

# New Developments in Mplus Version 7: Part 1

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Mplus  
[www.statmodel.com](http://www.statmodel.com)

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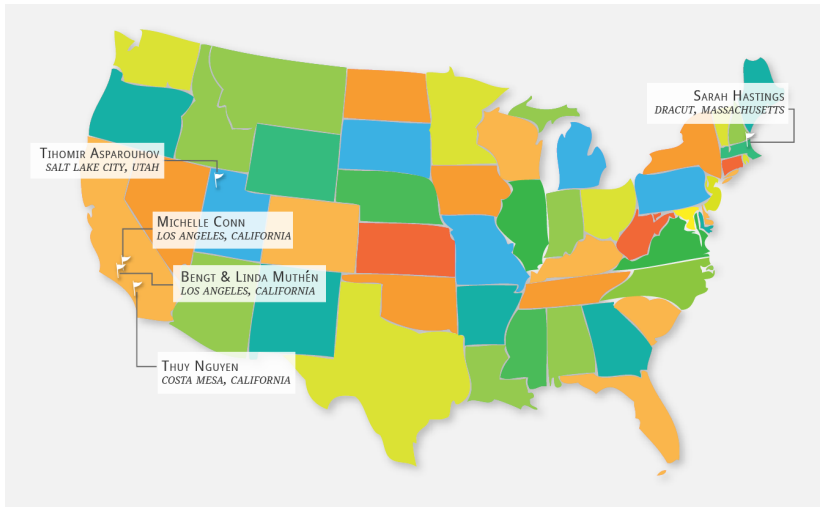
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# The Map of the Mplus Team



- Inefficient dissemination of statistical methods:
  - Many good methods contributions from biostatistics, psychometrics, etc are underutilized in practice
- Fragmented presentation of methods:
  - Technical descriptions in many different journals
  - Many different pieces of limited software
- Mplus: Integration of methods in one framework
  - Easy to use: Simple, non-technical language, graphics
  - Powerful: General modeling capabilities

Several programs in one

- Path analysis
- Exploratory factor analysis
- Structural equation modeling
- Item response theory analysis
- Growth modeling
- Mixture modeling (latent class analysis)
- Longitudinal mixture modeling (Markov, LTA, LCGA, GMM)
- Survival analysis (continuous- and discrete-time)
- Multilevel analysis
- Complex survey data analysis
- Bayesian analysis
- Monte Carlo simulation

Fully integrated in a general latent variable framework



5 big new features:

- 1 Surprise
- 2 Factor analysis
  - Bi-factor EFA rotations, bi-factor ESEM, two-tier modeling
  - Bayesian EFA and CFA (BSEM), bi-factor BSEM
- 3 Analysis of several groups with approx. measurement invariance
  - using a Bayes approach (multiple-group BSEM)
  - using a two-level analysis with random intercepts and loadings
- 4 Analysis of individual differences SEM using measurement parameters that vary across subjects
- 5 Mixture analysis
  - Using a proper 3-step analyze-classify-analyze approach to investigate covariates and distal outcomes
  - Latent transition analysis with new output, covariates influencing transition probabilities, and probability parameterization
  - Exploratory LCA using Bayesian analysis

5 more big features:

- 1 3-level SEM analysis, complex survey data handling, and multiple imputation
- 2 Cross-classified SEM analysis including random subjects and contexts (2 random modes)
- 3 IRT analysis with random items
- 4 Longitudinal analysis with approx. measurement invariance
  - using a Bayes approach (multiple-time point BSEM)
  - using cross-classified analysis of time and subjects
- 5 Analysis of changing membership over time

and 5 other new features:

- 1 Parallel analysis
- 2 LOOP plots (moderated mediation, cross-level interactions, etc)
- 3 Bayes plausible value factor score distribution plots for each subject
- 4 Two-tier algorithm
- 5 New convenience options: LOOP, DO, COV, DIFF, DO DIFF, MODEL=ALLFREE, auto-labeling, BY with random loadings, BITER = (minimum), TECH15, TECH16

- and if you don't see what you had on your wish list, stay tuned for

Version 7.1

Version 7.2

...

Version 8

- Part 1:
  - Hardware and Timings
  - New Options
  - Recap of Bayesian Analysis in Mplus
  - Advances in Factor Analysis
  - Advances in Multiple-Group Analysis: Invariance Across Groups
- Part 2:
  - Two-Level Random Loadings in IRT
  - Advances in Individual Differences Modeling: Invariance Across Subjects
  - Advances in Mixture Modeling
  - 3-Level Analysis, Complex Survey Data, and Multiple Imputation
  - Introductory Cross-Classified Analysis
- Part 3:
  - Advanced Cross-Classified Analysis: Two Random Modes
  - Random Items in IRT
  - Advances in Longitudinal Analysis: Invariance Across Time
  - Advances in Longitudinal Analysis: Growth Modeling with Many Time Points

Don't settle for using an outdated computer. Use 64-bit instead of 32-bit, use several processors instead of only 1 or 2, use fast CPUs.

- PC:

- Intel® Core™ i7-3770K 3.5GHz/3.9GHz Turbo 8MB L3 Cache HD 4000 (Intel's i7 processor released in April is also available for laptops)
- over-clocked to 4.2 to 4.5GHz
- 8 procs
- 32GB RAM
- 64-bit

- Mac Pro:

- Intel Xeon 3.33GHz, 6-core
- 12 procs
- 24GB RAM
- 64-bit

Bengt's PC as of June 2012: \$1,500 Dell XPS 8500, i7-3770 with 8 processors, CPU of 3.40 GHz, 12 GB RAM, 64-bit.

New option: PROCESSORS = a b; ! a = # processors, (b = # threads)  
Can be used with: STARTS = c d; ! New default: c = 20, d = 4.

Multiple processors:

- Programming parallelized code, executable distributes the computing over different processors
- Parallelized code implemented for numerical integration (sample split into parts), missing data patterns, but not for Bayes
- Speed increases as a increases in PROCESSORS = a;

Multiple threads:

- Used with STARTS = , typically for TYPE = MIXTURE
- Different starting value sets are analyzed in different threads using different processors, possibly using more than one processor per thread
- Using as many threads as processors is fastest, but choosing fewer threads than processors is less memory demanding

PROCESSORS = a b; ! a = # processors, (b = # threads)

Replaces PROCESSORS = a (STARTS);

- Analyses with STARTS= (typically mixtures): Using many threads saves time. Mplus makes best decision when saying PROCESSORS = a; although memory demand can be reduced by choosing  $b < a$ , for instance PROCESSORS = 8 2;
- Non-mixture analyses with numerical integration: Using many processors saves time, while number of threads has no effect
- Bayesian analysis: Time not dependent on number of threads or number of processors (PROCESSORS = 2 suitable for the default of 2 chains)



# Processors/Threads Available When?

- Multiple processors and multiple threads with random starts and multiple processors without random starts available when
  - TYPE=MIXTURE
  - Bayesian analysis with more than one chain if STVALUES=ML
  - Models that require numerical integration
- Multiple processors and multiple threads with random starts (w/o random starts, one processor is used) available when
  - TYPE=RANDOM
  - TYPE=TWOLEVEL and TYPE=THREELEVEL, continuous outcomes, ESTIMATOR= ML, MLR, and MLF without numerical integration
- Multiple processors but not multiple threads available when
  - Models with all continuous variables, missing data, and maximum likelihood estimation
  - Bayesian analysis with more than one chain
  - TYPE=TWOLEVEL, categorical outcomes, and ESTIMATOR=WLSMV

# Timing Examples: 3 Analyses

Example 1	Factor mixture model with 2 dimensions of integration, 2 classes, 18 binary outcomes and $n=3314$
Example 2	Factor mixture model with 2 dimensions of integration, 8 classes, 22 outcomes, and $n=842$
Example 3	Growth mixture model with 8 time points, count outcomes, 3 classes, starts = 32 8, 3 dimensions of integration, and $n=1314$

# Timing Examples: 3 Computers

Computer	Processor	RAM	Operating System
Sony Vaio (oldish laptop)	Intel Core 2 Duo CPU T8100 2.10 GHz	2046 MB	Windows Vista 32-bit
Dell Precision 490 (2 dual-core desktop)	Intel Xeon 5160 2x4MB Cache 1333-MHz data rate 3.0 GHz	4GB	Windows XP 32-bit
Dell XPS 8500 (new desktop June 2012)	i7-3770 8 processors 3.4GHz	12 GB	Windows 7 64-bit

# Timing Examples

Minutes: Seconds

	Example 1	Example 2	Example 3
Computer 1	4:40	14:04	38:04
Computer 2	0:57	8:29	13:09
Computer 3	0:38	2:30	5:20

# LOOP Option

LOOP is used in MODEL CONSTRAINT in conjunction with the PLOT option to create plots of one variable related to another, including a 95% confidence interval. An example:

**MODEL:** y ON x (p1);

**MODEL CONSTRAINT:**

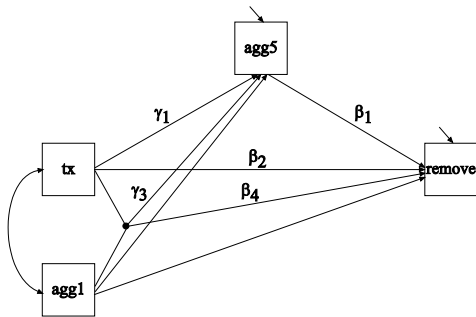
PLOT(ypred);

**LOOP(age, 20, 50, 1); ! 20 ≤ age ≤ 50 with steps of 1**

ypred = p1\*age;

- Plotting indirect effects with moderated mediation. Preacher, Rucker, Hayes (2007), MBR: Figure 3 - conditional indirect effect as a function of the moderator
- Plotting cross-level interactions in two-level modeling. Bauer & Curran (2005)
- Plotting sensitivity graphs for causal effect mediation modeling. Imai et al. (2010), Psych Methods; Muthén (2011)

# LOOP Example: Moderated Mediation of School Removal



$$remove = \beta_0 + \beta_1 agg5 + \beta_2 tx + \beta_3 agg1 + \beta_4 tx agg1 + \varepsilon_1, \quad (1)$$

$$agg5 = \gamma_0 + \gamma_1 tx + \gamma_2 agg1 + \gamma_3 tx agg1 + \varepsilon_2, \quad (2)$$

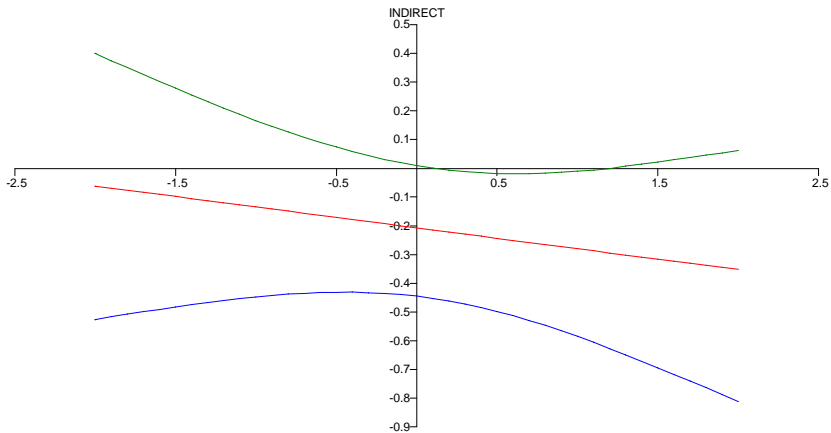
$$= \gamma_0 + (\gamma_1 + \gamma_3 agg1) tx + \gamma_2 agg1 + \varepsilon_2. \quad (3)$$

Indirect effect of tx on remove is  $\beta_1 (\gamma_1 + \gamma_3 agg1)$ , where agg1 moderates the effect of the treatment. Direct effect:  $\beta_2 + \beta_4 agg1$ .

