ESTIMATOR= WLSMV

Timing Examples: 3 Analyses

Example 1	Factor mixture model with 2 dimensions of integration, 2 classes, 18 binary outcomes and n=3314
Example 2	Factor mixture model with 2 dimensions of integration, 8 classes, 22 outcomes, and n=842
Example 3	Growth mixture model with 8 time points, count outcomes, 3 classes, starts = 32 8, 3 dimensions of integration, and n=1314

Timing Examples: 3 Computers

Computer	Processor	RAM	Operating System
Sony Vaio (oldish laptop)	Intel Core 2 Duo CPU T8100 2.10 GHz	2046 MB	Windows Vista 32-bit
Dell Precision 490 (2 dual-core desktop)	Intel Xeon 5160 2x4MB Cache 1333-MHz data rate 3.0 GHz	4GB	Windows XP 32-bit
Dell XPS 8500 (new desktop June 2012)	i7-3770 8 processors 3.4GHz	12 GB	Windows 7 64-bit

Timing Examples

Minutes: Seconds

	Example 1	Example 2	Example 3
Computer 1	4:40	14:04	38:04
Computer 2	0:57	8:29	13:09
Computer 3	0:38	2:30	5:20

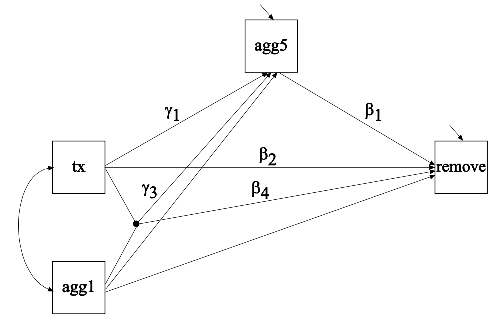
LOOP Option

LOOP is used in MODEL CONSTRAINT in conjunction with the PLOT option to create plots of one variable related to another, including a 95% confidence interval. An example:

MODEL: y ON x (p1);
MODEL CONSTRAINT:
PLOT(ypred);
LOOP(age, 20, 50, 1); ! 20 $\leq$ age $\leq$ 50 with steps of 1
ypred = p1*age;

- Plotting indirect effects with moderated mediation. Preacher, Rucker, Hayes (2007), MBR: Figure 3 - conditional indirect effect as a function of the moderator
- Plotting cross-level interactions in two-level modeling. Bauer & Curran (2005)
- Plotting sensitivity graphs for causal effect mediation modeling. Imai et al. (2010), Psych Methods; Muthén (2011)

LOOP Example: Moderated Mediation of School Removal



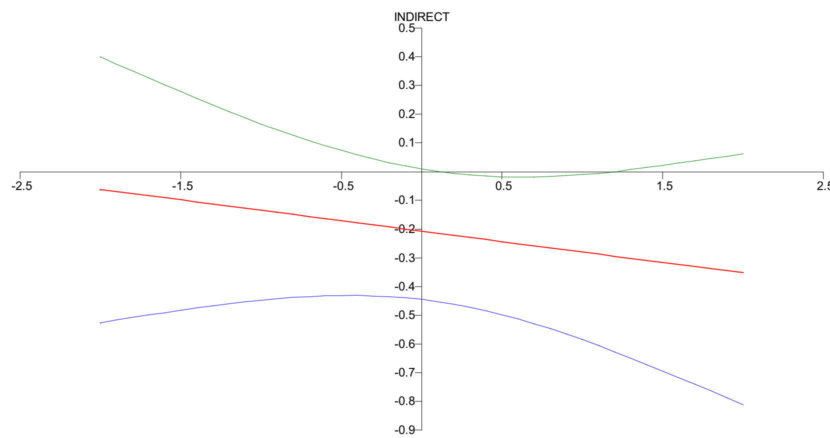
$$remove = \beta_0 + \beta_1\, agg5 + \beta_2\, tx + \beta_3\, agg1 + \beta_4\, tx\, agg1 + \varepsilon_1, \quad (1)$$
$$agg5 = \gamma_0 + \gamma_1\, tx + \gamma_2\, agg1 + \gamma_3\, tx\, agg1 + \varepsilon_2, \quad (2)$$
$$= \gamma_0 + (\gamma_1 + \gamma_3\, agg1)\, tx + \gamma_2\, agg1 + \varepsilon_2. \quad (3)$$

Indirect effect of tx on remove is $\beta_1\,(\gamma_1 + \gamma_3\, agg1)$, where agg1 moderates the effect of the treatment. Direct effect: $\beta_2 + \beta_4\, agg1$.

LOOP Example Continued

```
                inter = tx*agg1;
ANALYSIS:       ESTIMATOR = BAYES;
                PROCESSORS = 2; FBITER = 50000;
MODEL:          remove ON agg5 (beta1)
                tx (beta2)
                agg1 (beta3)
                inter (beta4);
                agg5 ON tx (gamma1)
                agg1 (gamma2)
                inter (gamma3);
MODEL CONSTRAINT:
                PLOT(indirect direct);
                ! let moderate represent the range of the agg1 moderator
                LOOP(moderate, -2, 2, 0.001);
                indirect = beta1*(gamma1+gamma3*moderate);
                direct = beta2+beta4*moderate;
PLOT:           TYPE = PLOT2;
```


Indirect Effect of Treatment as a Function of SD Units of the Moderator agg1



DO Option

Example: Two groups, 9 factor loadings in each group, expressing the 9 group differences (note that the longer symbol – denotes a ”dash”, namely a list, and the shorter symbol - denotes minus)

```
DO(1,9) diff# = lambda1# – lambda2#;
! Same as
! diff1 = lambda11 - lambda21;
! diff2 = lambda12 - lambda22:
! ...
! diff9 = lambda19 - lambda29;
```

- Useful in MODEL CONSTRAINT to create NEW parameters
- Useful in MODEL PRIORS for Bayesian analysis
- Can also be used in DEFINE

BY with Random Loadings

```
ANALYSIS:
TYPE = TWOLEVEL RANDOM;
ESTIMATOR = BAYES;

MODEL:
% WITHIN %
s1-s10 | f BY y1-y10;
% BETWEEN %
[s1-s10];
s1-s10;
```

Easier than a series of statements like:

```
s | y ON f;
```


Bayesian Analysis: A Review of the Mplus Implementation

Mplus conceptualization:

- Mplus was envisioned 15 years ago as both a frequentist and a Bayesian program
- Bayesian analysis firmly established and its use growing in mainstream statistics
- Much less use of Bayes outside statistics
- Bayesian analysis not sufficiently accessible in other programs
- Bayes provides a broader platform for further Mplus development

Bayesian Analysis

Why do we have to learn about Bayes?

- More can be learned about parameter estimates and model fit
- Better small-sample performance, large-sample theory not needed
- Priors can better reflect substantive hypotheses
- Analyses can be made less computationally demanding
  - Frequentists can see Bayes with non-informative priors as a computing algorithm to get answers that would be the same as ML if ML could have been done
- New types of models can be analyzed

Writings On The Bayes Implementation In Mplus

- Asparouhov & Muthén (2010). Bayesian analysis using Mplus: Technical implementation. Technical Report. Version 3.
- Asparouhov & Muthén (2010). Bayesian analysis of latent variable models using Mplus. Technical Report. Version 4.
- Asparouhov & Muthén (2010). Multiple imputation with Mplus. Technical Report. Version 2.
- Asparouhov & Muthén (2010). Plausible values for latent variable using Mplus. Technical Report.
- Muthén (2010). Bayesian analysis in Mplus: A brief introduction. Technical Report. Version 3.
- Muthén & Asparouhov (2010). Bayesian SEM: A more flexible representation of substantive theory.
- Asparouhov & Muthén (2011). Using Bayesian priors for more flexible latent class analysis.

Posted under Papers, Bayesian Analysis and Latent Class Analysis

Overview of Bayesian Features In Mplus

- Single-level, multilevel, and mixture models
- Continuous and categorical outcomes (probit link)
- Default non-informative priors or user-specified informative priors (MODEL PRIORS)
- Multiple chains using parallel processing (CHAIN)
- Convergence assessment using Gelman-Rubin potential scale reduction factors (PSR $\approx$ 1)
- Posterior parameter distributions with means, medians, modes, and credibility intervals (POINT)
- Posterior parameter trace plots
- Autocorrelation plots
- Posterior predictive checking plots

Multiple Imputation (DATA IMPUTATION)

- Carried out using Bayesian estimation to create several data sets where missing values have been imputed
- The multiple imputation data sets can be used for subsequent model estimation using ML or WLSMV
- The imputed data sets can be saved for subsequent analysis or analysis can be carried out in the same run
- Imputation can be done based on an unrestricted H1 model using three different algorithms including sequential regressions
- Imputation can also be done based on an H0 model specified in the MODEL command

Multiple Imputation (DATA IMPUTATION)

- The set of variables used in the imputation of the data do not need to be the same as the set of variables used in the analysis
- Single-level and multilevel data imputation are available
- Multiple imputation data can be read using TYPE=IMPUTATION in the DATA command

Plausible Values (PLAUSIBLE)

- Plausible values are multiple imputations for missing values corresponding to a latent variable
- Plausible values used in IRT contexts such as the ETS NAEP, The Nations Report Card (Mislevy et al., 1992)
- Available for both continuous and categorical latent variables (factors, random effects, latent classes)
- More informative than only an estimated factor score and its standard error or a class probability
- Plausible values are more accurate than factor scores
- Plausible values are given for each observation together with a summary over the imputed data sets for each observation and latent variable
- Multiple imputation and plausible values examples are given in the Users Guide, Chapter 11

Bayesian Analysis Using Mplus: An Ongoing Project

Features that are not yet implemented include:

- ESEM
- Logit link
- Censored, count, and nominal variables
- XWITH
- Weights
- c ON x in mixtures
- Mixture models with more than one categorical latent variable
- Two-level mixtures
- MODEL INDIRECT
- MODEL CONSTRAINT except for NEW parameters
- MODEL TEST

Overview of Bayes News in Version 7

- Multiple-group and multiple time point analysis with approximate measurement invariance
- 2-level analysis with random loadings
- 3-level analysis with continuous (ML and Bayes) and categorical outcomes (Bayes only)
- Cross-classified analysis (Bayes only)
- EFA
- Factor scores (plausible values)
- Kolmogorov-Smirnov convergence checking

News in Version 7: Bayesian EFA

- Bayesian estimation of exploratory factor analysis implemented in Mplus version 7 for models with continuous and categorical variables
- Asparouhov and Muthén (2012). Comparison of computational methods for high dimensional item factor analysis
- Asymptotically the Bayes EFA is the same as the ML solution
- Bayes EFA for categorical variable is a full information estimation method without using numerical integration and therefore feasible with any number of factors
- New in Mplus Version 7: Improved performance of ML-EFA for categorical variables, in particular high-dimensional EFA models with Montecarlo integration; improved unrotated starting values and standard errors

