

Mplus: A General Latent Variable Modeling Program

Bengt Muthén & Linda Muthén

1. Introduction

Using support from an NIAA SBIR contract, Bengt and Linda Muthén began the development of Mplus in 1995 with the goal of providing researchers with powerful new statistical modeling techniques. A wide gap was seen between new statistical methods presented in the statistical literature and the statistical methods used by researchers in substantively-oriented papers. The goal was to help bridge this gap with easy-to-use but powerful software. Version 1 of Mplus was released in November 1998; Version 2 was released in February 2001; Version 3 was released in March 2004; Version 4 was released in February 2006; Version 5 was released in November 2007, Version 6 was released in April 2010, and Version 7 was released in September 2012. The program is written in FORTRAN with graphical interface written in C, and diagramming capabilities written in Java. Tihomir Asparouhov is the statistical programmer and Thuy Nguyen is the interface programmer.

Due to not being limited to any one type of analysis such as IRT or SEM, Mplus is used by researchers in a wide variety of fields. It is especially popular among non-statisticians due to its ease of use and modeling flexibility, but is also used by biostatisticians due to its unique statistical capabilities.

IRT analysis in Mplus offers many unique features including multidimensional, two-level, three-level, and cross-classified analysis (Asparouhov & Muthén, 2012a, b), mixture modeling (Muthén, 2008), diagnostic classification modeling (Rupp et al., 2010), and multilevel mixture modeling (Asparouhov & Muthén, 2008a; Henry & Muthén, 2010). As of Mplus Version 7.4, 3PL, 4PL, and generalized Partial Credit Modeling (PCM) are also included (Asparouhov & Muthén, 2015). Due to the general modeling framework, this means, for example, that multilevel and mixture PCM with observed and latent predictors are available as well. Following is a full description of the Mplus functionality.

2. Functionality

Mplus is a statistical modeling program that provides researchers with a flexible tool to analyze their data. Mplus offers researchers a wide choice of models, estimators, and algorithms in a program that has an easy-to-use interface including a diagrammer and graphical displays of data and analysis results. Mplus allows the analysis of both cross-sectional and longitudinal data, single-level and multilevel data, data that come from different populations with either observed or unobserved heterogeneity, and data that contain missing values. Analyses can be carried out for observed variables that are

continuous, censored, binary, ordered categorical (ordinal), unordered categorical (nominal), counts, or combinations of these variable types. In addition, Mplus has extensive capabilities for Monte Carlo simulation studies, where data can be generated and analyzed according to most of the models included in the program.

2.1 The generality of latent variable modeling using Mplus

The use of latent variables is a common theme in many statistical analyses. Continuous latent variables appear not only as factors measured with errors in factor analysis, item response theory, and structural equation modeling, but also appear in the form of random effects in growth modeling, components of variation in complex survey data analysis and multilevel modeling, frailties and liabilities in survival and genetic analyses, latent response variables with missing data, priors in Bayesian analysis, and as counterfactuals and potential outcomes in causal analysis. In addition, categorical latent variables appear as latent classes in finite mixture analysis and latent transition analysis (Hidden Markov modeling), latent trajectory classes in growth mixture modeling, and latent response variables with missing data on categorical variables.

The Mplus modeling framework draws on to the unifying theme of latent variable modeling. The generality of the Mplus modeling framework comes from the unique use of both continuous and categorical latent variables. The general framework provides a way to break down barriers between seemingly disparate types of analyses. Researchers need to be able to move freely between analysis types to more easily answer their research questions. To provide answers to the often complex substantive questions, it is fruitful to use latent variable techniques to combine different analysis types. Several recent examples show the potential of such ideas, including the combination of item response bi-factor modeling and multilevel analysis (Muthén & Asparouhov, 2012a) to perform analysis of change, the combination of exploratory factor analysis (EFA) and structural equation modeling (SEM) to perform multiple-group EFA of measurement instruments (Asparouhov & Muthén, 2009), the combination of SEM with Bayesian analysis to perform a more flexible type of Bayesian confirmatory factor analysis (Muthén & Asparouhov, 2012b), the combination of latent class and survival analysis to study patient-reported outcomes and cancer survival (Muthén et al. 2009), the combination of growth mixture modeling with survival analysis to study non-ignorable dropout in antidepressant clinical trials (Muthén et al., 2011), and the combination of latent class and causal analysis to study randomized trial effects in the presence of non-compliance with

treatment (Sobel & Muthén, 2012). All these analyses can be carried out using the general latent variable modeling framework of the Mplus program.