# LOOP Example Continued

```
inter = tx*agg1;
ANALYSIS: ESTIMATOR = BAYES;
PROCESSORS = 2; FBITER = 50000;
MODEL:    remove ON agg5 (beta1)
          tx (beta2)
          agg1 (beta3)
          inter (beta4);
          agg5 ON tx (gamma1)
          agg1 (gamma2)
          inter (gamma3);
MODEL CONSTRAINT:
          PLOT(indirect direct);
          ! let moderate represent the range of the agg1 moderator
          LOOP(moderate, -2, 2, 0.001);
          indirect = beta1*(gamma1+gamma3*moderate);
          direct = beta2+beta4*moderate;
PLOT:    TYPE = PLOT2;
```

# Indirect Effect of Treatment as a Function of SD Units of the Moderator $agg1$





Example: Two groups, 9 factor loadings in each group, expressing the 9 group differences (note that the longer symbol – denotes a "dash", namely a list, and the shorter symbol - denotes minus)

DO(1,9) diff# = lambda1# – lambda2#;

! Same as

! diff1 = lambda11 - lambda21;

! diff2 = lambda12 - lambda22:

! ...

! diff9 = lambda19 - lambda29;

- Useful in MODEL CONSTRAINT to create NEW parameters
- Useful in MODEL PRIORS for Bayesian analysis
- Can also be used in DEFINE

ANALYSIS:

TYPE = TWOLEVEL RANDOM;

ESTIMATOR = BAYES;

MODEL:

% WITHIN %

**s1-s10 | f BY y1-y10;**

% BETWEEN %

[s1-s10];

s1-s10;

Easier than a series of statements like:

s | y ON f;

## Mplus conceptualization:

- Mplus was envisioned 15 years ago as both a frequentist and a Bayesian program
- Bayesian analysis firmly established and its use growing in mainstream statistics
- Much less use of Bayes outside statistics
- Bayesian analysis not sufficiently accessible in other programs
- Bayes provides a broader platform for further Mplus development

Why do we have to learn about Bayes?

- More can be learned about parameter estimates and model fit
- Better small-sample performance, large-sample theory not needed
- Priors can better reflect substantive hypotheses
- Analyses can be made less computationally demanding
  - Frequentists can see Bayes with non-informative priors as a computing algorithm to get answers that would be the same as ML if ML could have been done
- New types of models can be analyzed

- Asparouhov & Muthén (2010). Bayesian analysis using Mplus: Technical implementation. Technical Report. Version 3.
- Asparouhov & Muthén (2010). Bayesian analysis of latent variable models using Mplus. Technical Report. Version 4.
- Asparouhov & Muthén (2010). Multiple imputation with Mplus. Technical Report. Version 2.
- Asparouhov & Muthén (2010). Plausible values for latent variable using Mplus. Technical Report.
- Muthén (2010). Bayesian analysis in Mplus: A brief introduction. Technical Report. Version 3.
- Muthén & Asparouhov (2010). Bayesian SEM: A more flexible representation of substantive theory.
- Asparouhov & Muthén (2011). Using Bayesian priors for more flexible latent class analysis.

Posted under Papers, Bayesian Analysis and Latent Class Analysis

- Single-level, multilevel, and mixture models
- Continuous and categorical outcomes (probit link)
- Default non-informative priors or user-specified informative priors (MODEL PRIORS)
- Multiple chains using parallel processing (CHAIN)
- Convergence assessment using Gelman-Rubin potential scale reduction factors ( $PSR \approx 1$ )
- Posterior parameter distributions with means, medians, modes, and credibility intervals (POINT)
- Posterior parameter trace plots
- Autocorrelation plots
- Posterior predictive checking plots

# Multiple Imputation (DATA IMPUTATION)

- Carried out using Bayesian estimation to create several data sets where missing values have been imputed
- The multiple imputation data sets can be used for subsequent model estimation using ML or WLSMV
- The imputed data sets can be saved for subsequent analysis or analysis can be carried out in the same run
- Imputation can be done based on an unrestricted H1 model using three different algorithms including sequential regressions
- Imputation can also be done based on an H0 model specified in the MODEL command

- The set of variables used in the imputation of the data do not need to be the same as the set of variables used in the analysis
- Single-level and multilevel data imputation are available
- Multiple imputation data can be read using `TYPE=IMPUTATION` in the `DATA` command



# Plausible Values (PLAUSIBLE)

- Plausible values are multiple imputations for missing values corresponding to a latent variable
- Plausible values used in IRT contexts such as the ETS NAEP, The Nations Report Card (Mislevy et al., 1992)
- Available for both continuous and categorical latent variables (factors, random effects, latent classes)
- More informative than only an estimated factor score and its standard error or a class probability
- Plausible values are more accurate than factor scores
- Plausible values are given for each observation together with a summary over the imputed data sets for each observation and latent variable
- Multiple imputation and plausible values examples are given in the Users Guide, Chapter 11

Features that are not yet implemented include:

- ESEM
- Logit link
- Censored, count, and nominal variables
- XWITH
- Weights
- c ON x in mixtures
- Mixture models with more than one categorical latent variable
- Two-level mixtures
- MODEL INDIRECT
- MODEL CONSTRAINT except for NEW parameters
- MODEL TEST

- Multiple-group and multiple time point analysis with approximate measurement invariance
- 2-level analysis with random loadings
- 3-level analysis with continuous (ML and Bayes) and categorical outcomes (Bayes only)
- Cross-classified analysis (Bayes only)
- EFA
- Factor scores (plausible values)
- Kolmogorov-Smirnov convergence checking

- Bayesian estimation of exploratory factor analysis implemented in Mplus version 7 for models with continuous and categorical variables
- Asparouhov and Muthén (2012). Comparison of computational methods for high dimensional item factor analysis
- Asymptotically the Bayes EFA is the same as the ML solution
- Bayes EFA for categorical variable is a full information estimation method without using numerical integration and therefore feasible with any number of factors
- New in Mplus Version 7: Improved performance of ML-EFA for categorical variables, in particular high-dimensional EFA models with Montecarlo integration; improved unrotated starting values and standard errors

- The first step in the Bayesian estimation is the estimation of the unrotated model as a CFA model using the MCMC method
- Obtain posterior distribution for the unrotated solution
- To obtain the posterior distribution of the rotated parameters we simply rotate the generated unrotated parameters in every MCMC iteration, using oblique or orthogonal rotation
- No priors. Priors could be specified currently only for the unrotated solution
- If the unrotated estimation takes many iterations to converge, use THIN to reduce the number of rotations

- This MCMC estimation is complicated by identification issues that are similar to label switching in the Bayesian estimation of Mixture models
- There are two types of identification issues in the Bayes EFA estimation
- The first type is identification issues related to the unrotated parameters: loading sign switching
- Solution: constrain the sum of the loadings for each factor to be positive. Implemented in Mplus Version 7 for unrotated EFA and CFA. New in Mplus Version 7, leads to improved convergence in Bayesian SEM estimation

$$\sum_{i=1}^p \lambda_{ij} > 0$$

- The second type is identification issues related to the rotated parameters: loading sign switching and order of factor switching
- Solution: Align the signs  $s_j$  and factor order  $\sigma$  to minimize MSE between the current estimates  $\lambda$  and the average estimate from the previous MCMC iterations  $L$

$$\sum_{i,j} (s_j \lambda_{i\sigma(j)} - L_{ij})^2$$

- Minimize over all sign allocations  $s_j$  and factor permutations  $\sigma$

- Factor scores for the rotated solutions also available. Confidence intervals and posterior distribution plots
- Using the optimal rotation in each MCMC iteration we rotate the unrotated factors to obtain the posterior distribution of the rotated factors
- With continuous variables Bayes factor is computed to compare EFA with different number of factors. PPP value is computed with continuous or categorical variables



- Bayes factors is an easy and quick way to compare models using BIC

$$BF = \frac{P(H1)}{P(H0)} = \frac{\text{Exp}(-0.5BIC_{H1})}{\text{Exp}(-0.5BIC_{H0})}$$

- Values of  $BF$  greater than 3 are considered evidence in support of  $H1$
- New in Mplus Version 7: BIC is now included for all models with continuous items (single level and no mixtures)
- The above method can be used to easily compare nested and non-nested models

Absolute bias, coverage and log-likelihood for EFA model with 7 factors and 35 ordered polytomous variables.

Method	$\lambda_{11}$	$\lambda_{12}$	Log-Likelihood
Mplus Monte 500	.01(0.97)	.00(0.83)	-28580.3
Mplus Monte 5000	.01(0.96)	.00(0.87)	-28578.4
Mplus Bayes	.01(.90)	.00(.96)	-
Mplus WLSMV	.00(.94)	.00(.89)	-
IRTPRO MHRM	.00(.54)	.00(.65)	-28665.2

Average standard error, ratio between average standard error and standard deviation for the EFA model with with 7 factors and ordered polytomous variables.

Method	$\lambda_{11}$	$\lambda_{12}$
Mplus Monte 500	0.033(1.00)	0.031(0.72)
Mplus Monte 5000	0.033(0.99)	0.035(0.81)
Mplus Bayes	0.030(0.97)	0.032(0.98)
Mplus WLSMV	0.030(0.97)	0.038(0.85)
IRTPRO MHRM	0.012(0.42)	0.026(0.65)

Bayes EFA is the most accurate full information estimation method for high-dimensional EFA with categorical variables.

Example is based on Mplus User's Guide example 4.1 generated with 4 factors and 12 indicators.

```
DATA: FILE IS ex4.1.dat;  
VARIABLE: NAMES ARE y1-y12;  
ANALYSIS: TYPE = EFA 1 5; estimator=bayes;
```

We estimate EFA with 1, 2, 3, 4 or 5 factors.

Bayes factor results: The posterior probability that the number of factors is 4 is: 99.59%. However, this is a power result - there is enough information in the data to support 4 factors and not enough to support 5 factors. Use BITER = (10000)

```
POSTERIOR PROBABILITIES FOR ALL MODELS:  
1-FACTOR MODEL 0.0000  
2-FACTOR MODEL 0.0000  
3-FACTOR MODEL 0.0041  
4-FACTOR MODEL 0.9959  
5-FACTOR MODEL 0.0000
```

# Bayes EFA: Results

## EXPLORATORY FACTOR ANALYSIS WITH 4 FACTOR(S) :

Number of Free Parameters 66

### Bayesian Posterior Predictive Checking using Chi-Square

95% Confidence Interval for the Difference Between  
the Observed and the Replicated Chi-Square Values

-35.978 41.864

Posterior Predictive P-Value 0.376

### Information Criterion

Deviance (DIC) 3231.767

Estimated number of parameters (pD) 67.580

Bayesian (BIC) 3423.991

Bayes factor for the model with 4 factor(s) against  
the model with 3 factor(s) :

243.6367

Log of Bayes factor for the model with 4 factor(s)  
against the model with 3 factor(s) :

5.4957

# Bayes EFA: Results

GEOMIN ROTATED LOADINGS (\* significant at 5% level)

	1	2	3	4
Y1	0.622*	-0.008	0.028	-0.026
Y2	0.762*	-0.025	0.035	0.010
Y3	0.734*	0.055	-0.038	0.021
Y4	-0.112	0.592*	-0.008	-0.150
Y5	0.021	0.844*	0.049	0.007
Y6	0.053	0.639*	-0.029	-0.007
Y7	-0.009	0.037	0.788*	0.004
Y8	0.032	-0.059	0.646*	-0.115
Y9	0.008	0.040	0.602*	0.095
Y10	-0.042	0.065	0.005	0.671*
Y11	0.013	-0.078	-0.027	0.714*
Y12	0.051	-0.054	0.009	0.710*