Bayes EFA

- The first step in the Bayesian estimation is the estimation of the unrotated model as a CFA model using the MCMC method
- Obtain posterior distribution for the unrotated solution
- To obtain the posterior distribution of the rotated parameters we simply rotate the generated unrotated parameters in every MCMC iteration, using oblique or orthogonal rotation
- No priors. Priors could be specified currently only for the unrotated solution
- If the unrotated estimation takes many iterations to converge, use THIN to reduce the number of rotations

Bayes EFA

- This MCMC estimation is complicated by identification issues that are similar to label switching in the Bayesian estimation of Mixture models
- There are two types of identification issues in the Bayes EFA estimation
- The first type is identification issues related to the unrotated parameters: loading sign switching
- Solution: constrain the sum of the loadings for each factor to be positive. Implemented in Mplus Version 7 for unrotated EFA and CFA. New in Mplus Version 7, leads to improved convergence in Bayesian SEM estimation

$$\sum_{i=1}^{p} \lambda_{ij} > 0$$

Bayes EFA

- The second type is identification issues related to the rotated parameters: loading sign switching and order of factor switching
- Solution: Align the signs s_j and factor order σ to minimize MSE between the current estimates λ and the average estimate from the previous MCMC iterations L

$$\sum_{i,j} (s_j \lambda_{i\sigma(j)} - L_{ij})^2$$

- Minimize over all sign allocations s_j and factor permutations σ

Bayes EFA

- Factor scores for the rotated solutions also available. Confidence intervals and posterior distribution plots
- Using the optimal rotation in each MCMC iteration we rotate the unrotated factors to obtain the posterior distribution of the rotated factors
- With continuous variables Bayes factor is computed to compare EFA with different number of factors. PPP value is computed with continuous or categorical variables

Bayes Factors

- Bayes factors is an easy and quick way to compare models using BIC

$$BF = \frac{P(H1)}{P(H0)} = \frac{Exp(-0.5BIC_{H1})}{Exp(-0.5BIC_{H0})}$$

- Values of BF greater than 3 are considered evidence in support of $H1$
- New in Mplus Version 7: BIC is now included for all models with continuous items (single level and no mixtures)
- The above method can be used to easily compare nested and non-nested models

Bayes EFA: Simulation Study ($n = 500$)

Absolute bias, coverage and log-likelihood for EFA model with 7 factors and 35 ordered polytomous variables.

Method	λ_{11}	λ_{12}	Log-Likelihood
Mplus Monte 500	.01(0.97)	.00(0.83)	-28580.3
Mplus Monte 5000	.01(0.96)	.00(0.87)	-28578.4
Mplus Bayes	.01(.90)	.00(.96)	-
Mplus WLSMV	.00(.94)	.00(.89)	-
IRTPRO MHRM	.00(.54)	.00(.65)	-28665.2

Bayes EFA: Simulation Study ($n = 500$), Continued

Average standard error, ratio between average standard error and standard deviation for the EFA model with with 7 factors and ordered polytomous variables.

Method	λ_{11}	λ_{12}
Mplus Monte 500	0.033(1.00)	0.031(0.72)
Mplus Monte 5000	0.033(0.99)	0.035(0.81)
Mplus Bayes	0.030(0.97)	0.032(0.98)
Mplus WLSMV	0.030(0.97)	0.038(0.85)
IRTPRO MHRM	0.012(0.42)	0.026(0.65)

Bayes EFA is the most accurate full information estimation method for high-dimensional EFA with categorical variables.

Bayes EFA: Example

Example is based on Mplus User's Guide example 4.1 generated with 4 factors and 12 indicators.

```
DATA: FILE IS ex4.1.dat;
VARIABLE: NAMES ARE y1-y12;
ANALYSIS: TYPE = EFA 1 5; estimator=bayes;
```

We estimate EFA with 1, 2, 3, 4 or 5 factors.

Bayes EFA: Results

Bayes factor results: The posterior probability that the number of factors is 4 is: 99.59%. However, this is a power result - there is enough information in the data to support 4 factors and not enough to support 5 factors. Use BITER = (10000)

```
POSTERIOR PROBABILITIES FOR ALL MODELS:
1-FACTOR MODEL   0.0000
2-FACTOR MODEL   0.0000
3-FACTOR MODEL   0.0041
4-FACTOR MODEL   0.9959
5-FACTOR MODEL   0.0000
```


Bayes EFA: Results

```
EXPLORATORY FACTOR ANALYSIS WITH 4 FACTOR(S):


Number of Free Parameters                              66

Bayesian Posterior Predictive Checking using Chi-Square

          95% Confidence Interval for the Difference Between
          the Observed and the Replicated Chi-Square Values

                        -35.978            41.864

          Posterior Predictive P-Value       0.376

Information Criterion

          Deviance (DIC)                               3231.767
          Estimated number of parameters (pD)            67.580
          Bayesian (BIC)                               3423.991
          Bayes factor for the model with 4 factor(s) against
          the model with 3 factor(s):
                                                       243.6367
          Log of Bayes factor for the model with 4 factor(s)
          against the model with 3 factor(s):
                                                         5.4957
```


Bayes EFA: Results

GEOMIN ROTATED LOADINGS (* significant at 5% level)

	1	2	3	4
Y1	0.622*	-0.008	0.028	-0.026
Y2	0.762*	-0.025	0.035	0.010
Y3	0.734*	0.055	-0.038	0.021
Y4	-0.112	0.592*	-0.008	-0.150
Y5	0.021	0.844*	0.049	0.007
Y6	0.053	0.639*	-0.029	-0.007
Y7	-0.009	0.037	0.788*	0.004
Y8	0.032	-0.059	0.646*	-0.115
Y9	0.008	0.040	0.602*	0.095
Y10	-0.042	0.065	0.005	0.671*
Y11	0.013	-0.078	-0.027	0.714*
Y12	0.051	-0.054	0.009	0.710*

News in Version 7: Bayes Factor Scores Handling

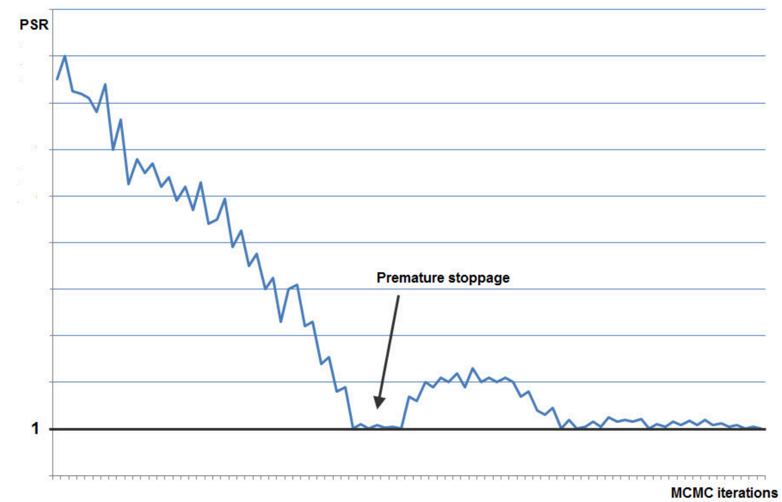
- New improved language for factor scores with Bayesian estimation. The same language as for other estimators
- SAVEDATA: FILE=fs.dat; SAVE=FS(300); FACTORS=factor names; This command specifies that 300 imputations will be used to estimate the factor scores and that plausible value distributions are available for plotting
- Posterior mean, median, confidence intervals, standard error, all imputed values, distribution plot for each factor score for each latent variable for any model estimated with the Bayes estimator
- Bayes factor score advantages: more accurate than ML factor scores in small sample size, Bayes factor score more accurate in secondary analysis such as for example computing correlations between factor

Bayes Factor Scores Example

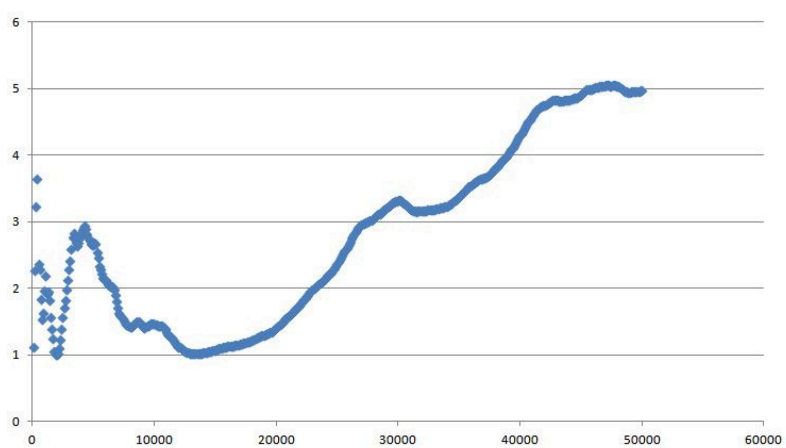
- Asparouhov & Muthén (2010). Plausible values for latent variables using Mplus
- Factor analysis with 3 indicators and 1 factor. Simulated data with N=45. True factor values are known. Bayes factor score estimates are more accurate. Bayes factor score SE are more accurate
- ML factor scores are particularly unreliable when Var(Y) is near 0

	ML	Bayes
MSE	0.636	0.563
Coverage	20%	89%
Average SE	0.109	0.484

PSR Convergence Issues: Premature Stoppage



PSR Convergence Issues: Premature Stoppages Due to Non-Identification



News in Version 7: Kolmogorov-Smirnov Convergence Test

- The Mplus default convergence criterion is the Potential Scale Reduction (PSR) criterion. The PSR is not sufficiently strict in certain cases, particularly when the model is not identified and an insufficient number of Bayes draws (iterations) has been used.
- Muthén & Asparouhov (2011). Bayesian SEM: A more flexible representation of substantive theory. With commentaries and rejoinder. Forthcoming in Psychological Methods
- A new more strict test of convergence is now implemented and reported as part of the Tech8 output
- The test is based on computing the Kolmogorov-Smirnov (K-S) test for equal sample distribution
- For each parameter the test uses 100 draws from each of the two MCMC chains and compares the two distributions

Kolmogorov-Smirnov Convergence Test

- If convergence has been achieved the two distributions should be similar and the K-S test would not reject the hypothesis that the distributions are equal
- The test is based on the K-S statistic

$$D = sup|F_1(x) - F_2(x)|$$

  where $F_1(x)$ and $F_2(x)$ are the sample distribution functions for chain 1 and 2
- Mplus computes D for each parameter and a p-value for the hypothesis that the two chains have the same distribution
- Large values of D and p-value< 0.05 indicate that convergence has not been achieved.
- Sometimes smaller values should be accepted because the mixing may be too slow to satisfy a strict criterion such as K-S. For more complex models p-values> 0.001 can be interpreted as confirmed convergence.

