Recent Advances in Modeling Short and Long Longitudinal Data

Bengt Muthén
Professor Emeritus, UCLA

Mplus: https://www.statmodel.com
bmuthen@statmodel.com

Tihomir Asparouhov
Mplus

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Section 1: Motivating Examples from Three Data Sets
- Stress, alcohol consumption, suicidal ideation, substance abuse, and negative affect (N = 270-1375)

PART 1: Short Longitudinal Data - Panel Data (T = 3-10)

Section 2: Brief Refresher of Longitudinal Modeling with Continuous Outcomes
- Dynamic models
- Models with auto-regressive residuals

Section 3: Brief Introduction to Multivariate Modeling with Categorical Outcomes
- Model specification, identification and estimation
- Model testing

Section 4: Applications of Binary RI-AR Modeling
- Suicidal ideation
- Abstinence
Section 5: Ordered Categorical (Ordinal) Outcomes

Two-Part Ordinal Modeling
- Model specification
- 5-category alcohol risk
- Treatment effects comparing regular and two-part ordinal

Section 6: Analysis of Two Processes: Cross-Lagged Panel Modeling
- Cross-lagged effects between a continuous stress outcome and a 5-category ordinal alcohol risk outcome

PART 2: Long Longitudinal Data - Intensive Longitudinal Data (T = 50-100)

Section 7: Two-level time series analysis, DSEM
- Two-level, cross-lagged modeling of positive and negative affect
- Two-level, two-part analysis of negative affect
- Daily cycles of positive affect
- Interaction effects in a randomized trial

Section 8: References
Section 1 Motivating Examples from Three Data Sets
Data from COMBINE, a 16-week, multisite randomized double-blind clinical trial comparing treatments of alcohol dependence (Anton et al., 2006, JAMA)

- N = 1,383. Mean age 44
- Measurement occasions: Baseline, week 1, week 2, week 4, week 6, week 8, week 10, week 12, week 16 and week 52 follow-up

Alcohol risk: Abstinence, low risk, medium risk, high risk, very high risk (risk levels based on amount of alcohol consumed)

Stress: Brief version of The Perceived Stress Scale

- Stress and alcohol use disorder (AUD). Stress causes drinking (Armeli et al., 2000 in J of Personality and Social Psych)

Covariates:

- Intervention - 9 groups (medication, placebo, and therapy), gender, race, age, education, marital status, employment
A 4-item version of The Perceived Stress Scale with scores of 0 to 16 has been used for analyses of the COMBINE data:

- McHugh et al. (2013). Positive affect and stress reactivity in alcohol-dependent outpatients. J. Studies in Alcohol and Drugs

- Can be treated as a continuous variable using non-normality robust ML

- Percentage at the lowest score of zero increases with time but does not exceed 15%
Distribution of the Alcohol Risk Variable

- WHO categories based on grams of pure alcohol per day (separate for males and females)
- Should not be treated as a continuous variable with linear relations because of the strong floor effect: Biases in correlations and regressions
- Can be analyzed as an ordered categorical (ordinal) with 5 categories. Floor effect not a problem
Binary vs ordinal:

- Traditionally, abstinence is the accepted outcome in treatment
- More recently, low-risk drinking is an alternative end point

Witkiewitz et al. (2017). Clinical validation of reduced alcohol consumption after treatment for alcohol dependence using the WHO risk drinking levels. Alcoholism: Clinical and Experimental Research
Suicidal ideation and substance abuse

Classic question of what influences what

Data from a preventive intervention study in Baltimore (Ialongo)

N = 737

T = 8: Ages 19-26

Covariates: Gender, race, lunch (poverty indicator)

References:


Musci et al. (2016). Suicide & Life Threatening Behavior

Thrul et al. (2021). Addiction
Suicidal Ideation and Substance Abuse: Binary Outcomes

- Number of suicidal thoughts or behaviors endorsed in the last year
- At age 19, 77% are at zero, 8% at 1: Dichotomize into 0 vs higher
  - At least one suicidal ideation and/or behaviors endorsed (Y)
  - At least one substance abuse or dependence criteria met across all substances assessed in the last year (Z)