- New improved language for factor scores with Bayesian estimation. The same language as for other estimators
- `SAVEDATA: FILE=fs.dat; SAVE=FS(300); FACTORS=factor names;` This command specifies that 300 imputations will be used to estimate the factor scores and that plausible value distributions are available for plotting
- Posterior mean, median, confidence intervals, standard error, all imputed values, distribution plot for each factor score for each latent variable for any model estimated with the Bayes estimator
- Bayes factor score advantages: more accurate than ML factor scores in small sample size, Bayes factor score more accurate in secondary analysis such as for example computing correlations between factor

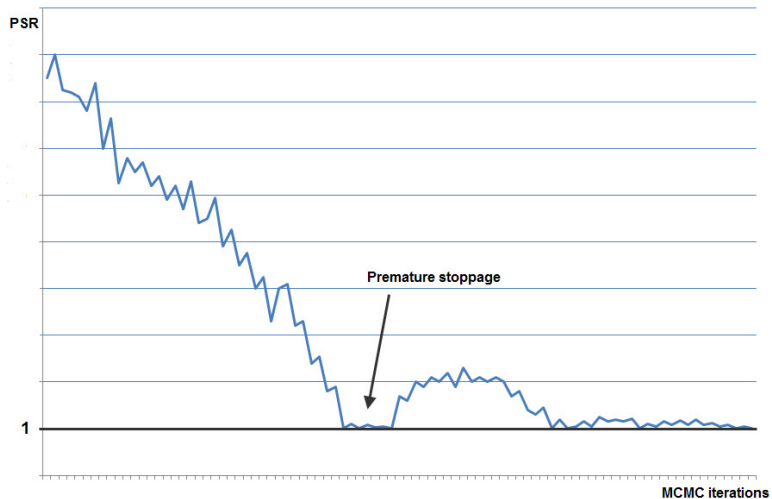


# Bayes Factor Scores Example

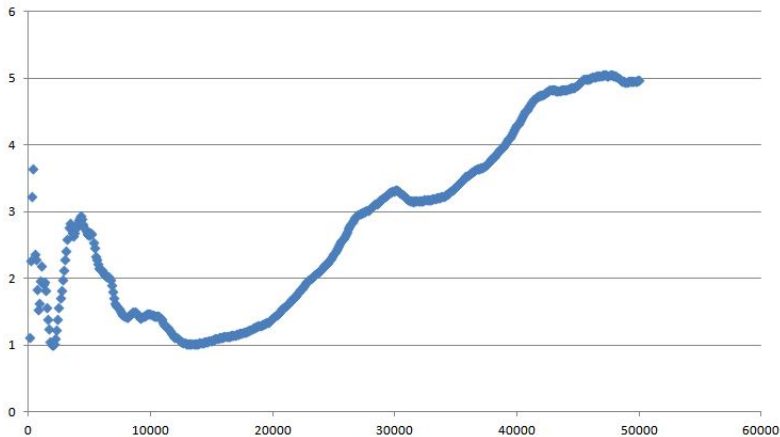
- Asparouhov & Muthén (2010). Plausible values for latent variables using Mplus
- Factor analysis with 3 indicators and 1 factor. Simulated data with  $N=45$ . True factor values are known. Bayes factor score estimates are more accurate. Bayes factor score SE are more accurate
- ML factor scores are particularly unreliable when  $\text{Var}(Y)$  is near 0

	ML	Bayes
MSE	0.636	0.563
Coverage	20%	89%
Average SE	0.109	0.484

# PSR Convergence Issues: Premature Stoppage



# PSR Convergence Issues: Premature Stoppages Due to Non-Identification



- The Mplus default convergence criterion is the Potential Scale Reduction (PSR) criterion. The PSR is not sufficiently strict in certain cases, particularly when the model is not identified and an insufficient number of Bayes draws (iterations) has been used.
- Muthén & Asparouhov (2011). Bayesian SEM: A more flexible representation of substantive theory. With commentaries and rejoinder. Forthcoming in Psychological Methods
- A new more strict test of convergence is now implemented and reported as part of the Tech8 output
- The test is based on computing the Kolmogorov-Smirnov (K-S) test for equal sample distribution
- For each parameter the test uses 100 draws from each of the two MCMC chains and compares the two distributions

# Kolmogorov-Smirnov Convergence Test

- If convergence has been achieved the two distributions should be similar and the K-S test would not reject the hypothesis that the distributions are equal
- The test is based on the K-S statistic

$$D = \sup |F_1(x) - F_2(x)|$$

where  $F_1(x)$  and  $F_2(x)$  are the sample distribution functions for chain 1 and 2

- Mplus computes  $D$  for each parameter and a p-value for the hypothesis that the two chains have the same distribution
- Large values of  $D$  and p-value  $< 0.05$  indicate that convergence has not been achieved.
- Sometimes smaller values should be accepted because the mixing may be too slow to satisfy a strict criterion such as K-S. For more complex models p-values  $> 0.001$  can be interpreted as confirmed convergence.

Consider the following unidentified factor analysis model which has a free factor mean.

```
VARIABLE:  NAMES = y1-y4;  
DATA:      FILE = kolm.dat;  
ANALYSIS:  ESTIMATOR = BAYES;  
MODEL:     f BY y1-y4*1;  
           f@1;  
           y1-y4*.5;  
           [f*0];
```

# Kolmogorov-Smirnov Convergence Test

After 1800 iterations the PSR criterion is satisfied. The K-S test however clearly rejects the convergence and also points out that the unidentified parameters are the indicator means as well as the factor mean.

Parameter	KS Statistic	P-value
Parameter 4	0.3900	0.0000

THE KOLMOGOROV-SMIRNOV DISTRIBUTION TEST FOR PARAMETER 4 HAS A P-VALUE 0.0000, INDICATING DISCREPANT POSTERIOR DISTRIBUTIONS IN THE DIFFERENT MCMC CHAINS. THIS MAY INDICATE NON-CONVERGENCE DUE TO AN INSUFFICIENT NUMBER OF MCMC ITERATIONS OR IT MAY INDICATE A NON-IDENTIFIED MODEL. SPECIFY A LARGER NUMBER OF MCMC ITERATIONS USING THE FBITER OPTION OF THE ANALYSIS COMMAND TO INVESTIGATE THE PROBLEM.

Parameter 1	0.3500	0.0000
Parameter 2	0.3300	0.0000
Parameter 13	0.3300	0.0000
Parameter 3	0.3200	0.0000
Parameter 6	0.1700	0.0994
Parameter 8	0.1500	0.1930
Parameter 12	0.1300	0.3439
Parameter 5	0.1300	0.3439
Parameter 7	0.1300	0.3439
Parameter 11	0.1200	0.4431
Parameter 10	0.1200	0.4431
Parameter 9	0.0900	0.7942

- Using the option `BITER=50000(20000)` we specify that the MCMC chain runs for a minimum of 20000 iterations and a maximum of 50000
- With this option the example does not converge, i.e., the PSR criterion agrees with the K-S criterion
- The PSR criterion tends to accidentally be satisfied if an insufficient number of Bayes draws (iterations) has been used. The K-S criterion should be checked especially when we are not clear if the model is identified.



- Bayes EFA
- BSEM CFA
- Bi-factor EFA
- Bi-factor ESEM
- Bi-factor BSEM

Data are from the classic 1939 factor analysis study by Holzinger and Swineford (1939). Twenty-six tests intended to measure a general factor and five specific factors were administered to seventh and eighth grade students in two schools, the Grant-White school ( $n = 145$ ) and the Pasteur school ( $n = 156$ ). Students from the Grant-White school came from homes where the parents were mostly American-born, whereas students from the Pasteur school came largely from working-class parents of whom many were foreign-born and where their native language was used at home.

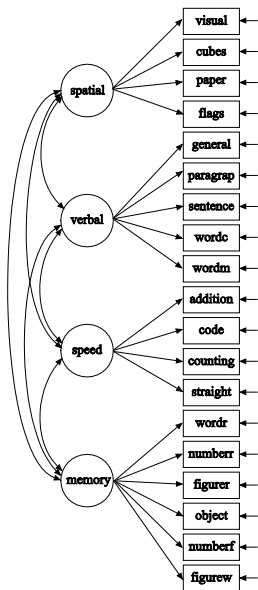
Source:

Holzinger, K. J. & Swineford, F. (1939). A study in factor analysis: The stability of a bi- factor solution. Supplementary Educational Monographs. Chicago, Ill.: The University of Chicago.

# A Factor Analysis Example: Holzinger-Swineford Data, Continued

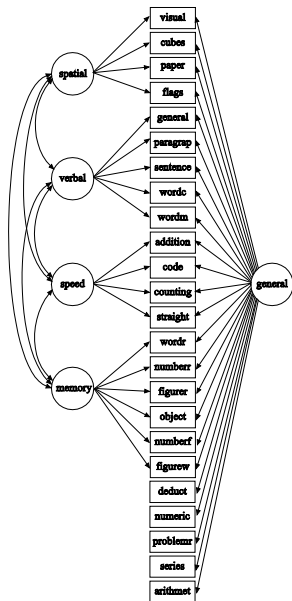
Current analyses:

- 19 tests hypothesized to measure four mental abilities: Spatial, verbal, speed, and memory
- 5 tests measuring a general ability (deduction, test taking ability)
- n=145 7th and 8th grade students from Grant-White elementary school
- n=156 7th and 8th grade students from the Pasteur elementary school



## CFA Factor Loading Pattern: 19 Variables

	Spatial	Verbal	Speed	Memory
visual	x	0	0	0
cubes	x	0	0	0
paper	x	0	0	0
flags	x	0	0	0
general	0	x	0	0
paragraf	0	x	0	0
sentence	0	x	0	0
wordc	0	x	0	0
wordm	0	x	0	0
addition	0	0	x	0
code	0	0	x	0
counting	0	0	x	0
straight	0	0	x	0
wordr	0	0	0	x
numberr	0	0	0	x
figurer	0	0	0	x
object	0	0	0	x
numberf	0	0	0	x
figurew	0	0	0	x



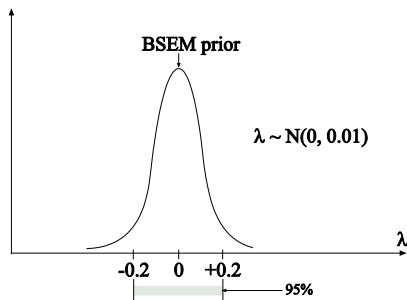
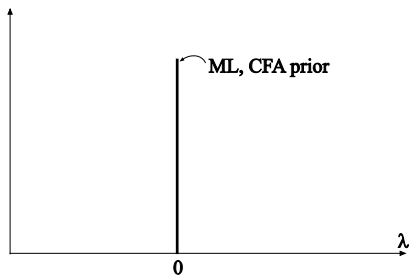
	General	Spatial	Verbal	Speed	Memory
visual	x	x	0	0	0
cubes	x	x	0	0	0
paper	x	x	0	0	0
flags	x	x	0	0	0
general	x	0	x	0	0
paragrap	x	0	x	0	0
sentence	x	0	x	0	0
wordc	x	0	x	0	0
wordm	x	0	x	0	0
addition	x	0	0	x	0
code	x	0	0	x	0
counting	x	0	0	x	0
straight	x	0	0	x	0
wordr	x	0	0	0	x
numberr	x	0	0	0	x
figurer	x	0	0	0	x
object	x	0	0	0	x
numberf	x	0	0	0	x
figurew	x	0	0	0	x
deduct	x	0	0	0	0
numeric	x	0	0	0	0
problemr	x	0	0	0	0
series	x	0	0	0	0
arithmet	x	0	0	0	0

Muthén & Asparouhov (2010). Bayesian SEM: A more flexible representation of substantive theory. Forthcoming in Psychological Methods, September 2012

- The BSEM paper
- 2 commentaries and a rejoinder
- Uses informative priors to estimate parameters that are not identified in ML