Kolmogorov-Smirnov Convergence Test

Consider the following unidentified factor analysis model which has a free factor mean.

```
VARIABLE:   NAMES = y1-y4;
DATA:       FILE = kolm.dat;
ANALYSIS:   ESTIMATOR = BAYES;
MODEL:
            f BY y1-y4*1;
            f@1;
            y1-y4*.5;
            [f*0];
```


Kolmogorov-Smirnov Convergence Test

After 1800 iterations the PSR criterion is satisfied. The K-S test however clearly rejects the convergence and also points out that the unidentified parameters are the indicator means as well as the factor mean.

```
Parameter   KS Statistic P-value
Parameter 4    0.3900    0.0000

THE KOLMOGOROV-SMIRNOV DISTRIBUTION TEST FOR PARAMETER 4 HAS A P-VALUE 0.0000,
INDICATING DISCREPANT POSTERIOR DISTRIBUTIONS IN THE DIFFERENT MCMC CHAINS.
THIS MAY INDICATE NON-CONVERGENCE DUE TO AN INSUFFICIENT NUMBER OF MCMC ITERATIONS OR IT MAY
INDICATE A NON-IDENTIFIED MODEL. SPECIFY A LARGER NUMBER OF MCMC ITERATIONS USING THE FBITER
OPTION OF THE ANALYSIS COMMAND TO INVESTIGATE THE PROBLEM.

Parameter 1    0.3500    0.0000
Parameter 2    0.3300    0.0000
Parameter 13    0.3300    0.0000
Parameter 3    0.3200    0.0000
Parameter 6    0.1700    0.0994
Parameter 8    0.1500    0.1930
Parameter 12    0.1300    0.3439
Parameter 5    0.1300    0.3439
Parameter 7    0.1300    0.3439
Parameter 11    0.1200    0.4431
Parameter 10    0.1200    0.4431
Parameter 9    0.0900    0.7942
```


Kolmogorov-Smirnov Convergence Test

- Using the option BITER=50000(20000) we specify that the MCMC chain runs for a minimum of 20000 iterations and a maximum of 50000
- With this option the example does not converge, i.e., the PSR criterion agrees with the K-S criterion
- The PSR criterion tends to accidentally be satisfied if an insufficient number of Bayes draws (iterations) has been used. The K-S criterion should be checked especially when we are not clear if the model is identified.

Advances in Factor Analysis

- Bayes EFA
- BSEM CFA
- Bi-factor EFA
- Bi-factor ESEM
- Bi-factor BSEM

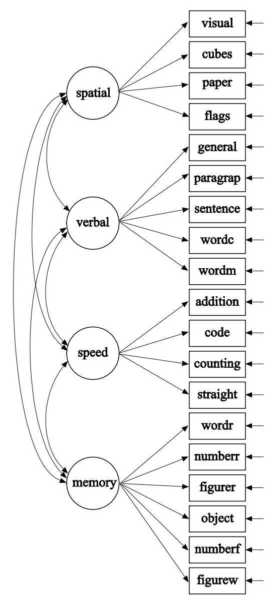
A Factor Analysis Example: Holzinger-Swineford Data

Data are from the classic 1939 factor analysis study by Holzinger and Swineford (1939). Twenty-six tests intended to measure a general factor and five specific factors were administered to seventh and eighth grade students in two schools, the Grant-White school ($n = 145$) and the Pasteur school ($n = 156$). Students from the Grant-White school came from homes where the parents were mostly American-born, whereas students from the Pasteur school came largely from working-class parents of whom many were foreign-born and where their native language was used at home.
Source:
Holzinger, K. J. & Swineford, F. (1939). A study in factor analysis: The stability of a bi- factor solution. Supplementary Educational Monographs. Chicago, Ill.: The University of Chicago.

A Factor Analysis Example: Holzinger-Swineford Data, Continued

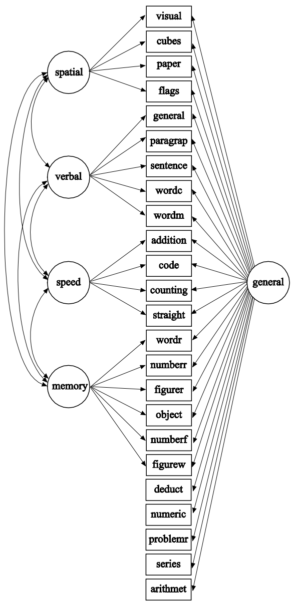
Current analyses:

- 19 tests hypothesized to measure four mental abilities: Spatial, verbal, speed, and memory
- 5 tests measuring a general ability (deduction, test taking ability)
- n=145 7th and 8th grade students from Grant-White elementary school
- n=156 7th and 8th grade students from the Pasteur elementary school



CFA Factor Loading Pattern: 19 Variables

	Spatial	Verbal	Speed	Memory
visual	x	0	0	0
cubes	x	0	0	0
paper	x	0	0	0
flags	x	0	0	0
general	0	x	0	0
paragrap	0	x	0	0
sentence	0	x	0	0
wordc	0	x	0	0
wordm	0	x	0	0
addition	0	0	x	0
code	0	0	x	0
counting	0	0	x	0
straight	0	0	x	0
wordr	0	0	0	x
numberr	0	0	0	x
figurer	0	0	0	x
object	0	0	0	x
numberf	0	0	0	x
figurew	0	0	0	x



	General	Spatial	Verbal	Speed	Memory
visual	x	x	0	0	0
cubes	x	x	0	0	0
paper	x	x	0	0	0
flags	x	x	0	0	0
general	x	0	x	0	0
paragrap	x	0	x	0	0
sentence	x	0	x	0	0
wordc	x	0	x	0	0
wordm	x	0	x	0	0
addition	x	0	0	x	0
code	x	0	0	x	0
counting	x	0	0	x	0
straight	x	0	0	x	0
wordr	x	0	0	0	x
numberr	x	0	0	0	x
figurer	x	0	0	0	x
object	x	0	0	0	x
numberf	x	0	0	0	x
figurew	x	0	0	0	x
deduct	x	0	0	0	0
numeric	x	0	0	0	0
problemr	x	0	0	0	0
series	x	0	0	0	0
arithmet	x	0	0	0	0

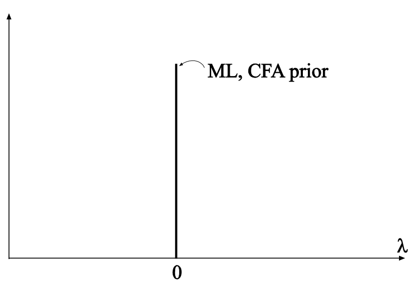
The BSEM CFA Approach

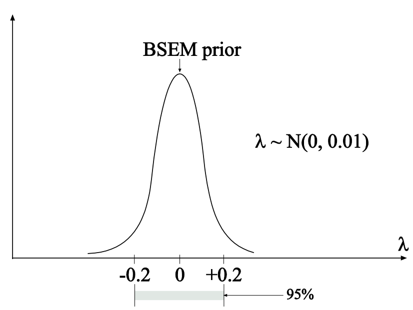
Muthén & Asparouhov (2010). Bayesian SEM: A more flexible representation of substantive theory. Forthcoming in Psychological Methods, September 2012

- The BSEM paper
- 2 commentaries and a rejoinder
- Uses informative priors to estimate parameters that are not identified in ML

ML CFA versus BESEM CFA

- ML CFA uses a very strong prior with an exact zero loading
- BSEM uses a zero-mean, small-variance prior for the loading:

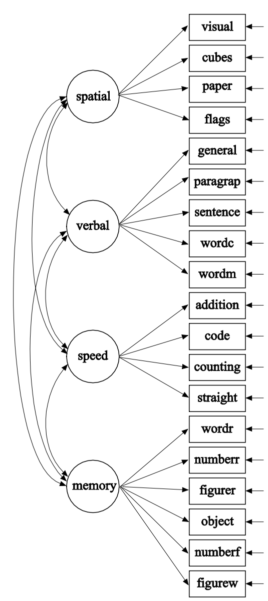




BSEM can be used to specify approximate zeros for

- Cross-loadings
- Residual correlations
- Direct effects from covariates
- Group and time differences in intercepts and loadings

BSEM CFA vs ML CFA: Holzinger-Swineford 19 Variables



CFA Factor Loading Pattern:

	Spatial	Verbal	Speed	Memory
visual	x	0	0	0
cubes	x	0	0	0
paper	x	0	0	0
flags	x	0	0	0
general	0	x	0	0
paragrap	0	x	0	0
sentence	0	x	0	0
wordc	0	x	0	0
wordm	0	x	0	0
addition	0	0	x	0
code	0	0	x	0
counting	0	0	x	0
straight	0	0	x	0
wordr	0	0	0	x
numberr	0	0	0	x
figurer	0	0	0	x
object	0	0	0	x
numberf	0	0	0	x
figurew	0	0	0	x

ML CFA Testing Results For Holzinger-Swineford Data For Grant-White (n =145) And Pasteur (n=156)

Model	χ^2	df	P-value	RMSEA	CFI
Grant-White					
CFA	216	146	0.000	0.057	0.930
EFA	110	101	0.248	0.025	0.991
Pasteur					
CFA	261	146	0.000	0.071	0.882
EFA	128	101	0.036	0.041	0.972

EFA has 6 (Grant-White) and 9 (Pasteur) significant cross-loadings

	Grant-White Factor Loading Patterns For EFA				Pasteur Factor Loading Pattern For EFA			
	Spatial	Verbal	Speed	Memory	Spatial	Verbal	Speed	Memory
visual	**0.628***	0.065	0.091	0.085	**0.580***	0.307*	-0.001	0.053
cubes	**0.485***	0.050	0.007	-0.003	**0.521***	0.027	-0.078	-0.059
paper	**0.406***	0.107	0.084	0.083	**0.484***	0.101	-0.016	-0.229*
flags	**0.579***	0.160	0.013	0.026	**0.687***	-0.051	0.067	0.101
general	0.042	**0.752***	0.126	-0.051	-0.043	**0.838***	0.042	-0.118
paragrap	0.021	**0.804***	-0.056	0.098	0.026	**0.800***	-0.006	0.069
sentence	-0.039	**0.844***	0.085	-0.057	-0.045	**0.911***	-0.054	-0.029
wordc	0.094	**0.556***	0.197*	0.019	0.098	**0.695***	0.008	0.083
wordm	0.004	**0.852***	-0.074	0.069	0.143*	**0.793***	0.029	-0.023
addition	-0.302*	0.029	**0.824***	0.078	-0.247*	0.067	**0.664***	0.026
code	0.012	0.050	**0.479***	0.279*	0.004	0.262*	**0.552***	0.082
counting	0.045	-0.159	**0.826***	-0.014	0.073	-0.034	**0.656***	-0.166
straight	0.346*	0.043	**0.570***	-0.055	0.266*	-0.034	**0.526***	-0.056
wordr	-0.024	0.117	-0.020	**0.523***	-0.005	0.020	-0.039	**0.726***
numberr	0.069	0.021	-0.026	**0.515***	-0.026	-0.057	-0.057	**0.604***
figurer	0.354*	-0.033	-0.077	**0.515***	0.329*	0.042	0.168	**0.403***
object	-0.195	0.045	0.154	**0.685***	-0.123	-0.005	0.333*	**0.469***
numberf	0.225	-0.127	0.246*	**0.450***	-0.014	0.092	0.092	**0.427***
figurew	0.069	0.099	0.058	**0.365***	0.139	0.013	0.237*	**0.291***

BSEM CFA for Holzinger-Swineford

- CFA: Cross-loadings fixed at zero - the model is rejected
- A more realistic hypothesis: Small cross-loadings allowed
- Cross-loadings are not all identified in terms of ML
- Different alternative: Bayesian CFA with informative priors for cross-loadings: $\lambda \sim$ N(0, 0.01).

