2.3 Comparing EFA methods

EFA Variations

- Hypothesis about the number of factors:
  - ANALYSIS: TYPE = EFA
  - ESEM (*1)
  - PSEM with GEOMIN priors
  - Second-order exploratory factor analysis (SEFA) using PSEM with GEOMIN priors
  - Bi-factor analysis using ROTATION = BI-GEOMIN and direct second-order exploratory factor analysis (DSEFA) using PSEM
- Hypothesis about the number of factors and key items:
  - ESEM with Target rotation
  - PSEM with ALF priors for cross loadings
- **Comparing EFA methods**
- Special models:
  - ESEM with PSEM priors for residual covariances
  - PSEM finding a small number of cross-loadings

Slide 55 returns to the EFA Variations overview. Here we turn to comparisons of exploratory second-order and bi-factor analyses based on simulation results. Target methods are also included.

Comparing EFA Methods: Data Generated from a 3-Factor SEFA Methods Sorted by MSE Performance

- Monte Carlo simulation study in Asparouhov & Muthén (2026)

Method	Rotation	Model	Framework	Number only?	Conv	Avg MSE	Avg Coverage
M1	Geomin	SEFA	PSEM	Yes	100%	0.0033	92.9%
M2	Target-alf	SEFA	PSEM	No	100%	0.0036	94.7%
M3	Geomin/alf	DSEFA	PSEM	Yes	100%	0.0039	94.6%
M4	Target-alf	Bifactor	PSEM	No	85%	0.0131	91.0%
M5	Target	Bifactor	ESEM	No	60%	0.0134	86.7%
M6	Geomin	EFA	PSEM	Yes	88%	0.0164	84.2%
M7	Bi-geomin	Bifactor	PSEM	No	81%	0.0191	86.0%
M5	Target	Bifactor	ESEM	No	100%	0.0222	34.1%
M8	Bi-geomin	Bifactor	ESEM	Yes	60%	0.0256	52.1%

- Number only? refers to specifying only the number of factors, not also key loadings
- M1 is the SEFA model discussed earlier
- M2 is PSEM ALF discussed earlier but adding a second-order factor
- M3 is the DSEFA model discussed earlier

Slides 56 and 57 show simulation results from the Asparouhov & Muthén 2026 paper: A unification of second-order and bi-factor EFA. The methods are listed in the order of performance from best to worst. The performance of the methods is judged by convergence percentage, average mean square error, and average coverage.

In the table on slide 56, the data are generated from a 3-factor SEFA and it is natural that this method, which is method M1, performs the best. Method M2 performs second best but needs not only specification of the number of factors but also locations of the targets.

Comparing EFA Methods: Data Generated from a 4-Factor Bi-Factor Model Methods Sorted by MSE Performance

- Monte Carlo simulation study in Asparouhov & Muthén (2026)

Method	Rotation	Model	Framework	Number only?	Conv	Avg MSE	Avg Coverage
M3	Geomin/alf	DSEFA	PSEM	Yes	100%	0.0069	93.8%
M4	Target-alf	Bifactor	PSEM	No	99%	0.0152	97.1%
M1	Geomin	SEFA	PSEM	Yes	98%	0.0167	93.4%
M5	Target	Bifactor	ESEM	No	97%	0.0255	87.2%
M8	Bi-geomin	Bifactor	ESEM	Yes	97%	0.0270	93.5%

- Number only? refers to specifying only the number of factors, not also key loadings
- M4 is the bi-factor PSEM ALF shown on the next slide
- M5 is the bi-factor target ESEM shown on slide 47

- M1 and M3 are among the top 3 in both tables and require only the number of factors
- M2 and M4 are among the top when key loadings are also specified

In the table of slide 57, the data are generated from a 4-factor bi-factor model so that method M3 performs best.

Drawing on the results from both tables, we conclude that the SEFA and DSEFA methods M1 and M3 are doing well - and importantly - require only the specification of the number of factors. Methods M2 and M4 do well when the location of key loadings are also specified. It is notable that M8 performs the worst in this simulation. It uses ROTATION = BI-GEOMIN which has to date been the most commonly used.

Input for Bi-Factor PSEM using ALF Cross-Loading Priors (M4)

```
ANALYSIS:     ESTIMATOR = MLR;
              ITERATIONS = 10000;
              CONVERGENCE = 0.000001;
              STARTS = 50;

MODEL:        ! 4 specific factors
              spatial BY
                visual-flags*1
                general-arithmet*0 (a1-a20);

              verbal BY
                visual-flags*0 (b1-b4)
                general-wordm*1
                addition-arithmet*0 (c1-c15);

              speed BY
                visual-wordm*0 (d1-d9)
                addition-straight*1
                wordr-arithmet*0 (e1-e11);

              memory BY
                visual-straight*0 (f1-f13)
                wordr-figurew*1
                deduct-arithmet*0 (g1-g5);

              ! general factor:
              gen BY visual-arithmet*;

              spatial-gen WITH spatial-gen@0;
              spatial-gen@1;

MODEL PRIORS: a1-g5 ~ ALF(0,1);

OUTPUT:       STANDARDIZED;
```


For completeness, slide 58 shows the input for bi-factor PSEM with ALF cross loadings priors. This is method M4 in the previous tables. The input refers to the 24-variable H&S data.