# ML CFA versus BESEM CFA

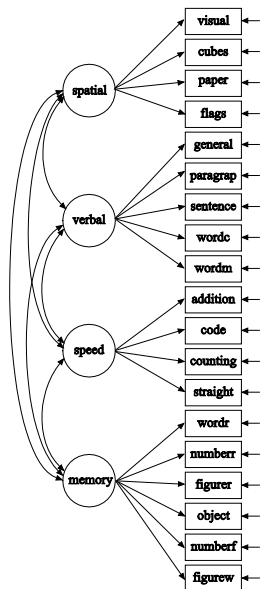
- ML CFA uses a very strong prior with an exact zero loading
- BSEM uses a zero-mean, small-variance prior for the loading:



BSEM can be used to specify approximate zeros for

- Cross-loadings
- Residual correlations
- Direct effects from covariates
- Group and time differences in intercepts and loadings

# BSEM CFA vs ML CFA: Holzinger-Swineford 19 Variables



## CFA Factor Loading Pattern:

	Spatial	Verbal	Speed	Memory
visual	x	0	0	0
cubes	x	0	0	0
paper	x	0	0	0
flags	x	0	0	0
general	0	x	0	0
paragraf	0	x	0	0
sentence	0	x	0	0
wordc	0	x	0	0
wordm	0	x	0	0
addition	0	0	x	0
code	0	0	x	0
counting	0	0	x	0
straight	0	0	x	0
wordr	0	0	0	x
numberr	0	0	0	x
figurer	0	0	0	x
object	0	0	0	x
numberf	0	0	0	x
figurew	0	0	0	x



# ML CFA Testing Results For Holzinger-Swineford Data For Grant-White (n =145) And Pasteur (n=156)

Model	$\chi^2$	df	P-value	RMSEA	CFI
Grant-White					
CFA	216	146	0.000	0.057	0.930
EFA	110	101	0.248	0.025	0.991
Pasteur					
CFA	261	146	0.000	0.071	0.882
EFA	128	101	0.036	0.041	0.972

EFA has 6 (Grant-White) and 9 (Pasteur) significant cross-loadings

	Grant-White Factor Loading Patterns For EFA				Pasteur Factor Loading Pattern For EFA			
	Spatial	Verbal	Speed	Memory	Spatial	Verbal	Speed	Memory
visual	<b>0.628*</b>	0.065	0.091	0.085	<b>0.580*</b>	0.307*	-0.001	0.053
cubes	<b>0.485*</b>	0.050	0.007	-0.003	<b>0.521*</b>	0.027	-0.078	-0.059
paper	<b>0.406*</b>	0.107	0.084	0.083	<b>0.484*</b>	0.101	-0.016	-0.229*
flags	<b>0.579*</b>	0.160	0.013	0.026	<b>0.687*</b>	-0.051	0.067	0.101
general	0.042	<b>0.752*</b>	0.126	-0.051	-0.043	<b>0.838*</b>	0.042	-0.118
paragrap	0.021	<b>0.804*</b>	-0.056	0.098	0.026	<b>0.800*</b>	-0.006	0.069
sentence	-0.039	<b>0.844*</b>	0.085	-0.057	-0.045	<b>0.911*</b>	-0.054	-0.029
wordc	0.094	<b>0.556*</b>	0.197*	0.019	0.098	<b>0.695*</b>	0.008	0.083
wordm	0.004	<b>0.852*</b>	-0.074	0.069	0.143*	<b>0.793*</b>	0.029	-0.023
addition	-0.302*	0.029	<b>0.824*</b>	0.078	-0.247*	0.067	<b>0.664*</b>	0.026
code	0.012	0.050	<b>0.479*</b>	0.279*	0.004	0.262*	<b>0.552*</b>	0.082
counting	0.045	-0.159	<b>0.826*</b>	-0.014	0.073	-0.034	<b>0.656*</b>	-0.166
straight	0.346*	0.043	<b>0.570*</b>	-0.055	0.266*	-0.034	<b>0.526*</b>	-0.056
wordr	-0.024	0.117	-0.020	<b>0.523*</b>	-0.005	0.020	-0.039	<b>0.726*</b>
numberr	0.069	0.021	-0.026	<b>0.515*</b>	-0.026	-0.057	-0.057	<b>0.604*</b>
figurer	0.354*	-0.033	-0.077	<b>0.515*</b>	0.329*	0.042	0.168	<b>0.403*</b>
object	-0.195	0.045	0.154	<b>0.685*</b>	-0.123	-0.005	0.333*	<b>0.469*</b>
numberf	0.225	-0.127	0.246*	<b>0.450*</b>	-0.014	0.092	0.092	<b>0.427*</b>
figurew	0.069	0.099	0.058	<b>0.365*</b>	0.139	0.013	0.237*	<b>0.291*</b>

- CFA: Cross-loadings fixed at zero - the model is rejected
- A more realistic hypothesis: Small cross-loadings allowed
- Cross-loadings are not all identified in terms of ML
- Different alternative: Bayesian CFA with informative priors for cross-loadings:  $\lambda \sim N(0, 0.01)$ .

This means that 95% of the prior is in the range -0.2 to 0.2

---

VARIABLE: NAMES = id female grade agey agem school  
! grade = 7/8  
! school = 0/1 for Grant-White/Pasteur  
visual cubes paper flags general paragra<sup>p</sup> sentence wordc  
wordm addition code counting straight wordr numberr figurer  
object numberf figurew deduct numeric problemr series arith-  
met;  
USEV = visual-figurew;  
USEOBS = school eq 0;  
DEFINE: STANDARDIZE visual-figurew;  
ANALYSIS: ESTIMATOR = BAYES;  
PROCESSORS = 2;  
FBITER = 10000;

---

# Input BSEM CFA 19 Items 4 Factors Crossloading Priors (Continued)

---

MODEL:        spatial BY visual\* cubes paper flags;  
              verbal BY general\* paragraf sentence wordc wordm;  
              speed BY addition\* code counting straight;  
              memory BY wordr\* numberr figurer object numberf figurew;  
              spatial-memory@1;  
              ! cross-loadings:  
              spatial BY general-figurew\*0 (a1-a15);  
              verbal BY visual-flags\*0 (b1-b4);  
              verbal BY addition-figurew\*0 (b5-b14);  
              speed BY visual-wordm\*0 (c1-c9);  
              speed BY wordr-figurew\*0 (c10-c15);  
              memory BY visual-straight\*0 (d1-d13);

MODEL PRIORS:

a1-d13 ~ N(0,.01);

OUTPUT:        TECH1 TECH8 STDY;

PLOT:            TYPE = PLOT2;

---

## ML analysis

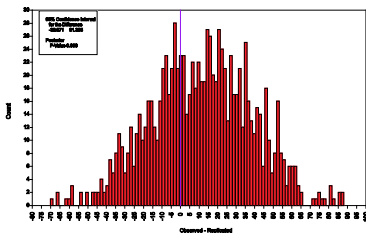
Model	$\chi^2$	Df	P-value	RMSEA	CFI
Grant-White					
CFA	216	146	0.000	0.057	0.930
EFA	110	101	0.248	0.025	0.991
Pasteur					
CFA	261	146	0.000	0.071	0.882
EFA	128	101	0.036	0.041	0.972

## Bayesian analysis

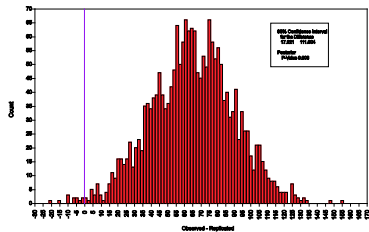
Model	Sample LRT	2.5% PP limit	97.5% PP limit	PP p-value
Grant-White				
CFA	219	12	112	0.006
CFA w/ cross-loadings	142	-39	61	0.361
Pasteur				
CFA	264	56	156	0.000
CFA w/ cross-loadings	156	-28	76	0.162

# Bayesian Posterior Predictive Checking For The CFA Model For Grant-White

CFA with small cross-loadings  
not rejected by Bayes PPC:  
 $p = 0.361$



Conventional CFA model  
rejected by Bayes PPC:  
 $p = 0.006$



	Grant-White Factor Loadings Using Informative Priors				Pasteur Factor Loadings Using Informative Priors			
	Spatial	Verbal	Speed	Memory	Spatial	Verbal	Speed	Memory
visual	<b>0.640*</b>	0.012	0.050	0.047	<b>0.633*</b>	0.145	0.027	0.039
cubes	<b>0.521*</b>	-0.008	-0.010	-0.012	<b>0.504*</b>	-0.027	-0.041	-0.030
paper	<b>0.456*</b>	0.040	0.041	0.047	<b>0.515*</b>	0.018	-0.024	-0.118
flags	<b>0.672*</b>	0.046	-0.020	0.005	<b>0.677*</b>	-0.095	0.026	0.093
general	0.037	<b>0.788*</b>	0.049	-0.040	-0.056	<b>0.856*</b>	0.027	-0.084
paragrap	-0.001	<b>0.837*</b>	-0.053	0.030	0.015	<b>0.801*</b>	-0.011	0.050
sentence	-0.045	<b>0.885*</b>	0.021	-0.055	-0.063	<b>0.925*</b>	-0.032	-0.036
wordc	0.053	<b>0.612*</b>	0.096	0.029	0.055	<b>0.694*</b>	0.013	0.063
wordm	-0.012	<b>0.886*</b>	-0.086	0.020	0.092	<b>0.803*</b>	0.001	0.012
addition	-0.172*	0.030	<b>0.795*</b>	0.004	-0.147	-0.004	<b>0.655*</b>	0.010
code	-0.002	0.054	<b>0.560*</b>	0.130	-0.004	0.111	<b>0.655*</b>	0.049
counting	0.013	-0.092	<b>0.828*</b>	-0.049	0.025	-0.058	<b>0.616*</b>	-0.057
straight	0.189*	0.043	<b>0.633*</b>	-0.035	0.132	-0.067	<b>0.558*</b>	0.001
wordr	-0.040	0.044	-0.031	<b>0.556*</b>	-0.058	0.006	-0.090	<b>0.731*</b>
numberr	0.003	-0.004	-0.038	<b>0.552*</b>	0.006	-0.098	-0.106	<b>0.634*</b>
figurer	0.132	-0.024	-0.049	<b>0.573*</b>	0.156*	0.027	0.064	<b>0.517*</b>
object	-0.139	0.014	0.029	<b>0.724*</b>	-0.097	0.007	0.122	<b>0.545*</b>
numberf	0.099	-0.071	0.095	<b>0.564*</b>	-0.029	0.041	0.003	<b>0.474*</b>
figurew	0.012	0.045	0.007	<b>0.445*</b>	0.049	0.018	0.085	<b>0.397*</b>

Number of significant cross-loadings: 2 for Grant-White and 1 for Pasteur



# Effects Of Using Different Variances For The Informative Priors Of The Cross-Loadings For The Holzinger-Swineford Data: Grant-White

Prior variance	95% cross-loading limit	PPP	Cross-loading (Posterior SD)	Factor corr. range
0.01	0.02	0.361	0.189 (.078)	0.443-0.557
0.02	0.28	0.441	0.248 (.096)	0.439-0.542
0.03	0.34	0.457	0.275 (.109)	0.432-0.530
0.04	0.39	0.455	0.292 (.120)	0.413-0.521
0.05	0.44	0.453	0.303 (.130)	0.404-0.513
0.06	0.48	0.447	0.309 (.139)	0.400-0.510
0.07	0.52	0.439	0.315 (.148)	0.395-0.508
0.08	0.55	0.439	0.319 (.156)	0.387-0.508
0.09	0.59	0.435	0.323 (.163)	0.378-0.506
1.00	0.62	0.427	0.327 (.171)	0.369-0.504

# Summary of Analyses of Holzinger-Swineford 19-Variable Data

- Conventional, frequentist, CFA model rejected
- Bayesian CFA with informative cross-loadings not rejected
- The Bayesian approach uses an intermediate hypothesis:
  - Less strict than conventional CFA
  - Stricter than EFA, where the hypothesis only concerns the number of factors
  - Cross-loadings shrunken towards zero; acceptable degree of shrinkage monitored by PPP
- Bayes modification indices obtained by estimated cross-loadings
- Factor correlations:  $EFA < BSEM < CFA$

- Target rotation: EFA rotation chosen to match zero target loadings using least-squares fitting
  - Similarities: Replaces mechanical rotation with judgement/hypotheses
  - Differences: For Target, specifying more than the necessary EFA restrictions does not affect fit and user-defined closeness to zero is replaced with least-squares fitting
- Results for Holzinger-Swineford data:
  - Results similar to EFA with 10 significant cross-loadings for Grant-White and 15 for Pasteur

- ESEM: Structural equation modeling with EFA measurement model (Asparouhov & Muthén (2009). Exploratory structural equation modeling. Structural Equation Modeling, 16, 397-438)
- Similarities: Both ESEM and BSEM can be used for measurement models in SEM
- Differences:
  - ESEM is EFA-oriented while BSEM is CFA-oriented
  - ESEM uses a mechanical rotation and the rotation is not based on information from other parts of the model
  - BSEM is applicable not only to measurement models

As popular today as in 1939.

