Unusual TLI values

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In SEM models the TLI fit index is defined as follows

$$TLI = \frac{\chi^2_B/DF_B - \chi^2_{H_0}/DF_{H_0}}{\chi^2_B/DF_B - 1}$$

where χ^2_B and DF_B are the chi-square statistic and the degrees of freedom for the baseline model while $\chi^2_{H_0}$ and DF_{H_0} are those quantities for the structural model.

TLI is typically compared to and used simultaneous with the CFI fit index

$$CFI = 1 - \frac{max(\chi^2_{H_0} - DF_{H_0}, 0)}{max(\chi^2_{H_0} - DF_{H_0}, \chi^2_B - DF_B, 0)}.$$

Alternative but equivalent definition of CFI is as follows

$$CFI = \frac{(\chi^2_B - DF_B) - (\chi^2_{H_0} - DF_{H_0})}{\chi^2_B - DF_B}$$

truncated to the interval from 0 to 1, i.e., if the CFI value comes out negative it is set to 0 and if the CFI value comes out greater than 1 it is set to 1. While the CFI definition ensures that the CFI value is always in the 0 to 1 range, the TLI definition does not provide this protection. Thus negative TLI values and values greater than 1 can occur.

Anderson and Gerbing (1984) conduct simulation studies and report that such out of range values are common for smaller sample sizes, see page 172 where the TLI index is called RHO. Here are two examples that have occurred in real data sets

Example 1. $\chi^2_{H_0} = 0.429$, $DF_{H_0} = 1$, $\chi^2_B = 5.619$, $DF_B = 5$, $N = 267$. In this case the TLI value is 5.609.

Example 2. $\chi^2_{H_0} = 11.898$, $DF_{H_0} = 1$, $\chi^2_B = 133.224$, $DF_B = 25$, $N = 79$. In this case the TLI value is -1.517.

In both examples the sample size is relatively small and thus it relates to the findings in Anderson and Gerbing (1984). In principle one can truncate the TLI to the 0 to 1 range the way CFI is truncated. This is a reasonable approach and is for example advocated by David Kenny, see http://davidakenny.net/cm/fit.htm. The truncated values can be reported instead of the out of range values. In example 1 the truncated TLI value will be 1 and the fit will be considered acceptable. In example 2 the truncated TLI value will be 0 and the fit will be unacceptable.

In addition, we should consider the appropriate practical use for the approximate fit indices. One can argue that when $DF_{H_0} = 1$ it would be hard to justify the use of approximate fit indices. If there is only one degree of freedom the most appropriate and straight forward test of fit is the T-test for the parameter that represents that degree of freedom. Where TLI and CFI excel is the situation where DF_{H_0} is large and the TLI and CFI indices can be viewed as methods for determining if the proposed model extracts the "majority" of information contained in the data without pursing the many small model misfits that typically are found in large DF_{H_0} models. The use of these fit indices is also most appropriate in the situations where the sample size is large and every small deviation from 0 becomes significant, i.e., the statistical significance concept becomes disconnected from the substantively significant concept. Anderson and Gerbing (1984) state "In very large samples, residuals of no practical significance can lead to statistical rejection of a model ...". The CFI and TLI indices resolve this issue and are useful in quantifying to what extent a model approximates the data. In small sample size however such a thing does not occur and we would recommenced using the chi-square test itself as a method for determining fit, while reserving the the fit indices to have more of a descriptive purpose. In conclusion, relying heavily on CFI and TLI in situations where the sample size is small or the DF_{H_0} of the SEM model is small is not recommended. The most appropriate use for CFI and TLI remains the situations of large sample size or large degrees of freedom.

References

[1] Anderson, J. & Gerbing, D. (1984) The effect of sampling error on convergence, improper solutions, and goodness-of-fit indices for maximum likelihood confirmatory factor analysis, Psychometrika, 155-173.