  This means that 95% of the prior is in the range -0.2 to 0.2

Input BSEM CFA 19 Items 4 Factors Crossloading Priors

```
VARIABLE:   NAMES = id female grade agey agem school
            ! grade = 7/8
            ! school = 0/1 for Grant-White/Pasteur
            visual cubes paper flags general paragrap sentence wordc
            wordm addition code counting straight wordr numberr figurer
            object numberf figurew deduct numeric problemr series arith-
            met;
            USEV = visual-figurew;
            USEOBS = school eq 0;
DEFINE:     STANDARDIZE visual-figurew;
ANALYSIS:   ESTIMATOR = BAYES;
            PROCESSORS = 2;
            FBITER = 10000;
```

Input BSEM CFA 19 Items 4 Factors Crossloading Priors (Continued)

```
MODEL:         spatial BY visual* cubes paper flags;
               verbal BY general* paragrap sentence wordc wordm;
               speed BY addition* code counting straight;
               memory BY wordr* numberr figurer object numberf figurew;
               spatial-memory@1;
               ! cross-loadings:
               spatial BY general-figurew*0 (a1-a15);
               verbal BY visual-flags*0 (b1-b4);
               verbal BY addition-figurew*0 (b5-b14);
               speed BY visual-wordm*0 (c1-c9);
               speed BY wordr-figurew*0 (c10-c15);
               memory BY visual-straight*0 (d1-d13);
MODEL PRIORS:
               a1-d13 ~ N(0,.01);
OUTPUT:        TECH1 TECH8 STDY;
PLOT:          TYPE = PLOT2;
```


ML analysis

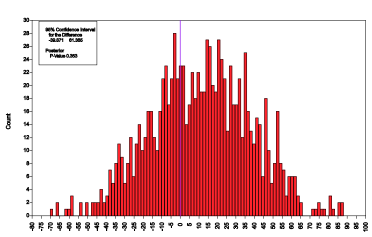
Model	χ^2	Df	P-value	RMSEA	CFI
Grant-White					
CFA	216	146	0.000	0.057	0.930
EFA	110	101	0.248	0.025	0.991
Pasteur					
CFA	261	146	0.000	0.071	0.882
EFA	128	101	0.036	0.041	0.972

Bayesian analysis

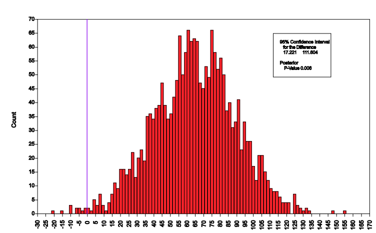
Model	Sample LRT	2.5% PP limit	97.5% PP limit	PP p-value
Grant-White				
CFA	219	12	112	0.006
CFA w/ cross-loadings	142	-39	61	0.361
Pasteur				
CFA	264	56	156	0.000
CFA w/ cross-loadings	156	-28	76	0.162

Bayesian Posterior Predictive Checking For The CFA Model For Grant-White

CFA with small cross-loadings not rejected by Bayes PPC: p = 0.361



Conventional CFA model rejected by Bayes PPC: p = 0.006:



	Grant-White Factor Loadings Using Informative Priors				Pasteur Factor Loadings Using Informative Priors			
	Spatial	Verbal	Speed	Memory	Spatial	Verbal	Speed	Memory
visual	**0.640***	0.012	0.050	0.047	**0.633***	0.145	0.027	0.039
cubes	**0.521***	-0.008	-0.010	-0.012	**0.504***	-0.027	-0.041	-0.030
paper	**0.456***	0.040	0.041	0.047	**0.515***	0.018	-0.024	-0.118
flags	**0.672***	0.046	-0.020	0.005	**0.677***	-0.095	0.026	0.093
general	0.037	**0.788***	0.049	-0.040	-0.056	**0.856***	0.027	-0.084
paragrap	-0.001	**0.837***	-0.053	0.030	0.015	**0.801***	-0.011	0.050
sentence	-0.045	**0.885***	0.021	-0.055	-0.063	**0.925***	-0.032	-0.036
wordc	0.053	**0.612***	0.096	0.029	0.055	**0.694***	0.013	0.063
wordm	-0.012	**0.886***	-0.086	0.020	0.092	**0.803***	0.001	0.012
addition	-0.172*	0.030	**0.795***	0.004	-0.147	-0.004	**0.655***	0.010
code	-0.002	0.054	**0.560***	0.130	-0.004	0.111	**0.655***	0.049
counting	0.013	-0.092	**0.828***	-0.049	0.025	-0.058	**0.616***	-0.057
straight	0.189*	0.043	**0.633***	-0.035	0.132	-0.067	**0.558***	0.001
wordr	-0.040	0.044	-0.031	**0.556***	-0.058	0.006	-0.090	**0.731***
numberr	0.003	-0.004	-0.038	**0.552***	0.006	-0.098	-0.106	**0.634***
figurer	0.132	-0.024	-0.049	**0.573***	0.156*	0.027	0.064	**0.517***
object	-0.139	0.014	0.029	**0.724***	-0.097	0.007	0.122	**0.545***
numberf	0.099	-0.071	0.095	**0.564***	-0.029	0.041	0.003	**0.474***
figurew	0.012	0.045	0.007	**0.445***	0.049	0.018	0.085	**0.397***

Number of significant cross-loadings: 2 for Grant-White and 1 for Pasteur

Effects Of Using Different Variances For The Informative Priors Of The Cross-Loadings For The Holzinger-Swineford Data: Grant-White

Prior variance	95% cross-loading limit	PPP	Cross-loading (Posterior SD)	Factor corr. range
0.01	0.02	0.361	0.189 (.078)	0.443-0.557
0.02	0.28	0.441	0.248 (.096)	0.439-0.542
0.03	0.34	0.457	0.275 (.109)	0.432-0.530
0.04	0.39	0.455	0.292 (.120)	0.413-0.521
0.05	0.44	0.453	0.303 (.130)	0.404-0.513
0.06	0.48	0.447	0.309 (.139)	0.400-0.510
0.07	0.52	0.439	0.315 (.148)	0.395-0.508
0.08	0.55	0.439	0.319 (.156)	0.387-0.508
0.09	0.59	0.435	0.323 (.163)	0.378-0.506
1.00	0.62	0.427	0.327 (.171)	0.369-0.504

Summary of Analyses of Holzinger-Swineford 19-Variable Data

- Conventional, frequentist, CFA model rejected
- Bayesian CFA with informative cross-loadings not rejected
- The Bayesian approach uses an intermediate hypothesis:
  - Less strict than conventional CFA
  - Stricter than EFA, where the hypothesis only concerns the number of factors
  - Cross-loadings shrunken towards zero; acceptable degree of shrinkage monitored by PPP
- Bayes modification indices obtained by estimated cross-loadings
- Factor correlations: EFA $<$ BSEM $<$ CFA

Comparing BSEM And Target Rotation

- Target rotation: EFA rotation chosen to match zero target loadings using least-squares fitting
  - Similarities: Replaces mechanical rotation with judgement/hypotheses
  - Differences: For Target, specifying more than the necessary EFA restrictions does not affect fit and user-defined closeness to zero is replaced with least-squares fitting
- Results for Holzinger-Swineford data:
  - Results similar to EFA with 10 significant cross-loadings for Grant-White and 15 for Pasteur

Comparing BSEM And ESEM

- ESEM: Structural equation modeling with EFA measurement model (Asparouhov & Muthén (2009). Exploratory structural equation modeling. Structural Equation Modeling, 16, 397-438)
- Similarities: Both ESEM and BSEM can be used for measurement models in SEM
- Differences:
  - ESEM is EFA-oriented while BSEM is CFA-oriented
  - ESEM uses a mechanical rotation and the rotation is not based on information from other parts of the model
  - BSEM is applicable not only to measurement models

Bi-Factor Modeling

As popular today as in 1939.

Gibbons, & Hedeker, (1992). Full-information item bi-factor analysis. Psychometrika, 57, 423436.
Reise, Morizot, & Hays (2007). The role of the bifactor model in resolving dimensionality issues in health outcomes measures. Quality of Life Research, 16, 1931.
Cai (2010). A two-tier full-information item factor analysis model with applications. Psychometrika, 75, 581-612.
Cai, Yang, Hansen (2011). Generalized full-information item bifactor analysis. Psychological Methods, 16, 221-248.