Gibbons, & Hedeker, (1992). Full-information item bi-factor analysis. *Psychometrika*, 57, 423-436.

Reise, Morizot, & Hays (2007). The role of the bifactor model in resolving dimensionality issues in health outcomes measures. *Quality of Life Research*, 16, 1931.

Cai (2010). A two-tier full-information item factor analysis model with applications. *Psychometrika*, 75, 581-612.

Cai, Yang, Hansen (2011). Generalized full-information item bifactor analysis. *Psychological Methods*, 16, 221-248.

- Testlet modeling, e.g. for PISA test items
- Longitudinal modeling with across-time correlation for residuals of the same item

New methods that do not use regular ML CFA:

- Bi-factor EFA (Jennrich & Bentler)
  - ROTATION = BI-GEOMIN
- Bi-factor ESEM
  - ROTATION = BI-GEOMIN (same as above)
  - Bi-factor ESEM with general CFA factor and ROTATION = GEOMIN for specific factors
- Bi-factor BSEM (no rotation)

```
TITLE:      this is an example of a bi-factor  
            exploratory factor analysis with  
            continuous factor indicators  
DATA:      FILE = ex4.7.dat;  
VARIABLE:  NAMES = y1-y10;  
ANALYSIS:  TYPE = EFA 2 3;  
            ROTATION = BI-GEOMIN;
```

The number of factors is the general factor plus the specific factors

```
TITLE:      this is an example of a bi-factor EFA
DATA:       FILE = ex5.29.dat;
VARIABLE:   NAMES = y1-y10;
ANALYSIS:   ROTATION = BI-GEOMIN;
MODEL:      fg f1 f2 BY y1-y10 (*1);
OUTPUT:     STDY;
```

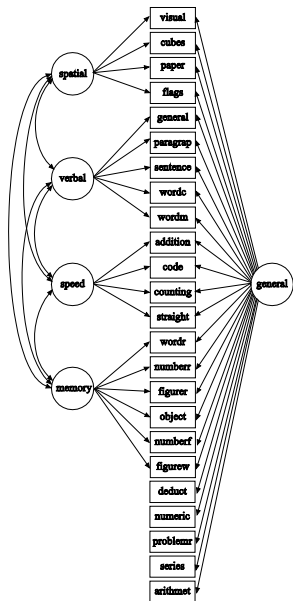


# Bi-Factor ESEM with CFA Factor and Regular Rotation: UG Ex5.30

```
TITLE:      this is an example of bi-factor EFA with
            two items loading on only the general
            factor
DATA:       FILE = ex5.30.dat;
VARIABLE:   NAMES = y1-y10;
ANALYSIS:   ROTATION = GEOMIN;
MODEL:      fg BY y1-y10*;
            fg@1;
            f1-f2 BY y1-y8 (*1);
            fg WITH f1-f2@0;
OUTPUT:     STDY;
```

```
TITLE:      this is an example of a Bayesian bi-factor
             CFA with two items loading on only the
             general factor and cross-loadings with
             zero-mean and small-variance priors

DATA:       FILE = ex5.31.dat;
VARIABLE:   NAMES = y1-y10;
ANALYSIS:   ESTIMATOR = BAYES;
             PROCESSORS = 2;
MODEL:      fg BY y1-y10*;
             fg@1;
             f1 BY y1-y4
             y5-y10 (f1xlam5-f1xlam10);
             f2 BY y5-y8
             y1-y4 y9-y10(f2xlam1-f2xlam6);
             fg WITH f1-f2@0;
MODEL PRIORS:
             f1xlam5-f2xlam6~N(0,0.01);
PLOT:       TYPE = PLOT2;
```



	General	Spatial	Verbal	Speed	Memory
visual	x	x	0	0	0
cubes	x	x	0	0	0
paper	x	x	0	0	0
flags	x	x	0	0	0
general	x	0	x	0	0
paragrap	x	0	x	0	0
sentence	x	0	x	0	0
wordc	x	0	x	0	0
wordm	x	0	x	0	0
addition	x	0	0	x	0
code	x	0	0	x	0
counting	x	0	0	x	0
straight	x	0	0	x	0
wordr	x	0	0	0	x
numberr	x	0	0	0	x
figurer	x	0	0	0	x
object	x	0	0	0	x
numberf	x	0	0	0	x
figurew	x	0	0	0	x
deduct	x	0	0	0	0
numeric	x	0	0	0	0
problemr	x	0	0	0	0
series	x	0	0	0	0
arithmet	x	0	0	0	0

# Holzinger-Swineford, 24 Variables: Input Excerpts for Bi-Factor EFA

```
ANALYSIS:  USEVARIABLES = visual - arithmet;  
           USEOBSERVATIONS = school EQ 0;  
           TYPE = EFA 5 5;  
           ROTATION = BI-GEOMIN;
```

# Bi-Factor EFA for Holzinger-Swineford's 24-variable Grant-White data

	General	Spatial	Verbal	Speed	Memory
visual	<b>0.621*</b>	<b>0.384*</b>	-0.065	0.072	0.002
cubes	<b>0.433*</b>	<b>0.207</b>	-0.103	-0.115	-0.118
paper	<b>0.430*</b>	<b>0.343*</b>	0.058	0.225	0.079
flags	<b>0.583*</b>	<b>0.311*</b>	-0.028	-0.077	-0.109
general	<b>0.610*</b>	-0.034	<b>0.524*</b>	0.001	-0.075
paragrap	<b>0.554*</b>	0.053	<b>0.618*</b>	0.012	0.102
sentence	<b>0.572*</b>	-0.037	<b>0.622*</b>	0.010	-0.064
wordc	<b>0.619*</b>	0.006	<b>0.354*</b>	0.038	-0.048
wordm	<b>0.582*</b>	-0.008	<b>0.603*</b>	-0.137	0.009
addition	<b>0.508*</b>	-0.528	-0.036	<b>0.327</b>	0.009
code	<b>0.532*</b>	-0.031	0.046	<b>0.428*</b>	0.310*
counting	<b>0.568*</b>	-0.229	-0.216*	<b>0.302</b>	-0.093
straight	<b>0.643*</b>	0.217	0.004	<b>0.526*</b>	-0.032

# Bi-Factor EFA for Holzinger-Swineford, Continued

	General	Spatial	Verbal	Speed	Memory
wordr	<b>0.349*</b>	0.018	0.077	0.032	<b>0.475*</b>
numberr	<b>0.352*</b>	0.037	-0.041	-0.052	<b>0.392*</b>
figurer	<b>0.495*</b>	0.221	-0.122	-0.033	<b>0.384*</b>
object	<b>0.422*</b>	-0.200	-0.010	-0.021	<b>0.497*</b>
numberf	<b>0.553*</b>	-0.041	-0.220*	0.003	<b>0.256*</b>
figurew	<b>0.414*</b>	-0.033	-0.003	-0.024	<b>0.246*</b>
deduct	<b>0.611*</b>	-0.001	0.089	-0.284*	0.036
numeric	<b>0.656*</b>	-0.021	-0.129	0.029	-0.023
problemr	<b>0.607*</b>	0.028	0.091	-0.227*	0.059
series	<b>0.714*</b>	0.023	0.034	-0.202	-0.067
arithmet	<b>0.638*</b>	-0.356*	0.092	-0.009	0.070

6 significant cross-loadings

## BI-GEOMIN Factor Correlations

---

	General	Spatial	Verbal	Speed	Memory
General	1.000				
Spatial	0.000	1.000			
Verbal	0.000	0.022	1.000		
Speed	0.000	-0.223*	-0.122*	1.000	
Memory	0.000	-0.037	0.068	-0.134	1.000

---

# Input for Bi-Factor ESEM with a General CFA Factor and Regular Rotation of the Specific Factors

```
ANALYSIS:  USEVARIABLES = visual-arithmet;  
           USEOBSERVATIONS = school EQ 0;  
           ROTATION = GEOMIN;  
MODEL:     fg BY visual-arithmet*;  
           fg@1;  
           s1-s4 BY visual-figurew (*1);  
           fg WITH s1-s4@0;  
OUTPUT:    STDY;
```



# Bi-Factor ESEM with a General CFA Factor and Regular Rotation of the Specific Factors

## STDY Standardization

	General	Spatial	Verbal	Speed	Memory
visual	<b>0.574*</b>	<b>0.450*</b>	-0.081	0.087	0.012
cubes	<b>0.425*</b>	<b>0.277*</b>	-0.140	-0.076	-0.132
paper	<b>0.382*</b>	<b>0.352*</b>	0.067	0.151	0.109
flags	<b>0.574*</b>	<b>0.356*</b>	-0.070	-0.057	-0.121
general	<b>0.652*</b>	-0.013	<b>0.460*</b>	0.034	-0.084
paragrap	<b>0.604*</b>	0.038	<b>0.580*</b>	-0.031	0.114
sentence	<b>0.614*</b>	-0.018	<b>0.574*</b>	0.042	-0.053
wordc	<b>0.628*</b>	0.040	<b>0.315*</b>	0.106	-0.032
wordm	<b>0.657*</b>	-0.042	<b>0.539*</b>	-0.126	0.001
addition	<b>0.448*</b>	-0.323*	-0.060	<b>0.537*</b>	0.010
code	<b>0.468*</b>	0.037	0.055	<b>0.450*</b>	0.313*
counting	<b>0.487*</b>	-0.059	-0.252*	<b>0.554*</b>	-0.112
straight	<b>0.517*</b>	0.394*	0.021	<b>0.606*</b>	-0.014

# Bi-Factor ESEM with a General CFA Factor and Regular Rotation of the Specific Factors, Continued

## STDY Standardization

	General	Spatial	Verbal	Speed	Memory
wordr	<b>0.368*</b>	-0.049	0.081	-0.006	<b>0.464*</b>
numberr	<b>0.378*</b>	-0.047	-0.051	-0.069	<b>0.365*</b>
figurer	<b>0.489*</b>	0.202	-0.116	-0.072	<b>0.390*</b>
object	<b>0.459*</b>	-0.270*	-0.033	0.034	<b>0.463*</b>
numberf	<b>0.549*</b>	0.005	-0.261*	0.078	<b>0.207</b>
figurew	<b>0.423*</b>	-0.018	-0.015	-0.004	<b>0.235*</b>
deduct	<b>0.643*</b>				
numeric	<b>0.612*</b>				
problemr	<b>0.638*</b>				
series	<b>0.717*</b>				
arithmet	<b>0.643*</b>				