<table>
<thead>
<tr>
<th>Age</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age 19</td>
<td>23.0%</td>
<td>19.9%</td>
</tr>
<tr>
<td>Age 20</td>
<td>18.2%</td>
<td>19.8%</td>
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<tr>
<td>Age 21</td>
<td>15.2%</td>
<td>15.3%</td>
</tr>
<tr>
<td>Age 22</td>
<td>19.0%</td>
<td>20.4%</td>
</tr>
<tr>
<td>Age 23</td>
<td>23.0%</td>
<td>18.5%</td>
</tr>
<tr>
<td>Age 24</td>
<td>22.3%</td>
<td>15.0%</td>
</tr>
<tr>
<td>Age 25</td>
<td>23.4%</td>
<td>14.3%</td>
</tr>
<tr>
<td>Age 26</td>
<td>21.1%</td>
<td>13.3%</td>
</tr>
</tbody>
</table>
Data from the older cohort of the Notre Dame Study of Health & Well-being (Cindy Bergeman): N = 271, T = 56 (daily measures on consecutive days)

10 NA items (5-category scale): afraid, ashamed, guilty, hostile, scared, upset, irritable, jittery, nervous, distressed

Question format: Today I felt... (1 = Not at all, 2 = A little, 3 = Moderately, 4 = Quite a bit, 5 = Extremely)
Negative Affect Sum of 10 Items

- 54% at lowest value - answering Not at all on all 10 items
- Not suitable for continuous variable analysis with linear models due to strong floor effect
- Can be treated as a semi-continuous variable
Section 2 Brief Refresher of Longitudinal Modeling with Continuous Outcomes
Dynamic Models

- Auto-Regression of lag 1 (AR1)


- Dynamic Random Intercept ARMA (1,1). Zyphur et al. (2020)
Statistical theory used in repeated measurement modeling:
- Laird & Ware (1982, Biometrics) random effect model
- Chi & Reinsel (1989, JASA) added auto-regressions among the residuals, \( \varepsilon_t = \beta \varepsilon_{t-1} + \delta_t \) (AR-1):

Special case of no trend: Random intercept plus first-order auto-regressions among the residuals (RI-AR1 modeling):

The figure corresponds to a single-level, wide format analysis suitable for short longitudinal data
Separation of between- and within-individual variation

RI-AR modeling is the univariate part of random intercept cross-lagged panel modeling (RI-CLPM; Hamaker et al., 2015)
This model is similar in spirit to RI-AR because of its separation of between- and within-individual variation also referred to as latent centering (centering using the random intercept i), but adds MA


Muthén & Asparouhov (2022). Mplus Web Talk No. 4, Part 1
Section 3 Brief Introduction to Multivariate Modeling with Categorical Outcomes
Three contexts:
- Item Response Theory (IRT): item difficulty and discrimination
- Factor analysis: item thresholds and factor loadings
- Random intercept (5 time points, single-level, wide format): factor loadings fixed at 1

Typical specification:
- Normally distributed latent variable
- Logistic or Probit regressions

ML, WLSMV, and Bayes estimation available in Mplus
Equivalent representation with continuous latent response variables $Y^*$:

- $Y^* >$ threshold results in $Y = 1$, otherwise $Y = 0$
- Specifying normally distributed $f$ together with probit regressions is the same as specifying normally distributed latent response variables $Y^*$ ($N + N = N$). Logistic regression does not give $Y^*$ normality
- Correlations between $Y^*$ variables for binary $Y$: Tetrachoric correlations used in WLSMV; ML, Bayes use raw data (full info)
The cross-sectional modeling of IRT and factor analysis focuses on the relationship between f and Y.

The longitudinal modeling of panel data analysis adds a focus on the relationship between Y’s at different time points.

- The random intercept may not account for all the Y* correlation.
  - Especially not when occasions are close in time.