- Testlet modeling, e.g. for PISA test items
- Longitudinal modeling with across-time correlation for residuals of the same item

Bi-Factor Modeling Without Using CFA

New methods that do not use regular ML CFA:

- Bi-factor EFA (Jennrich & Bentler)
  - ROTATION = BI-GEOMIN
- Bi-factor ESEM
  - ROTATION = BI-GEOMIN (same as above)
  - Bi-factor ESEM with general CFA factor and ROTATION = GEOMIN for specific factors
- Bi-factor BSEM (no rotation)

Bi-Factor EFA: UG Ex4.7

```
TITLE:     this is an example of a bi-factor
           exploratory factor analysis with
           continuous factor indicators
DATA:      FILE = ex4.7.dat;
VARIABLE:  NAMES = y1-y10;
ANALYSIS:  TYPE = EFA 2 3;
           ROTATION = BI-GEOMIN;
```

The number of factors is the general factor plus the specific factors

Bi-Factor ESEM: UG Ex5.29 Same as Bi-Factor EFA

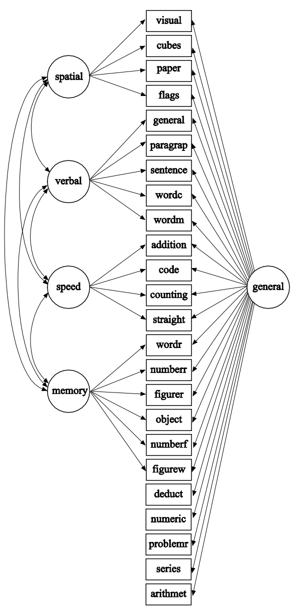
```
TITLE:     this is an example of a bi-factor EFA
DATA:      FILE = ex5.29.dat;
VARIABLE:  NAMES = y1-y10;
ANALYSIS:  ROTATION = BI-GEOMIN;
MODEL:     fg f1 f2 BY y1-y10 (*1);
OUTPUT:    STDY;
```

Bi-Factor ESEM with CFA Factor and Regular Rotation: UG Ex5.30

```
TITLE:     this is an example of bi-factor EFA with
           two items loading on only the general
           factor
DATA:      FILE = ex5.30.dat;
VARIABLE:  NAMES = y1-y10;
ANALYSIS:  ROTATION = GEOMIN;
MODEL:     fg BY y1-y10*;
           fg@1;
           f1-f2 BY y1-y8 (*1);
           fg WITH f1-f2@0;
OUTPUT:    STDY;
```


Bi-Factor BSEM (CFA-Like; No Rotation): UG Ex5.31

```
TITLE:     this is an example of a Bayesian bi-factor
           CFA with two items loading on only the
           general factor and cross-loadings with
           zero-mean and small-variance priors
DATA:      FILE = ex5.31.dat;
VARIABLE:  NAMES = y1-y10;
ANALYSIS:  ESTIMATOR = BAYES;
           PROCESSORS = 2;
MODEL:     fg BY y1-y10*;
           fg@1;
           f1 BY y1-y4
           y5-y10 (f1xlam5-f1xlam10);
           f2 BY y5-y8
           y1-y4 y9-y10(f2xlam1-f2xlam6);
           fg WITH f1-f2@0;
MODEL PRIORS:
           f1xlam5-f2xlam6~N(0,0.01);
PLOT:      TYPE = PLOT2;
```

	General	Spatial	Verbal	Speed	Memory
visual	x	x	0	0	0
cubes	x	x	0	0	0
paper	x	x	0	0	0
flags	x	x	0	0	0
general	x	0	x	0	0
paragrap	x	0	x	0	0
sentence	x	0	x	0	0
wordc	x	0	x	0	0
wordm	x	0	x	0	0
addition	x	0	0	x	0
code	x	0	0	x	0
counting	x	0	0	x	0
straight	x	0	0	x	0
wordr	x	0	0	0	x
numberr	x	0	0	0	x
figurer	x	0	0	0	x
object	x	0	0	0	x
numberf	x	0	0	0	x
figurew	x	0	0	0	x
deduct	x	0	0	0	0
numeric	x	0	0	0	0
problemr	x	0	0	0	0
series	x	0	0	0	0
arithmet	x	0	0	0	0

Holzinger-Swineford, 24 Variables: Input Excerpts for Bi-Factor EFA

```
           USEVARIABLES = visual - arithmet;
           USEOBSERVATIONS = school EQ 0;
ANALYSIS:  TYPE = EFA 5 5;
           ROTATION = BI-GEOMIN;
```

Bi-Factor EFA for Holzinger-Swineford's 24-variable Grant-White data

	General	Spatial	Verbal	Speed	Memory
visual	**0.621***	**0.384***	-0.065	0.072	0.002
cubes	**0.433***	**0.207**	-0.103	-0.115	-0.118
paper	**0.430***	**0.343***	0.058	0.225	0.079
flags	**0.583***	**0.311***	-0.028	-0.077	-0.109
general	**0.610***	-0.034	**0.524***	0.001	-0.075
paragrap	**0.554***	0.053	**0.618***	0.012	0.102
sentence	**0.572***	-0.037	**0.622***	0.010	-0.064
wordc	**0.619***	0.006	**0.354***	0.038	-0.048
wordm	**0.582***	-0.008	**0.603***	-0.137	0.009
addition	**0.508***	-0.528	-0.036	**0.327**	0.009
code	**0.532***	-0.031	0.046	**0.428***	0.310*
counting	**0.568***	-0.229	-0.216*	**0.302**	-0.093
straight	**0.643***	0.217	0.004	**0.526***	-0.032

Bi-Factor EFA for Holzinger-Swineford, Continued

	General	Spatial	Verbal	Speed	Memory
wordr	**0.349***	0.018	0.077	0.032	**0.475***
numberr	**0.352***	0.037	-0.041	-0.052	**0.392***
figurer	**0.495***	0.221	-0.122	-0.033	**0.384***
object	**0.422***	-0.200	-0.010	-0.021	**0.497***
numberf	**0.553***	-0.041	-0.220*	0.003	**0.256***
figurew	**0.414***	-0.033	-0.003	-0.024	**0.246***
deduct	**0.611***	-0.001	0.089	-0.284*	0.036
numeric	**0.656***	-0.021	-0.129	0.029	-0.023
problemr	**0.607***	0.028	0.091	-0.227*	0.059
series	**0.714***	0.023	0.034	-0.202	-0.067
arithmet	**0.638***	-0.356*	0.092	-0.009	0.070

6 significant cross-loadings

Bi-Factor EFA for Holzinger-Swineford, Continued

BI-GEOMIN Factor Correlations

	General	Spatial	Verbal	Speed	Memory
General	1.000				
Spatial	0.000	1.000			
Verbal	0.000	0.022	1.000		
Speed	0.000	-0.223*	-0.122*	1.000	
Memory	0.000	-0.037	0.068	-0.134	1.000

Input for Bi-Factor ESEM with a General CFA Factor and Regular Rotation of the Specific Factors

```
          USEVARIABLES = visual-arithmet;
          USEOBSERVATIONS = school EQ 0;
ANALYSIS: ROTATION = GEOMIN;
MODEL:
          fg BY visual-arithmet*;
          fg@1;
          s1-s4 BY visual-figurew (*1);
          fg WITH s1-s4@0;
OUTPUT:   STDY;
```

Bi-Factor ESEM with a General CFA Factor and Regular Rotation of the Specific Factors

STDY Standardization

	General	Spatial	Verbal	Speed	Memory
visual	**0.574***	**0.450***	-0.081	0.087	0.012
cubes	**0.425***	**0.277***	-0.140	-0.076	-0.132
paper	**0.382***	**0.352***	0.067	0.151	0.109
flags	**0.574***	**0.356***	-0.070	-0.057	-0.121
general	**0.652***	-0.013	**0.460***	0.034	-0.084
paragrap	**0.604***	0.038	**0.580***	-0.031	0.114
sentence	**0.614***	-0.018	**0.574***	0.042	-0.053
wordc	**0.628***	0.040	**0.315***	0.106	-0.032
wordm	**0.657***	-0.042	**0.539***	-0.126	0.001
addition	**0.448***	-0.323*	-0.060	**0.537***	0.010
code	**0.468***	0.037	0.055	**0.450***	0.313*
counting	**0.487***	-0.059	-0.252*	**0.554***	-0.112
straight	**0.517***	0.394*	0.021	**0.606***	-0.014

Bi-Factor ESEM with a General CFA Factor and Regular Rotation of the Specific Factors, Continued

STDY Standardization

	General	Spatial	Verbal	Speed	Memory
wordr	**0.368***	-0.049	0.081	-0.006	**0.464***
numberr	**0.378***	-0.047	-0.051	-0.069	**0.365***
figurer	**0.489***	0.202	-0.116	-0.072	**0.390***
object	**0.459***	-0.270*	-0.033	0.034	**0.463***
numberf	**0.549***	0.005	-0.261*	0.078	**0.207**
figurew	**0.423***	-0.018	-0.015	-0.004	**0.235***
deduct	**0.643***				
numeric	**0.612***				
problemr	**0.638***				
series	**0.717***				
arithmet	**0.643***				

6 significant cross-loadings

Input for Bi-Factor BSEM for Holzinger-Swineford

```
DATA:       FILE = H-S Combined.txt;
VARIABLE:   NAMES = id female grade agey agem school
            visual cubes paper flags general paragrap
            sentence wordc wordm addition code counting
            straight wordr numberr figurer object numberf
            figurew deduct numeric problemr series arithmet;
            USEVARIABLES = visual-arithmet;
            USEOBSERVATIONS = school EQ 0;
DEFINE:     STANDARDIZE visual-arithmet;
ANALYSIS:   ESTIMATOR = BAYES;
            PROCESSORS = 2;
            FBITER = 15000;
MODEL:      g BY visual-arithmet*;
            spatial BY visual-flags*
            general-arithmet*0 (s1-s20);
```


Input for Bi-Factor BSEM, Continued

```
               verbal BY visual-flags*0 (v1-v4)
               general-wordm*
               addition-arithmet*0 (v5-v19);
               speed BY visual-wordm*0 (sp1-sp9)
               addition-straight*
               wordr-arithmet*0 (sp10-sp20);
               memory BY visual-straight*0 (m1-m13)
               wordr-figurew*
               deduct-arithmet*0 (m14-m18);
               g-memory@1;
               g-memory WITH g-memory@0;
MODEL PRIORS:
               s1-s20~N(0,0.01);
               v1-v19~N(0,0.01);
               sp1-sp20~N(0,0.01);
               m1-m18~N(0,0.01);
OUTPUT:        TECH1 TECH8 STDYX;
PLOT:          TYPE = PLOT3;
```

Bi-Factor BSEM

STDY Standardization

	General	Spatial	Verbal	Speed	Memory
visual	**0.615***	**0.387***	-0.014	0.003	0.003
cubes	**0.434***	**0.271**	-0.041	-0.040	-0.040
paper	**0.409***	**0.359***	0.057	0.061	0.057
flags	**0.583***	**0.354***	0.017	-0.053	-0.033
general	**0.603***	0.021	**0.537***	0.040	-0.047
paragrap	**0.577***	0.009	**0.598***	-0.039	0.051
sentence	**0.576***	-0.025	**0.604***	0.009	-0.045
wordc	**0.619***	0.032	**0.361***	0.052	-0.016
wordm	**0.618***	-0.022	**0.593***	-0.102	0.000
addition	**0.463***	-0.194*	0.009	**0.641***	0.000
code	**0.521***	0.001	0.047	**0.398***	0.135
counting	**0.505***	0.004	-0.101	**0.542***	-0.042
straight	**0.587***	0.191*	0.029	**0.432***	-0.029

Bi-Factor BSEM, Continued

STDY Standardization

	General	Spatial	Verbal	Speed	Memory
wordr	**0.385***	-0.021	0.050	-0.004	**0.440***
numberr	**0.375***	0.013	-0.002	-0.021	**0.386***
figurer	**0.529***	0.102	-0.057	-0.051	**0.348***
object	**0.460***	-0.128	0.016	0.050	**0.463***
numberf	**0.560***	0.028	-0.104	0.083	**0.267***
figurew	**0.461***	-0.030	-0.003	0.002	**0.190**
deduct	**0.662***	0.002	0.068	-0.105	0.029
numeric	**0.672***	0.044	-0.067	0.099	-0.023
problemr	**0.665***	-0.002	0.054	-0.099	0.017
series	**0.741***	0.048	0.040	-0.044	-0.032
arithmet	**0.674***	-0.152*	0.066	0.159	0.022

3 significant cross-loadings, all for the Spatial factor.