6 significant cross-loadings

# Input for Bi-Factor BSEM for Holzinger-Swineford

```
DATA:          FILE = H-S Combined.txt;
VARIABLE:      NAMES = id female grade agey agem school
               visual cubes paper flags general paraprap
               sentence wordc wordm addition code counting
               straight wordr numberr figurer object numberf
               figurew deduct numeric problemr series arithmet;
               USEVARIABLES = visual-arithmet;
               USEOBSERVATIONS = school EQ 0;
DEFINE:        STANDARDIZE visual-arithmet;
ANALYSIS:      ESTIMATOR = BAYES;
               PROCESSORS = 2;
               FBITER = 15000;
MODEL:         g BY visual-arithmet*;
               spatial BY visual-flags*
               general-arithmet*0 (s1-s20);
```

```
verbal BY visual-flags*0 (v1-v4)
general-wordm*
addition-arithmet*0 (v5-v19);
speed BY visual-wordm*0 (sp1-sp9)
addition-straight*
wordr-arithmet*0 (sp10-sp20);
memory BY visual-straight*0 (m1-m13)
wordr-figurew*
deduct-arithmet*0 (m14-m18);
g-memory@1;
g-memory WITH g-memory@0;
```

## MODEL PRIORS:

```
s1-s20~N(0,0.01);
v1-v19~N(0,0.01);
sp1-sp20~N(0,0.01);
m1-m18~N(0,0.01);
```

## OUTPUT:

```
TECH1 TECH8 STDYX;
```

## PLOT:

```
TYPE = PLOT3;
```

# Bi-Factor BSEM

## STDY Standardization

	General	Spatial	Verbal	Speed	Memory
visual	<b>0.615*</b>	<b>0.387*</b>	-0.014	0.003	0.003
cubes	<b>0.434*</b>	<b>0.271</b>	-0.041	-0.040	-0.040
paper	<b>0.409*</b>	<b>0.359*</b>	0.057	0.061	0.057
flags	<b>0.583*</b>	<b>0.354*</b>	0.017	-0.053	-0.033
general	<b>0.603*</b>	0.021	<b>0.537*</b>	0.040	-0.047
paragrap	<b>0.577*</b>	0.009	<b>0.598*</b>	-0.039	0.051
sentence	<b>0.576*</b>	-0.025	<b>0.604*</b>	0.009	-0.045
wordc	<b>0.619*</b>	0.032	<b>0.361*</b>	0.052	-0.016
wordm	<b>0.618*</b>	-0.022	<b>0.593*</b>	-0.102	0.000
addition	<b>0.463*</b>	-0.194*	0.009	<b>0.641*</b>	0.000
code	<b>0.521*</b>	0.001	0.047	<b>0.398*</b>	0.135
counting	<b>0.505*</b>	0.004	-0.101	<b>0.542*</b>	-0.042
straight	<b>0.587*</b>	0.191*	0.029	<b>0.432*</b>	-0.029

# Bi-Factor BSEM, Continued

## STDY Standardization

	General	Spatial	Verbal	Speed	Memory
wordr	<b>0.385*</b>	-0.021	0.050	-0.004	<b>0.440*</b>
numberr	<b>0.375*</b>	0.013	-0.002	-0.021	<b>0.386*</b>
figurer	<b>0.529*</b>	0.102	-0.057	-0.051	<b>0.348*</b>
object	<b>0.460*</b>	-0.128	0.016	0.050	<b>0.463*</b>
numberf	<b>0.560*</b>	0.028	-0.104	0.083	<b>0.267*</b>
figurew	<b>0.461*</b>	-0.030	-0.003	0.002	<b>0.190</b>
deduct	<b>0.662*</b>	0.002	0.068	-0.105	0.029
numeric	<b>0.672*</b>	0.044	-0.067	0.099	-0.023
problemr	<b>0.665*</b>	-0.002	0.054	-0.099	0.017
series	<b>0.741*</b>	0.048	0.040	-0.044	-0.032
arithmet	<b>0.674*</b>	-0.152*	0.066	0.159	0.022

3 significant cross-loadings, all for the Spatial factor.

# Summary of Bi-Factor Analysis for Holzinger-Swineford Data for the Grant-White School

- Bi-factor CFA does not fit
- EFA with bi-factor rotation: 6 significant cross-loadings
- Bi-factor ESEM with general CFA factor and regular rotation for the specific factors: 6 significant cross-loadings
- Bi-factor BSEM with no rotation: 3 significant cross-loadings

Muthén & Asparouhov (2012): Rejoinder: Mastering a New Method suggests that BSEM bi-factor analysis outperforms all other bi-factor analyses of the classic Holzinger-Swineford data

- New in Mplus Version 7: Two-tier algorithm. New algorithm that reduces the dimension of integration with ML estimation of SEM with categorical variables
- Multiple factors that load on different indicators and are uncorrelated are reduced to 1 dimensional integration



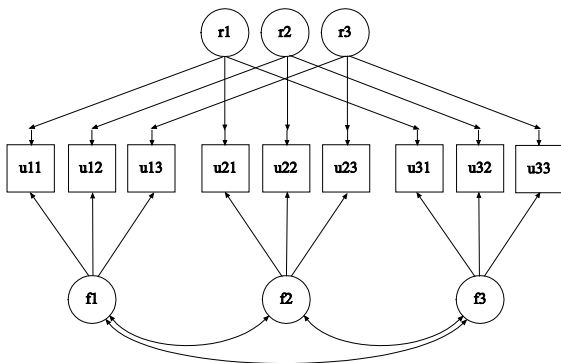
Table 1: Absolute bias, coverage and log-likelihood for the bifactor model.

Method	$\lambda_{11}$	$\lambda_{12}$	$\tau_{1,1}$	Log-Likelihood
Mplus Two Tier	.02(.95)	.02(.97)	.02(.97)	-31664.1
Mplus Monte 500	.03(.92)	.07(.92)	.02(.94)	-31760.8
Mplus Monte 5000	.03(.94)	.01(.95)	.01(.96)	-31678.5
Mplus Bayes	.11(.89)	.07(.93)	.16(.89)	-
Mplus WLSMV	.06(.93)	.05(.95)	.13(.93)	-
IRTPRO Two Tier	.02(.98)	.00(.96)	.01(.98)	-31680.7

Table 2: Average standard error, ratio between average standard error and standard deviation for the bifactor model.

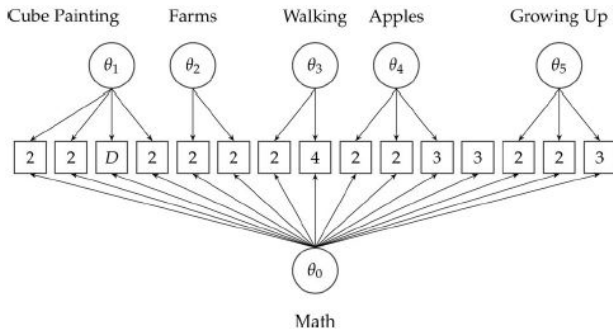
Method	$\lambda_{11}$	$\lambda_{12}$	$\tau_{1,1}$
Mplus Two Tier	0.12(0.97)	0.16(1.00)	0.19(1.10)
Mplus Monte 500	0.12(0.91)	0.15(0.99)	0.18(1.05)
Mplus Monte 5000	0.12(0.89)	0.15(0.94)	0.19(1.09)
Mplus Bayes	0.13(0.97)	0.16(1.01)	0.19(1.09)
Mplus WLSMV	0.13(0.97)	0.15(0.98)	0.18(1.08)
IRTPRO Two Tier	0.17(1.47)	0.16(1.01)	0.18(1.02)

# Two-Tier Modeling: A Longitudinal Example



With categorical outcomes, ML estimation leads to numerical integration that is difficult with many dimensions/factors. The two-tier algorithm reduces the 6 dimensions of integration to 4. Bi-factor CFA has a similar structure that benefits from two-tier computing.

# Two-Tier Modeling: Bi-Factor Model for PISA Math Items



The two-tier algorithm reduces the 6 dimensions of integration to 2.

Cai, Yang, & Hansen (2011) Generalized full-information item bifactor analysis. *Psychological Methods*, 16, 221-248

# Two-Tier Estimation

- The two-tier integration method can be estimated in Mplus prior to Version 7 as a two-level multiple group model where the general factors are between level factors, while the specific factors are within level factors and the multiple groups represent the different blocks of variables that are correlated beyond the general factors
- In Mplus Version 7 this is no longer necessary and the two-tier estimation can be used when the model is set up as a regular single level model
- The program will automatically determine if the model is a bifactor-like model that allows for more optimal two-tier integration and will set up the two-level multiple group model automatically. On the surface you can not tell if the model is estimated through a special two-level setup

# Advances in Multiple-Group Analysis: Invariance Across Groups

- An old dilemma
- Two new solutions

- Fixed mode:
  - Inference to only the groups in the sample
  - Small to medium number of groups
- Random mode:
  - Inference to a population of groups from which the current set of groups is a random sample
  - Medium to large number of groups

- New solution no. 1, suitable for a small to medium number of groups
  - A new BSEM approach where group is a fixed mode:  
Multiple-group BSEM
  - Approximate invariance allowed
- New solution no. 2, suitable for a medium to large number of groups
  - A new Bayes approach where group is a random mode
  - No limit on the number of groups



# New Options to Facilitate Multiple-Group BSEM (New Solution No. 1)

- COV
- DIFF
- DO DIFF
- MODEL=ALLFREE
- Auto labeling

When choosing Bayesian priors, parameters are often specified as independent, but can also be allowed to covary. This is accomplished with the COVARIANCE option in MODEL PRIORS. An example:

MODEL:

y ON x1(a)

x2(b);

**MODEL PRIORS:**

a ~ N(10, 4);

b ~ N(6, 1);

**COV(a,b)=0.5;**

which says that the prior bivariate distribution of a and b has a covariance of 0.5, which translates to a correlation of 0.25:

$$\frac{0.5}{\sqrt{(4)} \sqrt{(1)}} = 0.25$$

The COVARIANCE option can also be used to specify small differences between parameters. Note that

$$V(a - b) = V(a) + V(b) - 2 \text{cov}(a, b), \quad (4)$$

so that if  $V(a) = V(b) = 1000$ , using  $\text{cov}(a, b) = 999.995$  gives  $V(a - b) = 0.01$ . With a normal distribution, this means that the difference has a 95% chance of being between  $-0.196$  and  $+0.196$ , that is, in a small range around the zero mean.

Example: Two parameters  $a$  and  $b$  for which we want to apply zero-mean, small-variance priors

$a - b \sim N(0, 1000)$ ; ! non-informative priors  
COV(a, b)=999.995;

Used with MODEL PRIORS in Bayesian analysis to simplify specifying differences between parameters.

Example: The difference between the parameters a and b

$\text{DIFF}(a, b) \sim N(0, 0.01);$

This is the same as the two statements

$a - b \sim N(0, 1000);$  ! non-informative prior  
 $\text{COV}(a, b) = 999.995;$

DO DIFF is used to express parameter differences between large sets of parameters and groups/timepoints.