- Multivariate probit modeling allows linear regressions among normally distributed Y*’s such as in the below auto-regressive model (dynamic model in time series settings).
  - Auto-regression among the Y*’s or their residuals?
Binary case: $y_t^*$ continuous latent response variable at time $t$ with threshold $\tau_t$, $y_t^* > \tau_t \rightarrow y_t = 1$, otherwise $y_t = 0$

\[
y_{it}^* = \alpha_i + \hat{y}_{it}^*, \quad (1)
\]

\[
\hat{y}_{it}^* = \beta_t \hat{y}_{it-1}^* + \zeta_{it}; \quad t = 2, \ldots, T \quad (2)
\]

\[
\hat{y}_{i1}^* = \zeta_{i1}; \quad (3)
\]

Random intercept $\alpha_i \sim N(0, \psi)$ and residuals $\zeta_t \sim N(0, \theta_t)$

A maximum of $T - 1$ $\theta_t$ variances can be identified

- Empirical identification issue: Number of identifiable variances depends on the data (correlations across time, # time points)
- Default of residual variances fixed at 1 is often reasonable
 ML: AR leads to too many dimensions of numerical integration

WLSMV: Second-order, limited-information estimator (tetrachoric/polychoric correlations). Fast, good with low missingness. Does not handle MAR

Bayes: Full-information estimator (like ML) using raw data (not tetrachoric/polychoric correlations). Advantageous due to handling MAR (Asparouhov & Muthén, 2022: RSEM)

Models need to be identified in terms of the second-order information
RI and AR1 Impact on Correlations Across Time (T=10)

(a) RI variance \( (R^2) = 0.00 \)

(b) RI variance \( (R^2) = 0.25 \)

(c) RI variance \( (R^2) = 0.50 \)

(d) RI variance \( (R^2) = 0.75 \)

Correlation \( Y^*_1, Y^*_t = \psi + \beta^{t-1}(1 - \psi) \) where \( \psi \) is the random intercept variance, \( \beta \) is the constant auto-regression among the residuals, and \( Y^* \) variances are all 1.
Simulations for RI-AR1 with Binary Outcome

- Time-varying thresholds and ARs
- No Missing Data. WLSMV Estimation (Bayes results similar)
- Parameter values:
  - Autocorrelations = 0.3
  - Random intercept variance = 1, residual variances = 1: R-square due to random intercept = 0.5, R-square for residuals = 0.09
  - \( P(Y=1) = 0.20 \) (thresholds = 1.2)
- Fixed variances (default):
  - T=3 gets good results for N=500
  - T=4 gets good results for N=500
  - T=8 gets good results for N=500 (no improvement due to no time-invariant parameters)
- Free 1st variance:
  - T=3 not identified
  - T=4 gets good results for N=2000
  - T=8 gets good results for N=500 (more corr’s for 1st var)
- Free T-1 variances (often not a stable model in practice):
  - T=4 is not identified. T=5 gets good results for N=5000
  - T=8 gets good results for N=500
Two Kinds of Model Assessments for Categorical Outcomes

- Fit to correlations among a set of normal, continuous latent response variables $Y^*$ underlying the observed categorical $Y$’s (WLSMV and Bayes)
- Fit to the data in the form of response patterns, that is, a frequency table for all variables
  - A model may fit the $Y^*$ correlations but not the frequency table
  - Even a just-identified $Y^*$ model with free correlations may not fit the frequency table in some cases
Fit to correlations among a set of normal, continuous latent response variables $Y^*$ underlying the observed categorical Y’s (WLSMV and Bayes)

- The Muthén et al. (1997) WLSMV chi-square works well when the number of variables is not large and the sample size is not small: Suitable for cross-lagged panel modeling
- Bayes PPP idea: Using any fit statistic, compute the fit statistic for the observed data, generate a fit statistic distribution based on generated data from the estimated model, and find the proportion of cases where the latter is larger than the former
  - Bayes PPP for categorical variables: Based on chi-square test of overall model fit for $Y^*$s
  - Analogous to WLSMV chi-square test of estimated versus sample tetrachorics and polychorics
  - Low power for binary outcomes and less powerful than the WLSMV chi-square test (Asparouhov-Muthén, 2021a)
  - More powerful for polytomous variables
Frequency table test of model fit:

- With categorical variables, the model can be tested against data using Pearson and likelihood-ratio chi-square frequency table tests. Summing over the cells of the table:

\[
\text{Pearson} : \sum_j (o_j - e_j)^2 / e_j \\
\text{Likelihood ratio} : 2 \sum_j o_j \log(o_j / e_j)
\]