2.2 Modeling with continuous latent variables

Following are models with continuous latent variables that can be estimated using Mplus:

- Regression analysis
- Path analysis
- Exploratory factor analysis
- Confirmatory factor analysis
- Item response theory modeling
- Structural equation modeling
- Growth modeling
- Discrete-time survival analysis
- Continuous-time survival analysis

2.3 Modeling with categorical latent variables

Following are models with categorical latent variables that can be estimated using Mplus:

- Regression mixture modeling
- Path analysis mixture modeling
- Latent class analysis
- Latent class analysis with covariates and direct effects
- Confirmatory latent class analysis
- Latent class analysis with multiple categorical latent variables
- Loglinear modeling
- Non-parametric modeling of latent variable distributions
- Multiple group analysis
- Finite mixture modeling
- Complier Average Causal Effect (CACE) modeling
- Latent transition analysis and hidden Markov modeling
including mixtures and covariates
- Latent class growth analysis

- Discrete-time survival mixture analysis
- Continuous-time survival mixture analysis

2.4 Modeling with both continuous and categorical latent variables

The full modeling framework includes models with a combination of continuous and categorical latent variables. Following are models in the full modeling framework that can be estimated using Mplus:

- Latent class analysis with random effects
- Factor mixture modeling
- Structural equation mixture modeling
- Growth mixture modeling with latent trajectory classes
- Discrete-time survival mixture analysis
- Continuous-time survival mixture analysis

2.5 Modeling with complex survey and multilevel data

There are two approaches to the analysis of complex survey data in Mplus. One approach is to compute standard errors and a chi-square test of model fit taking into account stratification, non-independence of observations due to cluster sampling, and/or unequal probability of selection. Subpopulation analysis, replicate weights, and finite population correction are also available. With sampling weights, parameters are estimated by maximizing a weighted loglikelihood function. Standard error computations use a sandwich estimator. For this approach, observed outcome variables can be continuous, censored, binary, ordered categorical (ordinal), unordered categorical (nominal), counts, or combinations of these variable types.

A second approach is to specify a model for each level of the multilevel data thereby modeling the non-independence of observations due to cluster sampling. This is commonly referred to as multilevel modeling. The use of sampling weights in the estimation of parameters, standard errors, and the chi-square test of model fit is allowed. Both individual-level and cluster-level weights can be used. With sampling weights, parameters are estimated by maximizing a weighted loglikelihood function. Standard error computations use a sandwich estimator. For this approach, observed outcome

variables can be continuous, censored, binary, ordered categorical (ordinal), unordered categorical (nominal), counts, or combinations of these variable types.

The multilevel extension of the full modeling framework allows random intercepts and random slopes that vary across clusters in hierarchical data. Random slopes include the special case of random factor loadings. These random effects can be specified for any of the relationships of the full Mplus model for both independent and dependent variables and both observed and latent variables. Random effects representing across-cluster variation in intercepts and slopes or individual differences in growth can be combined with factors measured by multiple indicators on both the individual and cluster levels. In line with SEM, regressions among random effects, among factors, and between random effects and factors are allowed.

The two approaches described above can be combined. In addition to specifying a model for each level of the multilevel data thereby modeling the non-independence of observations due to cluster sampling, standard errors and a chi-square test of model fit are computed taking into account stratification, non-independence of observations due to cluster sampling, and/or unequal probability of selection. When there is clustering due to both primary and secondary sampling stages, the standard errors and chi-square test of

model fit are computed taking into account the clustering due to the primary sampling stage and clustering due to the secondary sampling stage is modeled.

2.6 Modeling with missing data

Mplus has several options for the estimation of models with missing data. Mplus provides maximum likelihood estimation under MCAR (missing completely at random), MAR (missing at random), and NMAR (not missing at random) for continuous, censored, binary, ordered categorical (ordinal), unordered categorical (nominal), counts, or combinations of these variable types (Little & Rubin, 2002). MAR means that missingness can be a function of observed covariates and observed outcomes. For censored and categorical outcomes using weighted least squares estimation, missingness is allowed to be a function of the observed covariates but not the observed outcomes (Asparouhov & Muthén, 2010). When there are no covariates in the model, this is analogous to pairwise present analysis. Non-ignorable missing data (NMAR) modeling is possible using maximum likelihood estimation where categorical outcomes are indicators

of missingness and where missingness can be predicted by continuous and categorical latent variables (Muthén, Jo, & Brown, 2003; Muthén et al., 2011).