Example: Group differences for 4 parameters in 3 groups. Let lam<sub>jk</sub> denote a factor loading for group/timepoint *j* and variable *k*:

DO(1,4) DIFF(lam1# – lam3#) ~ N(0,0.01);

! for variable 1 this results in approximate invariance across the 3 groups:

! lam<sub>11</sub> ≈ lam<sub>21</sub>

! lam<sub>11</sub> ≈ lam<sub>31</sub>

! lam<sub>21</sub> ≈ lam<sub>31</sub>

! etc. for the lam parameters for variables 2-4

# Auto Labeling Option for Multiple Groups

Used in conjunction with DO DIFF for multiple-group Bayes, which is carried out using TYPE=MIXTURE and KNOWNCLASS. For instance, with a factor measured by 4 variables in 3 groups:

**ANALYSIS:** TYPE = MIXTURE; ! 3 classes

ESTIMATOR = BAYES;

PROCESSORS = 2;

MODEL = ALLFREE;

**MODEL:**

%OVERALL%

f BY y1-y4\* (lam#\_1 – lam#\_4);

! the above gives labels for all 3 groups (group is #)

**MODEL PRIORS:**

DO(1,4) DIFF(lam1\_# – lam3\_#)~N(0,0.01);

Auto labeling saves an enormous amount of typing.

```
TITLE:      this is an example of a Bayesian
            multiple group model with approximate
            measurement invariance
DATA:       FILE = ex5.33.dat;
VARIABLE:   NAMES = u y1-y6 group;
            USEVARIABLES = y1-y6 group;
            CLASSES = c(10);
            KNOWNCLASS = c(group = 1-10);
ANALYSIS:   TYPE = MIXTURE;
            ESTIMATOR = BAYES;
            PROCESSORS = 2;
            MODEL = ALLFREE;
MODEL:      %OVERALL%
            f1 BY y1-y3* (lam#_1-lam#_3);
            f2 BY y4-y6* (lam#_4-lam#_6);
            [y1-y6] (nu#_1-nu#_6);
            %c#10%
            f1-f2@1;
            [f1-f2@0];
MODEL PRIORS:
            DO(1,6) DIFF(lam1_#-lam10_#)~N(0,0.01);
            DO(1,6) DIFF(nu1_#-nu10_#)~N(0,0.01);
OUTPUT:     TECH1 TECH8;
PLOT:       TYPE = PLOT2;
```

# New Solution No. 2: Group is Random Mode

## Two-level Factor Analysis with Random Loadings

Consider a single factor  $\eta$ . For factor indicator  $r$  ( $r = 1, 2, \dots, p$ ) for individual  $i$  in group (cluster)  $j$ ,

$$y_{rij} = v_{rj} + \lambda_{rj} \eta_{ij} + \varepsilon_{ij}, \quad (5)$$

$$\eta_{ij} = \eta_j + \zeta_{ij}, \text{ (this may be viewed as } \eta_{B_j} + \eta_{W_{ij}} \text{)} \quad (6)$$

$$v_{rj} = v_r + \delta_{v_j}, \quad (7)$$

$$\lambda_{rj} = \lambda_r + \delta_{\lambda_j}, \quad (8)$$

where  $v_r$  is the mean of the  $r^{\text{th}}$  intercept and  $\lambda_r$  is the mean of the  $r^{\text{th}}$  factor loading. Because the factor loadings are free, the factor metric is set by fixing  $V(\zeta_{ij}) = 1$  (the between-level variance  $V(\eta_j)$  is free). Note that the same loading is multiplying both the between- and within-level parts of the factor  $\eta$ .



# Two-Level Factor Analysis with Random Loadings: 3 Model Versions

$$y_{rij} = v_{rj} + \lambda_{rj} \eta_{ij} + \varepsilon_{ij}, \quad (9)$$

$$\eta_{ij} = \eta_j + \zeta_{ij}, \text{ (this may be viewed as } \eta_{B_j} + \eta_{W_{ij}} \text{)} \quad (10)$$

$$v_{rj} = v_r + \delta_{v_j}, \quad (11)$$

$$\lambda_{rj} = \lambda_r + \delta_{\lambda_j}, \quad (12)$$

A first alternative to this model is that  $V(\eta_j) = 0$  so that the factor with random loadings has only within-level variation. Instead, there can be a separate between-level factor with non-random loadings, measured by the random intercepts of the  $y$  indicators as in regular two-level factor analysis,  $y_{rj} = \lambda_{B_r} \eta_{B_j} + \zeta_{rj}$ , where  $y_{rj}$  is the between part of  $y_{rij}$ . A second alternative is that the  $\lambda_{B_r}$  loadings are equal to the means of the random loadings  $\lambda_r$ .

## Part 1: Random factor loadings (decomposition of the factor into within- and between-level parts)

```

TITLE:      this is an example of a two-level MIMIC
            model with continuous factor indicators,
            random factor loadings, two covariates on
            within, and one covariate on between
            with equal loadings across levels
DATA:      FILE = ex9.19.dat;
VARIABLE:  NAMES = y1-y4 x1 x2 w clus;
            WITHIN = x1 x2;
            BETWEEN = w;
            CLUSTER = clus;
ANALYSIS:  TYPE = TWOLEVEL RANDOM;
            ESTIMATOR = BAYES;
            PROCESSORS = 2;
            BITER = (1000);
MODEL:    %WITHIN%
            s1-s4 | f BY y1-y4;
            f@1;
            f ON x1 x2;
            %BETWEEN%
            f ON w;
            f;
PLOT:     TYPE = PLOT2;
OUTPUT:   TECH1 TECH8;

```

## Part 2: Random factor loadings and a separate between-level factor

```
MODEL:      %WITHIN%  
            s1-s4 | f BY y1-y4;  
            f@1;  
            f ON x1 x2;  
            %BETWEEN%  
            fb BY y1-y4;  
            fb ON w;
```

f@0; is the between-level default

Part 3: Random factor loadings and a separate between-level factor with loadings equal to the mean of the random loadings

```
MODEL:      %WITHIN%
            s1-s4 | f BY y1-y4;
            f@1;
            f ON x1 x2;
            %BETWEEN%
            fb BY y1-y4* (lam1-lam4);
            fb ON w;
            [s1-s4*1] (lam1-lam4);
```

# Monte Carlo Simulations for Groups as Random Mode: Two-Level Random Loadings Modeling

- The effect of treating random loadings as fixed parameters
  - Continuous variables
  - Categorical variables
- Small number of clusters/groups

# The Effect of Treating Random Loadings as Fixed Parameters with Continuous Variables

**Table:** Absolute bias and coverage for factor analysis model with random loadings - comparing random intercepts and loadings and v.s. random intercepts and fixed loadings models

parameter	Bayes	ML with fixed loadings
$\theta_1$	0.00(0.97)	0.20(0.23)
$\mu_1$	0.01(0.95)	0.14(0.66)
$\lambda_1$	0.01(0.96)	0.00(0.80)
$\theta_2$	0.02(0.89)	0.00(0.93)

Ignoring the random loadings leads to biased mean and variance parameters and poor coverage. The loading is unbiased but has poor coverage.

# The Effect of Treating Random Loadings as Fixed Parameters in Categorical Variables

**Table:** Absolute bias and coverage for factor analysis model with categorical data and random loadings - comparing random loadings and intercepts v.s. random intercepts and fixed loadings models

parameter	Bayes	WLSMV with fixed loadings
$\tau_1$	0.05(0.96)	0.17(0.63)
$\lambda_1$	0.03(0.92)	0.13(0.39)
$\theta_2$	0.05(0.91)	0.11(0.70)

Ignoring the random loadings leads to biased mean, loading and variance parameters and poor coverage.

- Many applications have small number of clusters/groups. How many variables and random effects can we use?
- Independent random effects model - works well even with 50 variables (100 random effects) and 10 clusters
- Weakly informative priors are needed to eliminate biases for cluster level variance parameters
- Correlated random effects model (1-factor model) - works only when "number of clusters > number of random effects". More than 10 clusters are needed with 5 variables or more.
- What happens if you ignore the correlation: standard error underestimation, decreased accuracy in cluster specific estimates
- BSEM: Muthén, B. and Asparouhov, T. (2012). Bayesian SEM: A more flexible representation of substantive theory. Forthcoming in Psychological Methods.
- Using BSEM with 1-factor model for the random effects and tiny priors  $N(1, \sigma)$  for the loadings resolves the problem.



Shortell et al. (1995). Assessing the impact of continuous quality improvement/total quality management: concept versus implementation. *Health Services Research*, 30, 377-401.

- Survey of 67 hospitals,  $n = 7168$  employee respondents, approximately 100/hospital
- 6 dimensions of an overall "quality improvement implementation" based on the Malcom Baldrige National Quality Award criteria
- Focus on 10 items measuring a leadership dimension

- Hospital as Fixed Mode:
  - Old approach: Conventional multiple-group factor analysis
  - New approach: BSEM multiple-group factor analysis
- Hospital as Random Mode:
  - Old approach: Conventional two-level factor analysis
  - New approach: Bayes random loadings two-level factor analysis (random factor variances also possible)

# Hospital as Fixed Mode: Conventional Multiple-Group Factor Analysis

Regular ML analysis:

```
USEVARIABLES = lead21-lead30! info31-info37  
! straqp38-straqp44 hru45-hru52 qm53-qm58 hosp;  
MISSING = ALL(-999);  
!CLUSTER = hosp;  
GROUPING = hosp (101 102 104 105 201 301-306  
308 310-314 316-320 322 401-403 405-409 412-416  
501-503 505-512 602-609 612-613 701 801 901-908);
```

```
ANALYSIS: ESTIMATOR = ML;  
PROCESSORS = 8;
```

MODEL:

```
lead BY lead21-lead30;
```

```
PLOT: TYPE = PLOT2;
```

```
OUTPUT: TECH1 TECH8 MODINDICES(ALL);
```

# Hospital as Fixed Mode: Conventional Multiple-Group Factor Analysis, Continued

Maximum-likelihood analysis with  $\chi^2$  test of model fit and modification indices.

Holding measurement parameters equal across groups/hospitals results in poor fit with many moderate-sized modification indices and none that sticks out as much larger than the others.

Conventional multiple-group factor analysis "fails".

# New Solution No. 1: Group as Fixed Mode using Multiple-Group BSEM BSEM Input Excerpts for Hospital Data

```
USEVARIABLES = lead21-lead30 hosp;  
MISSING = ALL (-999);  
CLASSES = c(67);  
KNOWNCLASS = c(hosp=101 102 104  
105 201 301-306 308 310-314 316-320  
322 401-403 405-409 412-416 501-503  
505-512 602-609 612-613 701 801  
901-908);
```

```
ANALYSIS: TYPE = MIXTURE;  
ESTIMATOR = BAYES;  
BITERATIONS = (2000); ! min number of Bayes iterations  
PROCESSORS = 2;  
MODEL = ALLFREE;  
! changes the mixture default of across-class equality of  
! nu, Lambda, Psi, and Theta parameters when using BY
```

Use the auto-labeling feature (# is the class number) in MODEL and specify approximate measurement equality for intercepts and factor loadings across classes/groups in MODEL PRIORS:

```
MODEL:          %OVERALL%
                lead BY lead21-lead30* (lam#_1-lam#_10);
                [lead21-lead30] (nu#_1-nu#_10);
                %c#67%
                lead@1; ! sets the metric of the factor

MODEL
PRIORS:         DO(1,10) DIFF(lam1_#-lam67_#)~N(0,0.05);
                DO(1,10) DIFF(nu1_#-nu67_#)~N(0,0.05);

PLOT:          TYPE = PLOT2;

OUTPUT:        TECH1 TECH8;
```

67-group BSEM requires 2142 parameters (!)