- There are typically too many frequency table cells with many cells having estimated frequencies close to zero, invalidating the tests: Pearson and Likelihood ratio tests disagree
- Example with 8 binary variables: \(2^8 = 256\) possible response patterns, where many patterns are probably not observed (zero cells in the frequency table) - the two tests disagree strongly
- Alternative checks: Fit for univariate and bivariate tables (higher frequencies) and most common response patterns
Section 4 Applications of Binary RI-AR Modeling
Multivariate Probit Analysis of Binary Suicidal Ideation Using Bayes (N=737, T=8)

- Bayes PPP (posterior predictive p-value)
- Number of significant residuals:
  - 20 most frequent response patterns. Freq. = 155 for all 0’s
  - 4*28 = 112 bivariate cells. 5% = 6

<table>
<thead>
<tr>
<th>Model</th>
<th># par’s</th>
<th>PPP</th>
<th># Significant Residuals</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Resp Pattern (max obs freq)</td>
<td>Bivar</td>
</tr>
<tr>
<td>1. Unrestr.</td>
<td>36</td>
<td>0.524</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2. AR1</td>
<td>15</td>
<td>0.133</td>
<td>2 (155)</td>
<td>22</td>
</tr>
<tr>
<td>3. AR2</td>
<td>21</td>
<td>0.378</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>4. RI</td>
<td>9</td>
<td>0.191</td>
<td>2 (25)</td>
<td>8</td>
</tr>
<tr>
<td>5. RI-AR1</td>
<td>16</td>
<td>0.422</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>6. RI-AR2</td>
<td>22</td>
<td>0.466</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

- 2 versus 5 shows importance of RI
- 4 versus 5 shows importance of AR
- ML can only estimate model 4, WLSMV can estimate all six models
Substantial random intercept variance = 0.756, S.E. (SD) = 0.102

STDYX estimates:

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>S.E.</th>
</tr>
</thead>
<tbody>
<tr>
<td>I BY</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y1</td>
<td>0.656</td>
<td>0.025</td>
</tr>
<tr>
<td>Y2</td>
<td>0.642</td>
<td>0.026</td>
</tr>
<tr>
<td>Y3</td>
<td>0.653</td>
<td>0.026</td>
</tr>
<tr>
<td>Y4</td>
<td>0.636</td>
<td>0.032</td>
</tr>
<tr>
<td>Y5</td>
<td>0.632</td>
<td>0.032</td>
</tr>
<tr>
<td>Y6</td>
<td>0.603</td>
<td>0.038</td>
</tr>
<tr>
<td>Y7</td>
<td>0.607</td>
<td>0.034</td>
</tr>
<tr>
<td>Y8</td>
<td>0.581</td>
<td>0.035</td>
</tr>
</tbody>
</table>

$R^2$ (Y4*) due to the random intercept = $0.636^2 = 0.404$

Corr (Y1*, Y8*) due to RI = $0.656 \times 0.581 = 0.381$
(total est corr = 0.383)

AR1 estimates (standardized):

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>S.E.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y2^ ON</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y1^</td>
<td>-0.231</td>
<td>0.131</td>
</tr>
<tr>
<td>Y3^ ON</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y2^</td>
<td>0.035</td>
<td>0.117</td>
</tr>
<tr>
<td>Y4^ ON</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y3^</td>
<td>0.304*</td>
<td>0.115</td>
</tr>
<tr>
<td>Y5^ ON</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y4^</td>
<td>0.339*</td>
<td>0.101</td>
</tr>
<tr>
<td>Y6^ ON</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y5^</td>
<td>0.488*</td>
<td>0.099</td>
</tr>
<tr>
<td>Y7^ ON</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y6^</td>
<td>0.473*</td>
<td>0.086</td>
</tr>
<tr>
<td>Y8^ ON</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y7^</td>
<td>0.565*</td>
<td>0.072</td>
</tr>
</tbody>
</table>
Multivariate Probit Analysis of Binary Abstinence Using Bayes (N=1375, T=8)