Summary of Bi-Factor Analysis for Holzinger-Swineford Data for the Grant-White School

- Bi-factor CFA does not fit
- EFA with bi-factor rotation: 6 significant cross-loadings
- Bi-factor ESEM with general CFA factor and regular rotation for the specific factors: 6 significant cross-loadings
- Bi-factor BSEM with no rotation: 3 significant cross-loadings

Muthén & Asparouhov (2012): Rejoinder: Mastering a New Method suggests that BSEM bi-factor analysis outperforms all other bi-factor analyses of the classic Holzinger-Swineford data

Two-Tier Modeling

- New in Mplus Version 7: Two-tier algorithm. New algorithm that reduces the dimension of integration with ML estimation of SEM with categorical variables
- Multiple factors that load on different indicators and are uncorrelated are reduced to 1 dimensional integration

Two-Tier Simulation Study for a Bi-Factor Model

Table 1: Absolute bias, coverage and log-likelihood for the bifactor model.

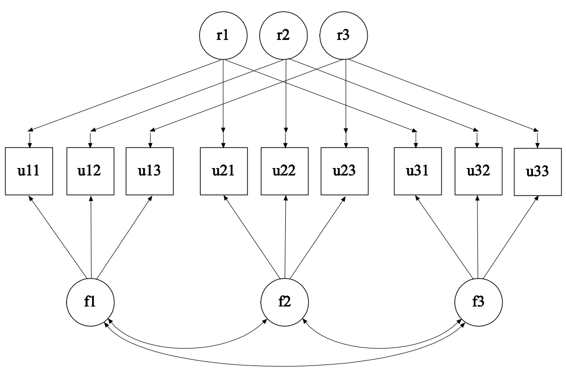
Method	λ_{11}	λ_{12}	$\tau_{1,1}$	Log-Likelihood
Mplus Two Tier	.02(.95)	.02(.97)	.02(.97)	-31664.1
Mplus Monte 500	.03(.92)	.07(.92)	.02(.94)	-31760.8
Mplus Monte 5000	.03(.94)	.01(.95)	.01(.96)	-31678.5
Mplus Bayes	.11(.89)	.07(.93)	.16(.89)	-
Mplus WLSMV	.06(.93)	.05(.95)	.13(.93)	-
IRTPRO Two Tier	.02(.98)	.00(.96)	.01(.98)	-31680.7

Two-Tier Simulation Study for a Bi-Factor Model, Cont'd

Table 2: Average standard error, ratio between average standard error and standard deviation for the bifactor model.

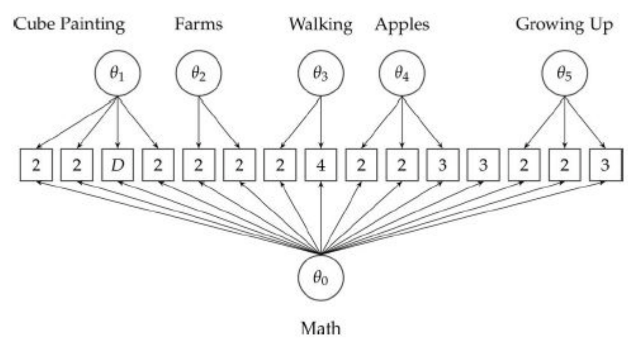
Method	λ_{11}	λ_{12}	$\tau_{1,1}$
Mplus Two Tier	0.12(0.97)	0.16(1.00)	0.19(1.10)
Mplus Monte 500	0.12(0.91)	0.15(0.99)	0.18(1.05)
Mplus Monte 5000	0.12(0.89)	0.15(0.94)	0.19(1.09)
Mplus Bayes	0.13(0.97)	0.16(1.01)	0.19(1.09)
Mplus WLSMV	0.13(0.97)	0.15(0.98)	0.18(1.08)
IRTPRO Two Tier	0.17(1.47)	0.16(1.01)	0.18(1.02)

Two-Tier Modeling: A Longitudinal Example



With categorical outcomes, ML estimation leads to numerical integration that is difficult with many dimensions/factors.
The two-tier algorithm reduces the 6 dimensions of integration to 4.
Bi-factor CFA has a similar structure that benefits from two-tier computing.

Two-Tier Modeling: Bi-Factor Model for PISA Math Items



The two-tier algorithm reduces the 6 dimensions of integration to 2.

Cai, Yang, & Hansen (2011) Generalized full-information item bifactor analysis. Psychological Methods, 16, 221-248

Two-Tier Estimation

- The two-tier integration method can be estimated in Mplus prior to Version 7 as a two-level multiple group model where the general factors are between level factors, while the specific factors are within level factors and the multiple groups represent the different blocks of variables that are correlated beyond the general factors
- In Mplus Version 7 this is no longer necessary and the two-tier estimation can be used when the model is set up as a regular single level model
- The program will automatically determine if the model is a bifactor-like model that allows for more optimal two-tier integration and will set up the two-level multiple group model automatically. On the surface you can not tell if the model is estimated through a special two-level setup

Advances in Multiple-Group Analysis: Invariance Across Groups

- An old dilemma
- Two new solutions

Fixed versus Random Groups

- Fixed mode:
  - Inference to only the groups in the sample
  - Small to medium number of groups
- Random mode:
  - Inference to a population of groups from which the current set of groups is a random sample
  - Medium to large number of groups

Two Solutions

- New solution no. 1, suitable for a small to medium number of groups
  - A new BSEM approach where group is a fixed mode: Multiple-group BSEM
  - Approximate invariance allowed
- New solution no. 2, suitable for a medium to large number of groups
  - A new Bayes approach where group is a random mode
  - No limit on the number of groups

New Options to Facilitate Multiple-Group BSEM (New Solution No. 1)

- COV
- DIFF
- DO DIFF
- MODEL=ALLFREE
- Auto labeling

COV Option

When choosing Bayesian priors, parameters are often specified as independent, but can also be allowed to covary. This is accomplished with the COVARIANCE option in MODEL PRIORS. An example:

MODEL:
y ON x1(a)
x2(b);
MODEL PRIORS:
a $\sim$ N(10, 4);
b $\sim$ N(6, 1);
COV(a,b)=0.5;

which says that the prior bivariate distribution of a and b has a covariance of 0.5, which translates to a correlation of 0.25:

$$\frac{0.5}{\sqrt{(4)}\,\sqrt{(1)}} = 0.25$$

COV Option Continued

The COVARIANCE option can also be used to specify small differences between parameters. Note that

$$V(a-b) = V(a) + V(b) - 2\,cov(a,b), \tag{4}$$

so that if $V(a) = V(b) = 1000$, using $cov(a,b) = 999.995$ gives $V(a-b) = 0.01$. With a normal distribution, this means that the difference has a 95% chance of being between -0.196 and $+0.196$, that is, in a small range around the zero mean.

Example: Two parameters a and b for which we want to apply zero-mean, small-variance priors

a – b $\sim$ N(0, 1000); ! non-informative priors
COV(a, b)=999.995;

DIFF Option

Used with MODEL PRIORS in Bayesian analysis to simplify specifying differences between parameters.

Example: The difference between the parameters a and b

DIFF(a, b) $\sim$ N(0, 0.01);

This is the same as the two statements

a – b $\sim$ N(0,1000); ! non-informative prior
COV(a, b)=999.995;

DO DIFF Option

DO DIFF is used to express parameter differences between large sets of parameters and groups/timepoints.

Example: Group differences for 4 parameters in 3 groups. Let lamjk denote a factor loading for group/timepoint j and variable k:

DO(1,4) DIFF(lam1# – lam3#) $\sim$ N(0,0.01);
! for variable 1 this results in approximate invariance across the 3 groups:
! lam11 $\approx$ lam21
! lam11 $\approx$ lam31
! lam21 $\approx$ lam31

! etc. for the lam parameters for variables 2-4

Auto Labeling Option for Multiple Groups

Used in conjunction with DO DIFF for multiple-group Bayes, which is carried out using TYPE=MIXTURE and KNOWNCLASS. For instance, with a factor measured by 4 variables in 3 groups:

ANALYSIS: TYPE = MIXTURE; ! 3 classes
ESTIMATOR = BAYES;
PROCESSORS = 2;
MODEL = ALLFREE;
MODEL:
%OVERALL%
f BY y1-y4* (lam#_1 – lam#_4);
! the above gives labels for all 3 groups (group is #)
MODEL PRIORS:
DO(1,4) DIFF(lam1_# – lam3_#)$\sim$N(0,0.01);

Auto labeling saves an enormous amount of typing.

New Solution No. 1: Group is Fixed Mode. UG Ex5.33

```
TITLE:     this is an example of a Bayesian
           multiple group model with approximate
           measurement invariance
DATA:      FILE = ex5.33.dat;
VARIABLE:  NAMES = u y1-y6 group;
           USEVARIABLES = y1-y6 group;
           CLASSES = c(10);
           KNOWNCLASS = c(group = 1-10);
ANALYSIS:  TYPE = MIXTURE;
           ESTIMATOR = BAYES;
           PROCESSORS = 2;
           MODEL = ALLFREE;
MODEL:     %OVERALL%
           f1 BY y1-y3* (lam#_1-lam#_3);
           f2 BY y4-y6* (lam#_4-lam#_6);
           [y1-y6] (nu#_1-nu#_6);
           %c#10%
           f1-f2@1;
           [f1-f2@0];
MODEL PRIORS:
           DO(1,6) DIFF(lam1_#-lam10_#)~N(0,0.01);
           DO(1,6) DIFF(nu1_#-nu10_#)~N(0,0.01);
OUTPUT:    TECH1 TECH8;
PLOT:      TYPE = PLOT2;
```


New Solution No. 2: Group is Random Mode Two-level Factor Analysis with Random Loadings

Consider a single factor η. For factor indicator r ($r = 1,2,\ldots p$) for individual i in group (cluster) j,

$$y_{rij} = \nu_{rj} + \lambda_{rj}\,\eta_{ij} + \varepsilon_{ij}, \tag{5}$$
$$\eta_{ij} = \eta_j + \zeta_{ij}, (\textit{this may be viewed as } \eta_{B_j} + \eta_{W_{ij}}) \tag{6}$$
$$\nu_{rj} = \nu_r + \delta_{\nu_j}, \tag{7}$$
$$\lambda_{rj} = \lambda_r + \delta_{\lambda_j}, \tag{8}$$

where ν_r is the mean of the r^{th} intercept and λ_r is the mean of the r^{th} factor loading. Because the factor loadings are free, the factor metric is set by fixing $V(\zeta_{ij}) = 1$ (the between-level variance $V(\eta_j)$ is free). Note that the same loading is multiplying both the between- and within-level parts of the factor η.