Computing time: 3:07:35

# BSEM Estimates for Hospital 67

Latent Class 67

LEAD	BY						
LEAD21		0.880	0.106	0.000	0.674	1.092	*
LEAD22		0.915	0.099	0.000	0.721	1.112	*
LEAD23		0.898	0.095	0.000	0.719	1.088	*
LEAD24		0.860	0.091	0.000	0.688	1.050	*
LEAD25		0.805	0.089	0.000	0.630	0.983	*
LEAD26		0.846	0.084	0.000	0.687	1.014	*
LEAD27		1.005	0.092	0.000	0.832	1.195	*
LEAD28		0.511	0.109	0.000	0.284	0.717	*
LEAD29		0.742	0.098	0.000	0.555	0.947	*
LEAD30		0.960	0.084	0.000	0.797	1.130	*
Means							
LEAD		0.000	0.000	1.000	0.000	0.000	
Intercepts							
LEAD21		3.628	0.087	0.000	3.465	3.800	*
LEAD22		3.496	0.087	0.000	3.335	3.670	*
LEAD23		3.298	0.087	0.000	3.129	3.468	*
LEAD24		3.682	0.079	0.000	3.527	3.836	*
LEAD25		3.776	0.075	0.000	3.626	3.918	*
LEAD26		3.687	0.072	0.000	3.544	3.821	*
LEAD27		3.608	0.082	0.000	3.457	3.769	*
LEAD28		3.433	0.090	0.000	3.257	3.608	*
LEAD29		3.320	0.081	0.000	3.158	3.476	*
LEAD30		3.550	0.075	0.000	3.404	3.693	*
Variances							
LEAD		1.000	0.000	0.000	1.000	1.000	



# BSEM Estimates for Hospital 66

Latent Class 66

LEAD	BY						
LEAD21		0.983	0.086	0.000	0.818	1.154	*
LEAD22		1.077	0.101	0.000	0.877	1.269	*
LEAD23		0.927	0.092	0.000	0.749	1.113	*
LEAD24		0.994	0.083	0.000	0.830	1.160	*
LEAD25		0.986	0.084	0.000	0.826	1.156	*
LEAD26		1.091	0.081	0.000	0.933	1.248	*
LEAD27		1.080	0.089	0.000	0.912	1.258	*
LEAD28		0.530	0.101	0.000	0.342	0.735	*
LEAD29		1.047	0.086	0.000	0.874	1.212	*
LEAD30		1.076	0.088	0.000	0.908	1.248	*
<b>Means</b>							
LEAD		-0.383	0.099	0.000	-0.567	-0.177	*
<b>Intercepts</b>							
LEAD21		3.701	0.087	0.000	3.528	3.863	*
LEAD22		3.460	0.096	0.000	3.274	3.642	*
LEAD23		3.524	0.090	0.000	3.344	3.694	*
LEAD24		3.702	0.083	0.000	3.533	3.859	*
LEAD25		3.739	0.087	0.000	3.571	3.909	*
LEAD26		3.762	0.085	0.000	3.578	3.922	*
LEAD27		3.598	0.092	0.000	3.407	3.770	*
LEAD28		3.431	0.099	0.000	3.238	3.635	*
LEAD29		3.423	0.085	0.000	3.263	3.593	*
LEAD30		3.613	0.090	0.000	3.430	3.778	*
<b>Variances</b>							
LEAD		0.677	0.118	0.000	0.491	0.943	*

# BSEM Measurement Intercept Differences Across Hospitals: Intercept for Item 3 for all 67 Hospitals

	Average	Std. Dev.	Deviations from the Mean				
3	3.401	0.042	NU1_3 0.140	NU2_3 -0.048	NU3_3 0.187*	NU4_3 -0.176	NU5_3 0.021
			...				
			NU41_3 -0.054	NU42_3 -0.079	NU43_3 0.034	NU44_3 -0.043	NU45_3 -0.219*
			NU46_3 -0.019	NU47_3 -0.062	NU48_3 0.142	NU49_3 -0.033	NU50_3 -0.028
			NU51_3 0.316*	NU52_3 0.063	NU53_3 0.107	NU54_3 0.103	NU55_3 -0.077
			NU56_3 0.031	NU57_3 -0.122	NU58_3 -0.004	NU59_3 0.035	NU60_3 0.048
			NU61_3 -0.006	NU62_3 0.103	NU63_3 -0.086	NU64_3 0.217*	NU65_3 -0.044
			NU66_3 0.123	NU67_3 -0.099			

# Displaying Non-Invariant Items for BSEM: Hospitals With Significant Differences Compared to the Mean (Prior $V = 0.01$ + Prior $V = 0.05$ )

---

Item	Loadings	Intercepts
1	48 + 67	50
2	24, 33, 45 + 47	33, 37, 40, 45, 47 + 24, 39, 50, 52, 57
3	-	45, 51, 64 + 3, 6
4	-	7, 34
5	- + 67	15 + 3, 34, 41, 53
6	- + 25, 67	52, 60 - 52
7	67 + 47	52 + 5, 11, 26
8	45 + 36	17, 60 + 1, 7, 26, 45, 53, 55, 59
9	- + 67	52 + 51
10	-	67

---

# Group as Random Mode: Conventional Two-Level Factor Analysis

- Recall random effects ANOVA (individual  $i$  in cluster  $j$ ):

$$y_{ij} = \nu + \eta_j + \varepsilon_{ij} = y_{B_j} + y_{W_j} \quad (13)$$

- Two-level factor analysis ( $r = 1, 2, \dots, p$  items; 1 factor on each level):

$$y_{rij} = \nu_r + \lambda_{B_r} \eta_{B_j} + \varepsilon_{B_{rj}} + \lambda_{W_{ij}} \eta_{W_{ij}} + \varepsilon_{W_{rij}} \quad (14)$$

- Alternative expression often used in 2-level IRT:

$$y_{rij} = \nu_r + \lambda_r \eta_{ij} + \varepsilon_{rij}, \quad (15)$$

$$\eta_{ij} = \eta_{B_j} + \eta_{W_{ij}}, \quad (16)$$

so that  $\lambda$  is the same for between and within.

# Input Excerpts for Hospital as Random Mode: Conventional Two-Level Factor Analysis

```
USEVARIABLES = lead21-lead30;  
MISSING = ALL (-999);  
CLUSTER = hosp;  
ANALYSIS: TYPE = TWOLEVEL;  
ESTIMATOR = ML;  
PROCESSORS = 8;  
MODEL: %WITHIN%  
leadw BY lead21-lead30* (lam1-lam10);  
leadw@1;  
%BETWEEN%  
leadb BY lead21-lead30* (lam1-lam10);  
leadb;  
OUTPUT: TECH1 TECH8 MODINDICES(ALL);
```

# Results for Hospital as Random Mode: Conventional Two-Level Factor Analysis

Equality of within- and between-level factor loadings cannot be rejected by  $\chi^2$  difference testing

10 % of the total variance in the leadership factor is due to between-hospital variation

No information about measurement invariance across hospitals

# New Solution No. 2: Hospital as Random Mode using Two-Level Analysis with Random Loadings

In the interest of time, this Hospital data analysis is not reported.

Instead, an application for binary items in PISA data for 40 countries is shown in the next part.

Davidov (2009). Measurement equivalence of nationalism and constructive patriotism in the ISSP: 34 countries in a comparative perspective. *Political Analysis*, 17, 64-82.

- Data from the ISSP 2003 National Identity Module
- 34 countries, n=45,546
- 5 measurements of nationalism and patriotism
- Expected 2-factor structure

Many other country comparisons, e.g. TIMMS



# Nationalism and Patriotism Data: Multiple-Group CFA with ML

Two-factor CFA with measurement invariance across all 34 countries:  
 $\chi^2(334) = 9669, p = 0 (!)$

Group-specific misfit evenly spread over the countries

Modification indices show a multitude of similarly large values

Alternatives:

- EFA for each group
- ESEM for all groups
- BSEM for all groups

# Nationalism and Patriotism Data: EFA for each Group

Australia ( $n = 2146$ ): EFA  $\chi^2 (1) = 0.402$

GEOMIN ROTATED LOADINGS (\* significant at 5% level)

	1	2
V21	0.821*	-0.005*
V22	0.569*	0.129*
V26	0.001	0.614*
V29	-0.041	0.503*
V35	0.223*	0.491*

GEOMIN FACTOR CORRELATIONS (\* significant at 5% level)

	1	2
1	1.000	
2	0.268*	1.000

# Nationalism and Patriotism Data: EFA for each Group

Germany ( $n = 1281$ ): EFA  $\chi^2 (1) = 0.841$

GEOMIN ROTATED LOADINGS (\* significant at 5% level)

	1	2
V21	1.159*	-0.001
V22	0.448*	0.159*
V26	0.002	0.650*
V29	-0.078*	0.662*
V35	0.022	0.606*

GEOMIN FACTOR CORRELATIONS (\* significant at 5% level)

	1	2
1	1.000	
2	0.110	1.000

ESTIMATED RESIDUAL VARIANCES

	V21	V22	V26	V29	V35
1	-0.343	0.759	0.577	0.567	0.630

# Nationalism and Patriotism Data: EFA for each Group

USA (n = 1210): No convergence for EFA

NO CONVERGENCE. NUMBER OF ITERATIONS EXCEEDED.  
PROBLEM OCCURRED IN EXPLORATORY FACTOR  
ANALYSIS WITH 2 FACTOR(S).

**ESEM:**

ANALYSIS:

MODEL:

f1-f2 BY v21-v35 (\*1);  
v21-v35 (var1-var5);

MODEL

CONSTRAINT:

DO(1,5) var#>0;

# Nationalism and Patriotism Data:

## ESEM Solution for USA (STDY), $\chi^2(1) = 4.763$

		Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
F1	BY				
	V21	1.000	0.000	15267.805	0.000
	V22	0.413	0.027	15.480	0.000
	V26	-0.020	0.035	-0.553	0.580
	V29	0.002	0.008	0.303	0.762
	V35	0.087	0.037	2.353	0.019
F2	BY				
	V21	-0.001	0.000	-4.245	0.000
	V22	0.224	0.035	6.466	0.000
	V26	0.641	0.042	15.418	0.000
	V29	0.503	0.035	14.249	0.000
	V35	0.551	0.040	13.939	0.000
F2	WITH				
	F1	0.270	0.049	5.489	0.000
Residual Variances					
	V21	0.000	0.000	24.485	0.000
	V22	0.729	0.024	30.424	0.000
	V26	0.595	0.049	12.075	0.000
	V29	0.747	0.035	21.467	0.000
	V35	0.663	0.040	16.703	0.000

# Nationalism and Patriotism Data: Multiple-Group ESEM

No convergence due to large negative residual variances for many countries

Constraining residual variances to be non-negative gives convergence but the measurement invariance model has  $\chi^2(496) = 13893$

# Nationalism and Patriotism Data: Multiple-Group BSEM

Multiple-group BSEM allows cross-loadings similar to ESEM, but also allows approximate measurement invariance

With a small number of variables, setting the metric by fixing factor variances at 1 may lead to poor mixing in Bayesian analysis. This occurs in these data.

An alternative is to fix the factor loading at 1 in one group and let the other groups' factor loading be approximately 1 by small-variance priors

# Nationalism and Patriotism Data:

## Multiple-Group BSEM Input

```
CLASSES = c(34);
KNOWNCLASS = c(cntry = 1 2 4 6-8 10-22 24-28
30-33 36 37 40-43);
ANALYSIS: TYPE = MIXTURE;
ESTIMATOR = BAYES;
PROCESSORS = 2;
MODEL = ALLFREE;
MODEL: %OVERALL%
nat BY v21* (lam#_1)
v22 (lam#_2);
pat BY v26* (lam#_3)
v29-v35* (lam#_4-lam#_5);
[v21-v35] (nu#_1-nu#_5);
pat BY v21-v22* (xlam#_1-xlam#_2);
nat BY v26-v35* (xlam#_3-xlam#_5);
```



# Nationalism and Patriotism Data: Multiple-Group BSEM Input, Continued

```
%c#1%  
nat BY v21@1;  
pat BY v26@1;  
[nat-pat@0];  
%c#34%  
[nat-pat];
```

## MODEL PRIORS:

```
DO(2,2) DIFF(lam1_#-lam34_#)~N(0,0.01);  
DO(4,5) DIFF(lam1_#-lam34_#)~N(0,0.01);  
DO(1,5) DIFF(nu1_#-nu34_#)~N(0,0.01);  
DO(2,34) lam#_1~N(1,0.01);  
DO(2,34) lam#_3~N(1,0.01);  
DO(1,5) xlam1_#-xlam34_#~N(0,0.01);
```

# 34-Country Factor Mean Estimates: Patriotic Factor. Measurement Difference Prior Variance 0.001 Versus 0.01