- Bayes PPP (posterior predictive p-value)
- Number of significant residuals:
  - 20 most frequent response patterns. Freq. = 312 for all 0’s
  - $4 \times 28 = 112$ bivariate cells. 5% = 6

<table>
<thead>
<tr>
<th>Model</th>
<th># par’s</th>
<th>PPP</th>
<th># Significant Residuals</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unrestr.</td>
<td>36</td>
<td>0.520</td>
<td>1 (38; z=1.98)</td>
<td>Good fit</td>
</tr>
<tr>
<td>AR1</td>
<td>15</td>
<td>0.082</td>
<td>1 (312)</td>
<td>Poor fit</td>
</tr>
<tr>
<td>AR2</td>
<td>21</td>
<td>0.474</td>
<td>0</td>
<td>Good fit</td>
</tr>
<tr>
<td>RI</td>
<td>9</td>
<td>0.000</td>
<td>8 (38)</td>
<td>Poor fit</td>
</tr>
<tr>
<td>RI-AR1</td>
<td>16</td>
<td>0.189</td>
<td>2 (38)</td>
<td>OK fit</td>
</tr>
<tr>
<td>RI-AR2</td>
<td>22</td>
<td>0.472</td>
<td>0</td>
<td>Good fit</td>
</tr>
</tbody>
</table>
Section 5 Ordered Categorical (Ordinal) Outcomes
Two-Part Ordinal Modeling
Brant (1990) chi-square test of the proportionality assumption of regular ordinal regression

- Chi-square (3) = 40 (p=0.000) rejects proportionality

Proportionality seems to hold for the 4 highest categories, suggesting a two-part model - one binary part and one 4-category ordinal part
Distribution of the Alcohol Risk Variable

- Binary vs ordinal:
  - Traditionally, abstinence is the accepted outcome in treatment
  - More recently, low-risk drinking is an alternative end point

- Witkiewitz et al. (2017). Clinical validation of reduced alcohol consumption after treatment for alcohol dependence using the WHO risk drinking levels. Alcoholism: Clinical and Experimental Research
The variable is split into two parts (Mplus DATA TWOPART):

\[
\begin{array}{ccc}
Y & U & V \\
>0 & 1 & \log Y \\
0 & 0 & 999 \\
999 & 999 & 999 \\
\end{array}
\]


Muthén (2001): Two-part growth mixture modeling

Muthén & Asparouhov (2022): Mplus Web Talk No. 4, Part 2: Cross-lagged modeling of categorical panel data
http://www.statmodel.com/Webtalk4P2.shtml

Mplus Version 8.8: Two-part ordinal multivariate probit model (TECH10 testing)

Hedeker (personal communication; longitudinal setting)
With censoring from below at zero and using probit regression with the event of $U = 1$ referring to a positive outcome, the two-part model can be expressed by a probit regression part and a continuous regression part typically with a log transformation,

$$P(U_i = 1|X_i) = \Phi\left(\frac{-\tau + \gamma_1 X_i}{\sqrt{\sigma^2}}\right), \quad (4)$$

$$V_i = \log Y_{U_i=1} = \beta_0 + \beta_1 X_i + \varepsilon_i. \quad (5)$$

Here, $\sigma^2 = 1$. Importantly, $\gamma_1$ and $\beta_1$ can be different and different $X$ variables can be involved. Logit can be used instead of probit.

Alternative models:

- Inflation model using a mixture with a zero latent class
- Heckman selection modeling
- Censored (tobit), censored-inflated
Regular ordinal probit regression with categories $c = 0, 1, 2, \ldots C-1$

$$P(Y_i = c | X_i) = \Phi[(\tau_c - \mu_i)/\sqrt{\sigma^2}] - \Phi[(\tau_{c-1} - \mu_i)/\sqrt{\sigma^2}], \quad (6)$$

