Estimation of a hierarchical Exploratory Structural Equation Model (ESEM) using ESEM-within-CFA

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Morin, Arens and Marsh (2016, also see Marsh, Morin, Parker, & Kaur, 2014; Morin, Marsh & Nagengast, 2013), based on earlier work by Marsh, Nagengast & Morin (2013) and Joreskög (1969; also see Muthén & Muthén, 2009, slides 133–146) introduced ESEM-Within-CFA (EWC) as a workaround some limitations of exploratory Structural Equation Models (ESEM). Essentially, EWC starts with an ESEM model, and re-express it in the CFA framework using the starts values generated from the initial ESEM model. More precisely, using the exact starts values generated form the initial ESEM model, the EWC model is estimated by adding m^2 constraints for identification purposes. To do so, selected parameter estimates are fixed to the values obtained from the ESEM solution. Typically:

(1) The *m* factor variances are fixed to 1 (in a single-group ESEM, for the first group in a multiple group solution, or the first time point for a longitudinal solution).

(2) A referent indicator is selected for each factor that has a large (target) loading for the factor it is designed to measure and small (non-target) cross-loadings. Then, for purposes of identification, these small cross-loadings are fixed (@) to their estimated values from the ESEM solution.

(3) For all other parameter estimates, the pattern of fixed and free estimates should be the same as in the selected ESEM solution.

(4) It should be noted that the mean structure from the EWC solution can be identified as in a standard CFA model (while using the ESEM start values when possible).

For most applications proposed by Morin and colleagues, this EWC approach works fine. However, we recently realized that this approach was suboptimal for the estimation of higher-order ESEM models reported in Morin, Arens and Marsh (2016). More precisely, to fully re-express the ESEM model, the first-order factor variances have to be set to a value of 1 [i.e., the constraints described under (1) above]. However, in the higher-order EWC models reported in Morin, Arens and Marsh (2016), the constraints described in (1) resulted in into a EWC solution in which the residual variance of the first-order factors were fixed to 1, rather than their variance. In addition, having the factor loading of the first indicator on the higher-order factor fixed to 1 created further interference with the model estimation, taking it even further from the initial ESEM model. More precisely, if we consider one of the first-order factor (F1) and the Higher-order factor (HF):

Var(F1)= residual variance (F1) + loading_HF^2 * Var(HF)

So that if the constraints described above are imposed:
Var(F1)= 1 + 1 * Var(HF)

To circumvent this issue, we proposed an alternative (and simpler) EWC solution in which the variance of the first-order factors can be freely estimated:

(1) A referent indicator is selected for each factor that has a large (target) loading for the factor it is designed to measure and small (non-target) cross-loadings. Then, for purposes of identification, these large main loadings and small cross-loadings are fixed (@) to their estimated values from the ESEM solution. All factor variances are freely estimated. Note here that it is important to use the referent indicator approach. Fixing some other arbitrary m^2 loadings may compromise the efficiency of the estimation.

(2) For all other parameter estimates, the pattern of fixed and free estimates should be the same as in the selected ESEM solution.
(3) It should be noted that the mean structure from the EWC solution can be identified as in a standard CFA model (while using the ESEM start values when possible).

For most other contexts, the typical EWC approach would work. For instance, Morin, Arens and Marsh (2016) show how to re-express a partial mediation ESEM model with EWC in order to obtain bias-corrected bootstrap confidence intervals. In this example, the residual variance of the outcome ESEM factors has to be fixed to 1 in the EWC model to ensure its equivalence with the ESEM model in which they are also fixed to 1. Likewise, Morin, Marsh and Nagengast (2013) illustrated how EWC could be used to estimate latent change scores in the context of a longitudinal ESEM solution. In this context, the residual variance of the Time 2 factors should not be fixed to 1 in the EWC solution. However, this constraint is not required for this model to match the ESEM solution in which, due to measurement invariance, the variance of the Time 2 factors was allowed to be freely estimated. Still, we urge readers to carefully consider their specific EWC application in order to find the approach that best work with their data.