With missing data, the standard errors for the parameter estimates are computed using the observed information matrix (Kenward & Molenberghs, 1998). Bootstrap standard errors and confidence intervals are also available with missing data.

Mplus provides multiple imputation of missing data using Bayesian analysis (Rubin, 1987; Schafer, 1997). Both the unrestricted H1 model and a restricted H0 model can be used for imputation. Multiple data sets generated using multiple imputation can be analyzed using a special feature of Mplus. Parameter estimates are averaged over the set of analyses, and standard errors are computed using the average of the standard errors over the set of analyses and the between analysis parameter estimate variation (Rubin, 1987; Schafer, 1997). A chi-square test of overall model fit is provided (Asparouhov & Muthén, 2008b; Enders, 2010).

2.7 Estimators and algorithms

Mplus provides both Bayesian and frequentist inference. Bayesian analysis uses Markov chain Monte Carlo (MCMC) algorithms. Posterior distributions can be monitored by trace and autocorrelation plots. Convergence can be monitored by the Gelman-Rubin potential scaling reduction using parallel computing in multiple MCMC chains. Posterior predictive checks are provided.

Frequentist analysis uses maximum likelihood and weighted least squares estimators. Mplus provides maximum likelihood estimation for all models. With censored and categorical outcomes, an alternative weighted least squares estimator is also available. For all types of outcomes, robust estimation of standard errors and robust chi-square tests of model fit are provided. These procedures take into account non-normality of outcomes and non-independence of observations due to cluster sampling. Robust standard errors are computed using the sandwich estimator. Robust chi-square tests of model fit are computed using mean and mean and variance adjustments as well as a likelihood-based approach. Bootstrap standard errors are available for most models. The optimization algorithms use one or a combination of the following: Quasi-Newton, Fisher scoring, Newton-Raphson, and the Expectation Maximization (EM) algorithm (Dempster et al., 1977). Linear and non-linear parameter constraints are allowed. With maximum likelihood estimation and categorical outcomes, models with continuous latent variables

and missing data for dependent variables require numerical integration in the computations. The numerical integration is carried out with or without adaptive quadrature in combination with rectangular integration, Gauss-Hermite integration, or Monte Carlo integration.

2.8 Monte Carlo simulation capabilities

Mplus has extensive Monte Carlo facilities both for data generation and data analysis. Several types of data can be generated: simple random samples, clustered (multilevel) data, missing data, discrete- and continuous-time survival data, and data from populations that are observed (multiple groups) or unobserved (latent classes). Data generation models can include random effects and interactions between continuous latent variables and between categorical latent variables. Outcome variables can be generated as continuous, censored, binary, ordered categorical (ordinal), unordered categorical (nominal), counts, or combinations of these variable types. In addition, two-part (semicontinuous) variables and time-to-event variables can be generated. Independent variables can be generated as binary or continuous. All or some of the Monte Carlo generated data sets can be saved.

The analysis model can be different from the data generation model. For example, variables can be generated as categorical and analyzed as continuous or generated as a three-class model and analyzed as a two-class model. In some situations, a special external Monte Carlo feature is needed to generate data by one model and analyze it by a different model. For example, variables can be generated using a clustered design and analyzed ignoring the clustering. Data generated outside of Mplus can also be analyzed using this special external Monte Carlo feature.

Other special Monte Carlo features include saving parameter estimates from the analysis of real data to be used as population and/or coverage values for data generation in a Monte Carlo simulation study. In addition, analysis results from each replication of a Monte Carlo simulation study can be saved in an external file.

2.9 Graphics

Mplus includes a dialog-based, post-processing graphics module that provides graphical displays of observed data and analysis results including outliers and influential observations.