Two-part ordinal with categories $c = 0, 1, 2, \ldots C-1$ uses probit regression with the event of $U = 1$ referring to a positive outcome and ordinal probit for the positive categories $c = 1, 2, \ldots C - 1$,

$$P(U_i = 1 | X_i) = \Phi[(-\tau + \mu_i)/\sqrt{\sigma^2}], \quad (7)$$

$$P(V_i = c | X_i) = \Phi[(\tau_c - \mu_{pi})/\sqrt{\sigma_p^2}] - \Phi[(\tau_{c-1} - \mu_{pi})/\sqrt{\sigma_p^2}], \quad (8)$$

In the multivariate response case, the variances $\sigma^2$ and $\sigma_p^2$ contain key model parameters such as random intercept variance and auto-regression coefficients.
Comparison of regular and twopart ordinal analysis:
  - Modeling of the alcohol risk outcome
  - Modeling the treatment effects on the alcohol risk outcome
    - Possibly different effects on binary and ordinal parts
Testing regular versus two-part ordinal random intercept models for weeks 1 - 16 \((T = 8)\) for the 5-category alcohol risk outcome

- Most frequent pattern of all 0’s has frequency 312
- Bivariate tables have \(5 \times 5 \times \frac{8 \times (8-1)}{2} = 700\) cells. 5\% = 35
<table>
<thead>
<tr>
<th>Estimator/Model</th>
<th># par’s</th>
<th>Fit</th>
<th>Resp Pattern (Max Obs Freq)</th>
<th>Bivariate</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Regular Ordinal</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ML, probit, RI</td>
<td>33</td>
<td>-10471</td>
<td>21180</td>
<td>11 (312)</td>
</tr>
<tr>
<td>Bayes, Unrestr.</td>
<td>60</td>
<td>0.498</td>
<td>[-29 27 ]</td>
<td>5 (312)</td>
</tr>
<tr>
<td>Bayes, RI</td>
<td>33</td>
<td>0.000</td>
<td>[398 530]</td>
<td>7 (312)</td>
</tr>
<tr>
<td>Bayes, RI, AR1</td>
<td>40</td>
<td>0.000</td>
<td>[31 107]</td>
<td>6 (312)</td>
</tr>
<tr>
<td>Bayes, RI, AR2</td>
<td>46</td>
<td>0.129</td>
<td>[11 44]</td>
<td>5 (312)</td>
</tr>
<tr>
<td><strong>Two-Part Ordinal</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ML, probit, RI</td>
<td>35</td>
<td>-9749</td>
<td>19750</td>
<td>4 (25)</td>
</tr>
<tr>
<td>Bayes, Unrestr.</td>
<td>144</td>
<td>0.472</td>
<td>[-46 52 ]</td>
<td>1 (12)</td>
</tr>
<tr>
<td>Bayes, RI</td>
<td>35</td>
<td>0.000</td>
<td>[77 199]</td>
<td>4 (25)</td>
</tr>
<tr>
<td>Bayes, RI, AR1</td>
<td>49</td>
<td>0.110</td>
<td>[-16 85 ]</td>
<td>1 (12)</td>
</tr>
<tr>
<td>Bayes, RI, AR2</td>
<td>61</td>
<td>0.219</td>
<td>[-27 70]</td>
<td>1 (12)</td>
</tr>
</tbody>
</table>
Assessing Treatment Effects: Regular Versus Two-Part Ordinal RI-AR1 Models for the 5-Category Alcohol Risk Outcome

<table>
<thead>
<tr>
<th>Tx Effects</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
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<td>8</td>
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<tr>
<td>9</td>
</tr>
</tbody>
</table>

2 = acamprosate, 3 = naltrexone, 4 = naltrexone + acamprosate, 5 = placebo + behavioral intervention, 6 = acamprosate and combined behavioral intervention, 7 = naltrexone and combined behavioral intervention, 8 = naltrexone + acamprosate + behavioral intervention, 9 = combined behavioral intervention with no mediations
Section 6 Analysis of Two Processes: Cross-Lagged Panel Modeling
Continuous outcomes:

Categorical outcomes:
- Muthén & Asparouhov (2022). Mplus Web Talk No. 4, Part 2
Returning to the question in the introduction: Does stress influence alcohol risk or the other way around?