Two-Level Factor Analysis with Random Loadings: 3 Model Versions

$$y_{rij} = \nu_{rj} + \lambda_{rj}\,\eta_{ij} + \varepsilon_{ij}, \tag{9}$$
$$\eta_{ij} = \eta_j + \zeta_{ij}, (\textit{this may be viewed as } \eta_{B_j} + \eta_{W_{ij}}) \tag{10}$$
$$\nu_{rj} = \nu_r + \delta_{\nu_j}, \tag{11}$$
$$\lambda_{rj} = \lambda_r + \delta_{\lambda_j}, \tag{12}$$

A first alternative to this model is that $V(\eta_j) = 0$ so that the factor with random loadings has only within-level variation. Instead, there can be a separate between-level factor with non-random loadings, measured by the random intercepts of the y indicators as in regular two-level factor analysis, $y_{rj} = \lambda_{B_r}\,\eta_{B_j} + \zeta_{rj}$, where y_{rj} is the between part of y_{rij}. A second alternative is that the λ_{B_r} loadings are equal to the means of the random loadings λ_r.

New Solution No. 2: Group is Random Mode. UG Ex9.19

Part 1: Random factor loadings (decomposition of the factor into within- and between-level parts)

```
TITLE:     this is an example of a two-level MIMIC
           model with continuous factor indicators,
           random factor loadings, two covariates on
           within, and one covariate on between
           with equal loadings across levels
DATA:      FILE = ex9.19.dat;
VARIABLE:  NAMES = y1-y4 x1 x2 w clus;
           WITHIN = x1 x2;
           BETWEEN = w;
           CLUSTER = clus;
ANALYSIS:  TYPE = TWOLEVEL RANDOM;
           ESTIMATOR = BAYES;
           PROCESSORS = 2;
           BITER = (1000);
MODEL:     %WITHIN%
           s1-s4 | f BY y1-y4;
           f@1;
           f ON x1 x2;
           %BETWEEN%
           f ON w;
           f;
PLOT:      TYPE = PLOT2;
OUTPUT:    TECH1 TECH8;
```


New Solution No. 2: Group is Random Mode. UG Ex9.19

Part 2: Random factor loadings and a separate between-level factor

```
MODEL:     %WITHIN%
           s1-s4 | f BY y1-y4;
           f@1;
           f ON x1 x2;
           %BETWEEN%
           fb BY y1-y4;
           fb ON w;
```

f@0; is the between-level default

New Solution No. 2: Group is Random Mode. UG Ex9.19

Part 3: Random factor loadings and a separate between-level factor with loadings equal to the mean of the random loadings

```
MODEL:     %WITHIN%
           s1-s4 | f BY y1-y4;
           f@1;
           f ON x1 x2;
           %BETWEEN%
           fb BY y1-y4* (lam1-lam4);
           fb ON w;
           [s1-s4*1] (lam1-lam4);
```

Monte Carlo Simulations for Groups as Random Mode: Two-Level Random Loadings Modeling

- The effect of treating random loadings as fixed parameters
  - Continuous variables
  - Categorical variables
- Small number of clusters/groups

The Effect of Treating Random Loadings as Fixed Parameters with Continuous Variables

Table: Absolute bias and coverage for factor analysis model with random loadings - comparing random intercepts and loadings and v.s. random intercepts and fixed loadings models

parameter	Bayes	ML with fixed loadings
θ_1	0.00(0.97)	0.20(0.23)
μ_1	0.01(0.95)	0.14(0.66)
λ_1	0.01(0.96)	0.00(0.80)
θ_2	0.02(0.89)	0.00(0.93)

Ignoring the random loadings leads to biased mean and variance parameters and poor coverage. The loading is unbiased but has poor coverage.

The Effect of Treating Random Loadings as Fixed Parameters in Categorical Variables

Table: Absolute bias and coverage for factor analysis model with categorical data and random loadings - comparing random loadings and intercepts v.s. random intercepts and fixed loadings models

parameter	Bayes	WLSMV with fixed loadings
τ_1	0.05(0.96)	0.17(0.63)
λ_1	0.03(0.92)	0.13(0.39)
θ_2	0.05(0.91)	0.11(0.70)

Ignoring the random loadings leads to biased mean, loading and variance parameters and poor coverage.

Random Loadings with Small Number of Clusters/Groups

- Many applications have small number of clusters/groups. How many variables and random effects can we use?
- Independent random effects model - works well even with 50 variables (100 random effects) and 10 clusters
- Weakly informative priors are needed to eliminate biases for cluster level variance parameters
- Correlated random effects model (1-factor model) - works only when "number of clusters > number of random effects". More than 10 clusters are needed with 5 variables or more.
- What happens if you ignore the correlation: standard error underestimation, decreased accuracy in cluster specific estimates
- BSEM: Muthén, B. and Asparouhov, T. (2012). Bayesian SEM: A more flexible representation of substantive theory. Forthcoming in Psychological Methods.
- Using BSEM with 1-factor model for the random effects and tiny priors $N(1,\sigma)$ for the loadings resolves the problem.

Hospital Data Example

Shortell et al. (1995). Assessing the impact of continuous quality improvement/total quality management: concept versus implementation. Health Services Research, 30, 377-401.

- Survey of 67 hospitals, $n = 7168$ employee respondents, approximately 100/hospital
- 6 dimensions of an overall ”quality improvement implementation” based on the Malcom Baldrige National Quality Award critera
- Focus on 10 items measuring a leadership dimension

Hospital Data: Old and New Factor Analysis Alternatives

- Hospital as Fixed Mode:
  - Old approach: Conventional multiple-group factor analysis
  - New approach: BSEM multiple-group factor analysis
- Hospital as Random Mode:
  - Old approach: Conventional two-level factor analysis
  - New approach: Bayes random loadings two-level factor analysis (random factor variances also possible)

Hospital as Fixed Mode: Conventional Multiple-Group Factor Analysis

Regular ML analysis:

```
           USEVARIABLES = lead21-lead30! info31-info37
           ! straqp38-straqp44 hru45-hru52 qm53-qm58 hosp;
           MISSING = ALL(-999);
           !CLUSTER = hosp;
           GROUPING = hosp (101 102 104 105 201 301-306
           308 310-314 316-320 322 401-403 405-409 412-416
           501-503 505-512 602-609 612-613 701 801 901-908);
ANALYSIS:  ESTIMATOR = ML;
           PROCESSORS = 8;
MODEL:
           lead BY lead21-lead30;
PLOT:      TYPE = PLOT2;
OUTPUT:    TECH1 TECH8 MODINDICES(ALL);
```


Hospital as Fixed Mode: Conventional Multiple-Group Factor Analysis, Continued

Maximum-likelihood analysis with χ^2 test of model fit and modification indices.

Holding measurement parameters equal across groups/hospitals results in poor fit with many moderate-sized modification indices and none that sticks out as much larger than the others.

Conventional multiple-group factor analysis ”fails”.

New Solution No. 1: Group as Fixed Mode using Multiple-Group BSEM
BSEM Input Excerpts for Hospital Data

```
           USEVARIABLES = lead21-lead30 hosp;
           MISSING = ALL (-999);
           CLASSES = c(67);
           KNOWNCLASS = c(hosp=101 102 104
           105 201 301-306 308 310-314 316-320
           322 401-403 405-409 412-416 501-503
           505-512 602-609 612-613 701 801
           901-908);
ANALYSIS:  TYPE = MIXTURE;
           ESTIMATOR = BAYES;
           BITERATIONS = (2000); ! min number of Bayes iterations
           PROCESSORS = 2;
           MODEL = ALLFREE;
           ! changes the mixture default of across-class equality of
           ! nu, Lambda, Psi, and Theta parameters when using BY
```


BSEM Input Excerpts for Hospital Data, Continued

Use the auto-labeling feature (# is the class number) in MODEL and specify approximate measurement equality for intercepts and factor loadings across classes/groups in MODEL PRIORS:

```
MODEL:     %OVERALL%
           lead BY lead21-lead30* (lam#_1-lam#_10);
           [lead21-lead30] (nu#_1-nu#_10);
           %c#67%
           lead@1; ! sets the metric of the factor
MODEL
PRIORS:    DO(1,10) DIFF(lam1_#-lam67_#)~N(0,0.05);
           DO(1,10) DIFF(nu1_#-nu67_#)~N(0,0.05);
PLOT:      TYPE = PLOT2;
OUTPUT:    TECH1 TECH8;
```


BSEM Analysis of Hospital Data

67-group BSEM requires 2142 parameters (!)

Computing time: 3:07:35

BSEM Estimates for Hospital 67

```
Latent Class 67

 LEAD     BY
    LEAD21             0.880      0.106     0.000      0.674      1.092      *
    LEAD22             0.915      0.099     0.000      0.721      1.112      *
    LEAD23             0.898      0.095     0.000      0.719      1.088      *
    LEAD24             0.860      0.091     0.000      0.688      1.050      *
    LEAD25             0.805      0.089     0.000      0.630      0.983      *
    LEAD26             0.846      0.084     0.000      0.687      1.014      *
    LEAD27             1.005      0.092     0.000      0.832      1.195      *
    LEAD28             0.511      0.109     0.000      0.284      0.717      *
    LEAD29             0.742      0.098     0.000      0.555      0.947      *
    LEAD30             0.960      0.084     0.000      0.797      1.130      *

 Means
    LEAD               0.000      0.000     1.000      0.000      0.000

 Intercepts
    LEAD21             3.628      0.087     0.000      3.465      3.800      *
    LEAD22             3.496      0.087     0.000      3.335      3.670      *
    LEAD23             3.298      0.087     0.000      3.129      3.468      *
    LEAD24             3.682      0.079     0.000      3.527      3.836      *
    LEAD25             3.776      0.075     0.000      3.626      3.918      *
    LEAD26             3.687      0.072     0.000      3.544      3.821      *
    LEAD27             3.608      0.082     0.000      3.457      3.769      *
    LEAD28             3.433      0.090     0.000      3.257      3.608      *
    LEAD29             3.320      0.081     0.000      3.158      3.476      *
    LEAD30             3.550      0.075     0.000      3.404      3.693      *

 Variances
    LEAD               1.000      0.000     0.000      1.000      1.000
```