These graphical displays can be viewed after the Mplus analysis is completed. They include histograms, scatterplots, plots of individual observed and estimated values, plots of sample and estimated means and proportions/probabilities, plots of estimated probabilities for a categorical latent variable as a function of its covariates, plots of item characteristic curves and information curves, plots of survival and hazard curves, plots of missing data statistics, plots of user-specified functions, and plots related to Bayesian estimation. These are available for the total sample, by group, by class, and adjusted for covariates. The graphical displays can be edited and exported as a DIB, EMF, or JPEG file. In addition, the data for each graphical display can be saved in an external file for use by another graphics program.

2.10 Diagrammer

The Diagrammer can be used to draw an input diagram, to automatically create an output diagram, and to automatically create a diagram using an Mplus input without an analysis or data. To draw an input diagram, the Diagrammer is accessed through the Open Diagrammer menu option of the Diagram menu in the Mplus Editor. The Diagrammer uses a set of drawing tools and pop-up menus to draw a diagram. When an input diagram

is drawn, a partial input is created which can be edited before the analysis. To automatically create an output diagram, an input is created in the Mplus Editor. The output diagram is automatically created when the analysis is completed. This diagram can be edited and used in a new analysis. The Diagrammer can be used as a drawing tool by using an input without an analysis or data.

2.11 LTA calculator

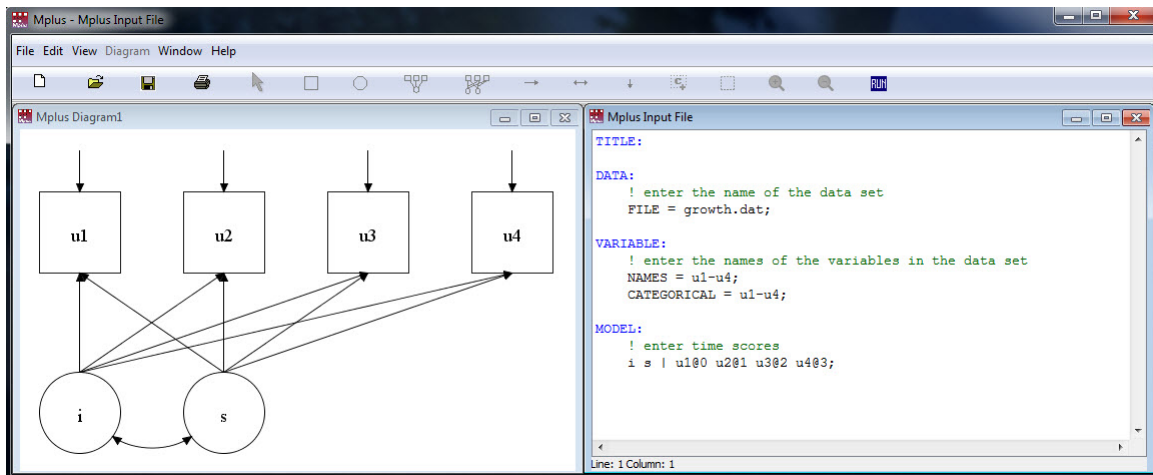
Conditional probabilities, including latent transition probabilities, for different values of a set of covariates can be computed using the LTA Calculator. It is accessed by choosing LTA calculator from the Mplus menu of the Mplus Editor.

3. User interface

The Mplus interface is provided by the Mplus Editor. A model can be specified either in terms of the Mplus language or using a model diagram. Model specification using the language is shown in the Mplus Editor screen shot below for a mixture IRT model. The language is discussed in the next section.

```
Mplus - ex7.27.inp
File Edit View Diagram Window Help
ex7.27.inp
TITLE:  this is an example of a factor (IRT) mixture analysis
        with binary latent class and factor indicators
DATA:   FILE = ex7.27.dat;
VARIABLE: NAMES = u1-u8;
         CATEGORICAL = u1-u8;
         CLASSES = c (2);
ANALYSIS: TYPE = MIXTURE;
          ALGORITHM = INTEGRATION;
          STARTS = 50 5;
MODEL:  %OVERALL%
        f BY u1-u8;
        [f@0];
        %c#1%
        f BY u1@1 u2-u8;
        f;
        [u1$1-u8$1];
        %c#2%
        f BY u1@1 u2-u8;
        f;
        [u1$1-u8$1];
OUTPUT: TECH1 TECH8;
Line: 21 Column: 22
```

Model specification using a model diagram is shown in the figure below. A model can be conveniently drawn in the left part of the screen, producing Mplus input statements in the right part of the screen. A diagram is also produced as the result of an analysis. Furthermore, a diagram can be produced without analysis or data by specifying a model using the Mplus language, providing a drawing tool.