Stress causes drinking (Armeli et al., 2000 in J of Personality and Social Psych)

The alcohol treatment setting may produce a different picture

RI-CLPM with one continuous and one categorical outcome
Bivariate Analysis of Stress and Alcohol Risk

Week 1 - Week 16 (N=1375, T=8)

Average Correlations as a Function of the Time Distance

- RI-AR1 model chosen for stress - continuous outcome
  - Asparouhov & Muthén (2022). RSEM paper
- Two-part ordinal RI-AR1 model chosen for 5-category alcohol risk
- 3 correlated random intercepts, 3 processes
Number of significant cross-lagged effects (out of 7 possible):
- Stress regressed on binary part of alcohol risk: 1
- Stress regressed on ordinal part of alcohol risk: 6
- Binary part of alcohol risk regressed on stress: 0
- Ordinal part of alcohol risk regressed on stress: 3 (small effects)

Most of the cross-lagged effect is alcohol risk increasing stress, not the other way around
- Abstinence or not has little effect on stress
- Higher degree of alcohol risk (non-abstinence) has a significant effect on stress
- Stress has a minor effect on higher degree of alcohol risk
Muthén & Asparouhov (2022): Mplus Web Talk No. 4, Part 2: Cross-lagged modeling of categorical panel data
http://www.statmodel.com/Webtalk4P2.shtml

Other models:
- Observed Y instead of latent Y* as predictor
- Reciprocal interaction
- Bivariate latent transition analysis (Mover-Stayer LTA, RI-LTA)
Section 7 Part 2: Long Longitudinal Data
- Intensive Longitudinal Data
Two-Level Time Series Analysis, DSEM
Frequent observations, large T (20, 50, 100, 1000): Daily diary data, ecological momentary assessments, experience sampling methods, wearables

Within level = time, between level = individual. Variation in within level parameters across individuals can be characterized by many random effects (continuous latent variables) not only random intercepts: Mean/intercepts (level), variance, auto-correlation, slopes, amplitude

Modeling with a large number of random effects is made possible by Bayesian estimation. Mplus implementation presented in:


More references and short course videos at:
http://www.statmodel.com/TimeSeries
Cross-lagged modeling with large T and Bayes estimation can in principle allow for more random effects than just intercepts.

Between-Within decomposition where the W variables correspond to the residuals in the earlier figures (hat variables in Mplus):

Hamaker, Asparouhov et al. (2018). At the frontiers of modeling intensive longitudinal data: Dynamic structural equation models for the affective measurements from the COGITO study. Multivariate Behavioral Research.
Data from a study designed to detect at-risk mood profiles related to depression in adolescents

- de Haan-Rietdijk, Voelkle, Keisers, Hamaker (2017). Discrete-vs. continuous-time modeling of unequally spaced experience sampling method data. frontiers in Psychology

- ESM questionnaires measuring positive and negative affect in Dutch adolescents (age 12 to 16)

- N = 233, several measures per day for 7 days
Random intercepts only model

PA ON PA&1 refers to the lag-1 auto-regression of \( PA_t \) on \( PA_{t-1} \). Because it appears on the Within level, it is the residual of PA, that is, the latent variable centered version of PA - or in other words, what is left in PA after the random intercept has been subtracted.

Standardized cross-lagged effects:

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>S.D.</th>
<th>Lower 2.5%</th>
<th>Upper 2.5%</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Within Level</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PA ON</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>PA&amp;1</td>
<td>0.374</td>
<td>0.016</td>
<td>0.340</td>
<td>0.403</td>
<td>*</td>
</tr>
<tr>
<td>NA&amp;1</td>
<td>-0.013</td>
<td>0.015</td>
<td>-0.041</td>
<td>0.020</td>
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<tr>
<td>NA ON</td>
<td></td>
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<td></td>
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<tr>
<td>NA&amp;1</td>
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<td>0.016</td>
<td>0.223</td>
<td>0.286</td>
<td>*</td>
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<tr>
<td>PA&amp;1</td>
<td>-0.023</td>
<td>0.017</td>
<td>-0.055</td>
<td>0.010</td>
<td></td>
</tr>
</tbody>
</table>

Small or insignificant cross-lagged effects are often found for PA-NA
Negative Affect Distribution