BSEM Estimates for Hospital 66

```
Latent Class 66

 LEAD     BY
    LEAD21           0.983      0.086     0.000      0.818      1.154      *
    LEAD22           1.077      0.101     0.000      0.877      1.269      *
    LEAD23           0.927      0.092     0.000      0.749      1.113      *
    LEAD24           0.994      0.083     0.000      0.830      1.160      *
    LEAD25           0.986      0.084     0.000      0.826      1.156      *
    LEAD26           1.091      0.081     0.000      0.933      1.248      *
    LEAD27           1.080      0.089     0.000      0.912      1.258      *
    LEAD28           0.530      0.101     0.000      0.342      0.735      *
    LEAD29           1.047      0.086     0.000      0.874      1.212      *
    LEAD30           1.076      0.088     0.000      0.908      1.248      *

 Means
    LEAD            -0.383      0.099     0.000     -0.567     -0.177      *

 Intercepts
    LEAD21           3.701      0.087     0.000      3.528      3.863      *
    LEAD22           3.460      0.096     0.000      3.274      3.642      *
    LEAD23           3.524      0.090     0.000      3.344      3.694      *
    LEAD24           3.702      0.083     0.000      3.533      3.859      *
    LEAD25           3.739      0.087     0.000      3.571      3.909      *
    LEAD26           3.762      0.085     0.000      3.578      3.922      *
    LEAD27           3.598      0.092     0.000      3.407      3.770      *
    LEAD28           3.431      0.099     0.000      3.238      3.635      *
    LEAD29           3.423      0.085     0.000      3.263      3.593      *
    LEAD30           3.613      0.090     0.000      3.430      3.778      *

 Variances
    LEAD             0.677      0.118     0.000      0.491      0.943      *
```


BSEM Measurement Intercept Differences Across Hospitals: Intercept for Item 3 for all 67 Hospitals

```
          Average   Std. Dev.    Deviations from the Mean

                                  NU1_3      NU2_3      NU3_3      NU4_3      NU5_3
   3        3.401      0.042      0.140     -0.048      0.187*    -0.176      0.021

                                  ...

                                  NU41_3     NU42_3     NU43_3     NU44_3     NU45_3
                                 -0.054     -0.079      0.034     -0.043     -0.219*

                                  NU46_3     NU47_3     NU48_3     NU49_3     NU50_3
                                 -0.019     -0.062      0.142     -0.033     -0.028

                                  NU51_3     NU52_3     NU53_3     NU54_3     NU55_3
                                  0.316*     0.063      0.107      0.103     -0.077

                                  NU56_3     NU57_3     NU58_3     NU59_3     NU60_3
                                  0.031     -0.122     -0.004      0.035      0.048

                                  NU61_3     NU62_3     NU63_3     NU64_3     NU65_3
                                 -0.006      0.103     -0.086      0.217*    -0.044

                                  NU66_3     NU67_3
                                  0.123     -0.099
```


Displaying Non-Invariant Items for BSEM: Hospitals With Significant Differences Compared to the Mean (Prior V = 0.01 + Prior V = 0.05)

Item	Loadings	Intercepts
1	48 + 67	50
2	24, 33, 45 + 47	33, 37, 40, 45, 47 + 24, 39, 50, 52, 57
3	-	45, 51, 64 + 3, 6
4	-	7, 34
5	- + 67	15 + 3, 34, 41, 53
6	- + 25, 67	52, 60 - 52
7	67 + 47	52 + 5, 11, 26
8	45 + 36	17, 60 + 1, 7, 26, 45, 53, 55, 59
9	- + 67	52 + 51
10	-	67

Group as Random Mode: Conventional Two-Level Factor Analysis

- Recall random effects ANOVA (individual i in cluster j):

$$y_{ij} = \nu + \eta_j + \varepsilon_{ij} = y_{B_j} + y_{W_j} \tag{13}$$

- Two-level factor analysis ($r = 1, 2, \ldots, p$ items; 1 factor on each level):

$$y_{rij} = \nu_r + \lambda_{B_r}\, \eta_{B_j} + \varepsilon_{B_{rj}} + \lambda_{W_{ij}}\, \eta_{W_{ij}} + \varepsilon_{W_{rij}} \tag{14}$$

- Alternative expression often used in 2-level IRT:

$$y_{rij} = \nu_r + \lambda_r\, \eta_{ij} + \varepsilon_{rij}, \tag{15}$$

$$\eta_{ij} = \eta_{B_j} + \eta_{W_{ij}}, \tag{16}$$

so that λ is the same for between and within.

Input Excerpts for Hospital as Random Mode: Conventional Two-Level Factor Analysis

```
            USEVARIABLES = lead21-lead30;
            MISSING = ALL (-999);
            CLUSTER = hosp;
ANALYSIS:   TYPE = TWOLEVEL;
            ESTIMATOR = ML;
            PROCESSORS = 8;
MODEL:      %WITHIN%
            leadw BY lead21-lead30* (lam1-lam10);
            leadw@1;
            %BETWEEN%
            leadb BY lead21-lead30* (lam1-lam10);
            leadb;
OUTPUT:     TECH1 TECH8 MODINDICES(ALL);
```


Results for Hospital as Random Mode: Conventional Two-Level Factor Analysis

Equality of within- and between-level factor loadings cannot be rejected by χ^2 difference testing

10 % of the total variance in the leadership factor is due to between-hospital variation

No information about measurement invariance across hospitals

New Solution No. 2: Hospital as Random Mode using Two-Level Analysis with Random Loadings

In the interest of time, this Hospital data analysis is not reported.

Instead, an application for binary items in PISA data for 40 countries is shown in the next part.

Multiple-Group BSEM: Cross-Cultural Comparisons

Davidov (2009). Measurement equivalence of nationalism and constructive patriotism in the ISSP: 34 countries in a comparative perspective. Political Analysis,17, 64-82.

- Data from the ISSP 2003 National Identity Module
- 34 countries, n=45,546
- 5 measurements of nationalism and patriotism
- Expected 2-factor structure

Many other country comparisons, e.g. TIMMS

Nationalism and Patriotism Data: Multiple-Group CFA with ML

Two-factor CFA with measurement invariance across all 34 countries: χ^2 (334) = 9669, p = 0 (!)

Group-specific misfit evenly spread over the countries

Modification indices show a multitude of similarly large values

Alternatives:

- EFA for each group

- ESEM for all groups

- BSEM for all groups

Nationalism and Patriotism Data: EFA for each Group

Australia ($n = 2146$): EFA χ^2 (1) = 0.402

```
            GEOMIN ROTATED LOADINGS (* significant at 5% level)
                  1             2
              ________      ________
 V21            0.821*       -0.005*
 V22            0.569*        0.129*
 V26            0.001         0.614*
 V29           -0.041         0.503*
 V35            0.223*        0.491*

            GEOMIN FACTOR CORRELATIONS (* significant at 5% level)
                  1             2
              ________      ________
      1         1.000
      2         0.268*        1.000
```


Nationalism and Patriotism Data: EFA for each Group

Germany ($n = 1281$): EFA χ^2 (1) = 0.841

```
           GEOMIN ROTATED LOADINGS (* significant at 5% level)
                  1             2
              ________      ________
 V21            1.159*       -0.001
 V22            0.448*        0.159*
 V26            0.002         0.650*
 V29           -0.078*        0.662*
 V35            0.022         0.606*

           GEOMIN FACTOR CORRELATIONS (* significant at 5% level)
                  1             2
              ________      ________
      1         1.000
      2         0.110         1.000

           ESTIMATED RESIDUAL VARIANCES
              V21           V22           V26           V29           V35
              ________      ________      ________      ________      ________
      1        -0.343         0.759         0.577         0.567         0.630
```


Nationalism and Patriotism Data: EFA for each Group

USA (n = 1210): No convergence for EFA

NO CONVERGENCE. NUMBER OF ITERATIONS EXCEEDED. PROBLEM OCCURRED IN EXPLORATORY FACTOR ANALYSIS WITH 2 FACTOR(S).

ESEM:

```
ANALYSIS:
MODEL:
                f1-f2 BY v21-v35 (*1);
                v21-v35 (var1-var5);
MODEL
CONSTRAINT:
                DO(1,5) var#>0;
```

Nationalism and Patriotism Data: ESEM Solution for USA (STDY), $\chi^2(1) = 4.763$

```
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 F1       BY
    V21                1.000      0.000  15267.805      0.000
    V22                0.413      0.027     15.480      0.000
    V26               -0.020      0.035     -0.553      0.580
    V29                0.002      0.008      0.303      0.762
    V35                0.087      0.037      2.353      0.019

 F2       BY
    V21               -0.001      0.000     -4.245      0.000
    V22                0.224      0.035      6.466      0.000
    V26                0.641      0.042     15.418      0.000
    V29                0.503      0.035     14.249      0.000
    V35                0.551      0.040     13.939      0.000

 F2       WITH
    F1                 0.270      0.049      5.489      0.000

 Residual Variances
    V21                0.000      0.000     24.485      0.000
    V22                0.729      0.024     30.424      0.000
    V26                0.595      0.049     12.075      0.000
    V29                0.747      0.035     21.467      0.000
    V35                0.663      0.040     16.703      0.000
```


Nationalism and Patriotism Data: Multiple-Group ESEM

No convergence due to large negative residual variances for many countries

Constraining residual variances to be non-negative gives convergence but the measurement invariance model has $\chi^2(496) = 13893$

Nationalism and Patriotism Data: Multiple-Group BSEM

Multiple-group BSEM allows cross-loadings similar to ESEM, but also allows approximate measurement invariance

With a small number of variables, setting the metric by fixing factor variances at 1 may lead to poor mixing in Bayesian analysis. This occurs in these data.

An alternative is to fix the factor loading at 1 in one group and let the other groups' factor loading be approximately 1 by small-variance priors

Nationalism and Patriotism Data: Multiple-Group BSEM Input

```
            CLASSES = c(34);
            KNOWNCLASS = c(cntry = 1 2 4 6-8 10-22 24-28
            30-33 36 37 40-43);
ANALYSIS:   TYPE = MIXTURE;
            ESTIMATOR = BAYES;
            PROCESSORS = 2;
            MODEL = ALLFREE;
MODEL:      %OVERALL%
            nat BY v21* (lam#_1)
            v22 (lam#_2);
            pat BY v26* (lam#_3)
            v29-v35* (lam#_4-lam#_5);
            [v21-v35] (nu#_1-nu#_5);
            pat BY v21-v22* (xlam#_1-xlam#_2);
            nat BY v26-v35* (xlam#_3-xlam#_5);
```

Nationalism and Patriotism Data: Multiple-Group BSEM Input, Continued

```
%c#1%
nat BY v21@1;
pat BY v26@1;
[nat-pat@0];
%c#34%
[nat-pat];
```

MODEL PRIORS:

```
DO(2,2) DIFF(lam1_#-lam34_#)~N(0,0.01);
DO(4,5) DIFF(lam1_#-lam34_#)~N(0,0.01);
DO(1,5) DIFF(nu1_#-nu34_#)~N(0,0.01);
DO(2,34) lam#_1~N(1,0.01);
DO(2,34) lam#_3~N(1,0.01);
DO(1,5) xlam1_#-xlam34_#~N(0,0.01);
```


34-Country Factor Mean Estimates: Patriotic Factor. Measurement Difference Prior Variance 0.001 Versus 0.01