The revised approach proposed here would lead to new sample inputs for the examples provided by Morin, Arens and Marsh (2016) on pages 8 (middle) and X of their online supplemental materials, as well as different fit indices associated with their Higher-Order EWC solutions:

Table 1.
Adjusted Goodness of Fit Statistics and Information Criteria for the Re-Estimated Higher-Order ESEM models

Model	χ^2	df	CFI	TLI	RMSEA	RMSEA 90% CI	AIC	CAIC	BIC	SBIC
Simulated data	100.432*	33	.996	.991	.036	.028; .044	40021	403385	403328	40146
SDQ-I	5813.763*	2071	.949	.929	.030	.029; .031	360300	366425	365494	362536
SDQ-I without method control	6219.130*	2113	.944	.924	.032	.031; .032	360835	366684	365795	362971

Note. df = Degrees of freedom; CFI = comparative fit index; TLI = Tucker-Lewis index; RMSEA = root mean square error of approximation; CI = confidence interval; AIC = Akaike information criterion; CAIC = Constant AIC; BIC = Bayesian information criterion; ABIC = Sample size adjusted BIC. ESEM models were conducted with target oblique rotation. * All χ^2 values are significant ($p < .01$).

Title: Hierarchical ESEM using ESEM-Within-CFA (Simulated Data)

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! The previous ESEM model is re-expressed using CFA. No rotation is necessary.
! The model section uses the exact values of the non-standardized loadings and cross loadings
! estimated from the previous model as starts values (using *). First-order factor variances are also freely
! estimated, whereas the variance of the higher-order factor is fixed to 1 for identification purposes.
! For the first-order factors, one item per factor has all loadings and cross loadings
! constrained to be exactly equal to their ESEM values (using @).
! These 3 factors define a higher-order factor HF, with all higher-order loadings freely estimated..
Model:
f1 BY x1*0.74674; f1 BY x2*0.80372; f1 BY x3@0.80739; f1 BY x4*0.83759;
f1 BY y1*-0.05015; f1 BY y2*0.20610; f1 BY y3*-0.09183; f1 BY y4@0.03835;
f1 BY z1*0.22881; f1 BY z2*0.02457; f1 BY z3*0.01376; f1 BY z4@-0.13088;
f2 BY y1*0.79513; f2 BY y2*0.80701; f2 BY y3*0.95053; f2 BY y4@0.90008;
f2 BY x1*-0.12434;  f2 BY x2*0.15514; f2 BY x3@-0.07168; f2 BY x4*0.08193;
f2 BY z1*0.06430; f2 BY z2*0.31927; f2 BY z3*-0.14645; f2 BY z4@-0.00922;
f3 BY z1*0.71349; f3 BY z2*0.66022; f3 BY z3*0.96202; f3 BY z4@0.95145;
f3 BY x1*0.11211; f3 BY x2*-0.13255; f3 BY x3@0.15235; f3 BY x4*-0.02669;
f3 BY y1*0.16258; f3 BY y2*-0.02858; f3 BY y3*0.07700; f3 BY y4@-0.01649;
f1-f3*1;
HF BY F1*1 F2 F3;
HF@1;
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Title: Hierarchical ESEM Model of the SDQ-I Using ESEM-Within-CFA (Real Data)
! […] Analysis and Model sections only

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Model:
esteem BY sdq_1*-0.03101; esteem BY sdq_2*0.01217; esteem BY sdq_3*-0.09151;
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esteem BY sdq_7*-0.06562; esteem BY sdq_8*0.07463; esteem BY sdq_9*0.03260;
esteem BY sdq_10*-0.01524; esteem BY sdq_11*-0.05772; esteem BY sdq_12*0.12921;
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esteem BY sdq_16@-0.02130; esteem BY sdq_17*0.02080; esteem BY sdq_18*-0.03832;
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esteem BY sdq_25*-0.03904; esteem BY sdq_26*0.01240; esteem BY sdq_27@-0.03013;
esteem BY sdq_28*-0.00719; esteem BY sdq_29*0.22094; esteem BY sdq_30*0.18339;
esteem BY sdq_31*0.11053; esteem BY sdq_32*0.07525; esteem BY sdq_33*0.08772;
esteem BY sdq_34*-0.02310; esteem BY sdq_35*-0.05219; esteem BY sdq_36*0.14200;
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esteem BY sdq_40*-0.00166; esteem BY sdq_41*-0.01108; esteem BY sdq_42*-0.00535;
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! Higher-order factor
general by esteem*1 peer appear phy parent
schocom schoaff Germcom Germaff  MathAff MathCom ;
general@1;
! Method Factor (negative items)
MF BY SDQ_30* SDQ_17 SDQ_12 SDQ_21 SDQ_47 SDQ_23 SDQ_33 SDQ_65
SDQ_75 SDQ_6 SDQ_37 SDQ_61; MF@1; [MF@0];
MF WITH general@0 esteem@0 peer@0  appear@0 phy@0 parent@0 schocom@0
schoaff@0 Germcom@0 Germaff@0 MathAff@0 MathCom@0;
! correlated uniquenesses between parallel worded items
SDQ_11 with SDQ_51 SDQ_71; SDQ_51 with SDQ_71;
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