- 60% at the lowest value
- A possible reason for small cross-lagged effects?
- Two-part modeling motivated and can be done in DSEM
  - Binary part uses multivariate probit
  - Positive part specified as continuous-normal using a log transformation
NU refers to the Y* (probit regression) for the binary part of negative affect $\geq$ floor, POS refers to the continuous part (linear regression)

Standardized cross-lagged effects:

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<th>Lower 2.5%</th>
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<td></td>
</tr>
<tr>
<td>PA&amp;1</td>
<td>0.340</td>
<td>0.018</td>
<td>0.304</td>
<td>0.374</td>
<td>*</td>
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<tr>
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<td>0.020</td>
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<td>-0.108</td>
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<td>0.020</td>
<td>-0.059</td>
<td>0.021</td>
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<tr>
<td>NU ON</td>
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<tr>
<td>NU&amp;1</td>
<td>0.460</td>
<td>0.030</td>
<td>0.399</td>
<td>0.513</td>
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<td>PA&amp;1</td>
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<td>0.022</td>
<td>-0.119</td>
<td>-0.032</td>
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<td>NPOS ON</td>
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<tr>
<td>NPOS&amp;1</td>
<td>0.310</td>
<td>0.034</td>
<td>0.240</td>
<td>0.373</td>
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<td>PA&amp;1</td>
<td>-0.001</td>
<td>0.025</td>
<td>-0.052</td>
<td>0.045</td>
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</tr>
</tbody>
</table>
Daily Cycles of Positive Affect: Observed Averages Over Individuals Tuesday - Monday
- **Biological cycles**
  - 24-hour cycles: Circadian rhythm such as heart rate
- **Behavioral cycles**
  - Weekly drinking pattern
- **Environmental cycles**
  - Monthly temperature fluctuations
f(t) = A cos (2\pi \omega t + \phi)

= -A sin \phi \sin (2\pi \omega t) + A cos \phi \cos (2\pi \omega t)

\beta_1 \beta_2
\beta_1
\beta_2
x_1 x_2

Amplitude = A = \sqrt{\beta_1^2 + \beta_2^2}

Phase = \phi = tan^{-1}(-\beta_1/\beta_2)

- \omega is a frequency index defined as cycles per unit. With 8 measurements per day, a daily cycle is obtained by \omega = 1/8
- Random effects for amplitude and phase are of interest and can be obtained via random effects for \beta_1, \beta_2
- 2-component model (cosinor) with 4 random slopes used for blood pressure. Madden et al. (2018) in Statistics in Medicine
- Dummy variables can be added for weekdays or weekend

- The estimated model uses two-level dynamic structural equation modeling (DSEM) allowing for individual differences in daily cycles

- Muthén et al. (2022). In preparation: DSEM with daily cycles modeled by sine-cosine curve with random effects
Are the two outcomes related after accounting for the daily cycles?

Residual dynamic SEM (RDSEM)

Randomized Trial


- Trial Design (N = 119):
  - Baseline: 6 days Experience Sampling (ESM) using 10 beeps/day via digital wristwatch
  - Randomization into 8 weeks of treatment or control
  - Post treatment: 6 days ESM again
  - Total T = 60 pre + 60 post
  - Positive affect
Preliminary findings for intervention effects on the random effects of the post-intervention time series for momentary positive emotions:

- Positive effect on level (intercept); higher effect on level for persons with higher pre-intervention level
- Positive effect on level for persons with high pre-intervention auto correlation
Section 8 References
Further DSEM References

- Schultzberg & Muthén (2018). Number of subjects and time points needed for multilevel time series analysis: A simulation study of dynamic structural equation modeling. Structural Equation Modeling

- McNeish & Hamaker (2020). A primer on two-level dynamic structural equation models for intensive longitudinal data in Mplus. Psych Methods


- Mplus Web Talk No. 5. Forthcoming. Focus on using Mplus
Further References

  http://www.statmodel.com/download/RSEM.pdf
  http://www.statmodel.com/bmuthen/articles/Article_045.pdf
  http://www.statmodel.com/download/Article_075.pdf


Zyphur et al. (2020). From data to causes II: Comparing approaches to panel data analysis. Organizational Research Methods