4. Sample input and output

The user language for Mplus consists of a set of ten commands each of which has several options. The default options for Mplus have been chosen so that user input can be minimized for the most common types of analyses. For most analyses, only a small subset of the Mplus commands is needed. Complicated models can be easily described using the Mplus language. The ten commands of Mplus are:

- TITLE
- DATA (required)
- VARIABLE (required)

- DEFINE
- ANALYSIS
- MODEL
- OUTPUT
- SAVEDATA
- PLOT
- MONTECARLO

The TITLE command is used to provide a title for the analysis. The DATA command is used to provide information about the data set to be analyzed. The VARIABLE command is used to provide information about the variables in the data set to be analyzed. The DEFINE command is used to transform existing variables and create new variables. The ANALYSIS command is used to describe the technical details of the analysis. The MODEL command is used to describe the model to be estimated. The OUTPUT command is used to request additional output not included as the default. The SAVEDATA command is used to save the analysis data, auxiliary data, and a variety of analysis results. The PLOT command is used to request graphical displays of observed data and analysis results. The MONTECARLO command is used to specify the details of a Monte Carlo simulation study.

The Mplus commands may come in any order. The DATA and VARIABLE commands are required for all analyses. All commands must begin on a new line and must be followed by a colon. Semicolons separate command options. There can be more than one option per line. The records in the input setup must be no longer than 90 columns. They can contain upper and/or lower case letters and tabs.

Commands, options, and option settings can be shortened for convenience. Commands and options can be shortened to four or more letters. Option settings can be referred to by either the complete word or the part of the word shown in bold type in the command boxes in each chapter.

Comments can be included anywhere in the input setup. A comment is designated by an exclamation point. Anything on a line following an exclamation point is treated as a user comment and is ignored by the program.

The keywords IS, ARE, and = can be used interchangeably in all commands except DEFINE, MODEL CONSTRAINT, and MODEL TEST. Items in a list can be separated by blanks or commas.

Mplus uses a hyphen (-) to indicate a list of variables or numbers. The use of this feature is discussed in each section for which it is appropriate. There is also a special keyword ALL which can be used to indicate all variables. This keyword is discussed with the options that use it.

5. Performance

Following is information about system requirements for Mplus. Mplus Version 7 is available for 32-bit and 64-bit operating systems.

- Operating Systems
 - Microsoft Windows7/8
 - Mac OS X 10.8 or later
 - Linux (tested on the following distributions: Ubuntu, RedHat, Fedora, Debian, and Gentoo)
- At least 1 GB of RAM

- At least 120 MB of hard disk space

Memory requirements for Mplus (both RAM and virtual memory) depend on the type of analysis and the amount of data. For example, analyses using numerical integration with large samples can be computationally heavy and may require a large amount of memory. There is no limit on the file size. Computing time varies widely as a function of models and estimators.

6. Support

For information about Mplus, see www.statmodel.com. The Mplus User's Guide is posted on the web site together with all its examples, including the Monte Carlo version for each example, showing how the data were generated. The web site has a series of video recordings and handouts from nine different short courses given at the Johns Hopkins University. A large number of papers are posted that use Mplus features. Mplus Discussion is a forum where anyone can post Mplus-related questions and get answers from the Mplus team. Mplus support is available for licensed users.

7. Availability

Mplus is available in several versions differing with respect to modeling generality. The Mplus Base and Combination Add-On program covers all types of analyses. The Mplus Base and Mixture Add-On program does not include multilevel modeling. The Mplus Base and Multilevel Add-On program does not include mixture modeling. The Mplus Base program version does not include multilevel or mixture modeling. Typical IRT modeling can be carried out in the Mplus Base program. There is a free demo version suitable for teaching which can be downloaded from the web site. It is the Mplus Base and Combination Add-On program limited by the number of analysis variables. A student version is available at a third of the full price for each of the four Mplus versions. The different prices are shown at <http://www.statmodel.com/orderonline/>.

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Figure captions:

Figure 1. Mplus model specification for a mixture IRT model.

Figure 2. Growth model specification using a model diagram.

