Perturbation of Starting Values

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1 Perturbation

We provide perturbation for all parameters except variance covariance parameters. The perturbed starting value \( v_i \) is obtained by the following formula

\[
v_i = w_i + sr_i b_i
\]

where \( w_i \) is the original unperturbed starting value, \( r_i \) is a uniformly distributed random number with values on the interval \([-0.5, 0.5]\), \( s \) is a scale variable and \( b \) is the base scale for that parameter. The scale variable \( s \) take a default value 5, and defines the strength of the perturbation. A larger \( s \) value will trigger a more aggressive local maxima search, a smaller value would trigger a search that uses mostly reasonable starting values and has a better chance of finding a local maximum with log-likelihood larger then the original log-likelihood. For different data and models different \( s \) values would produce the most efficient search. Typically if \( s \) is too small the search produces the same local maxima as the original starting value. In such cases the user might need to increase the \( s \) value. If \( s \) is too large the search will produce many local maxima that are solutions with much smaller log-likelihood, this is due to the extreme nature of the starting values, which in term produce extreme local solutions. Such solutions are generally not of interest and in such cases the user might need to decrease the scale value \( s \) to make the search within some more reasonable region. The base scale value \( b_i \) is computed as follows. For all parameters except mean and variance parameters \( b_i = 2 \). For variance and covariance parameters \( b_i = 0 \) and for mean parameters, i.e., intercept parameter for a variable

\[
b_i = 2 \max\{1, \sqrt{\text{var}}\}
\]
where \( \text{var} \) is the original starting value for the variance parameter (in the same class) of that variable.

The above construction is used when the seed generating the random numbers is an odd integer. When the seed is an even number, we take a different approach for the parameters that are thresholds and means of the latent class variable. This alternative approach produces better results in certain examples. The approach is as follows. For the thresholds of a given observed variable \( U \) is used the following construction. If \( U \) has \( T + 1 \) categories, i.e., it has \( T \) thresholds in a given class, we generate \( r_{1}, \ldots, r_{T} \) random numbers from the interval \([0, 1]\). Let \( r_{(1)}, \ldots, r_{(T)} \) be the ordered statistics for these random numbers, i.e., let \( r_{(1)} \) be the smallest number etc. We use as a starting value for the threshold \( i \): 
\[
\tau_{i} = \log(\frac{\tau_{(i)}}{1 - \tau_{(i)}}).
\]
Note that unlike the previous construction these starting values are independent of the starting values provided by the user. Similarly, we provide starting values for the means of \( C \). If \( C \) takes \( K \) values we generate \( K - 1 \) random numbers from the interval \([0, 1]\) and the starting value for \( \alpha_{i,k} = \log(\frac{(r_{(k)} - r_{(k-1)})}{(1 - r_{(K-1)})}) \), where we set \( r_{(0)} = 0 \). These formulas set the distribution of \( U \) and \( C \) to be defined by the random numbers, i.e., \( P(U \leq i) = P(C \leq i) = r_{(i)} \).

### 2 Control Structure for the Perturbations

There are two modes that the perturbations can be done: multiple perturbations and single perturbation. Usually the first stage is multiple perturbations, an exploratory stage, where the user provides a seed (\( \text{pseed} = \) ), which is used to generate seeds for the individual perturbations (\( \text{spseed} \)). These seeds (\( \text{spseed} \)) are reported next to the log-likelihood and can be used to rerun an individual perturbation in a single perturbation mode.

In multiple perturbation mode we complete the computation only for selected perturbation. In the first stage of the computation we compute all (\( \text{spert} = \)) perturbation runs until an increased convergence criteria (\( \text{pconv} = \), with default 1, applies to all of the convergence quantities, namely mconv, logcriteria, rlogcriteria) is satisfied or the maximum number of iterations is reached (\( \text{piter} = \), default 20). The class count convergence criteria is ignored. In the second stage we complete the computation only for those (\( \text{fpert} = \)) that produced the highest log-likelihood values during the first stage. Another control is (\( \text{pscale} = \) perturbation scale value).