New Developments in Mplus Version 7:
Part 1

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Mplus
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The Map of the Mplus Team

Mplus Background

- Inefficient dissemination of statistical methods:
  - Many good methods contributions from biostatistics, psychometrics, etc are underutilized in practice
- Fragmented presentation of methods:
  - Technical descriptions in many different journals
  - Many different pieces of limited software
- Mplus: Integration of methods in one framework
  - Easy to use: Simple, non-technical language, graphics
  - Powerful: General modeling capabilities

Mplus

Several programs in one

- Path analysis
- Exploratory factor analysis
- Structural equation modeling
- Item response theory analysis
- Growth modeling
- Mixture modeling (latent class analysis)
- Longitudinal mixture modeling (Markov, LTA, LCGA, GMM)
- Survival analysis (continuous- and discrete-time)
- Multilevel analysis
- Complex survey data analysis
- Bayesian analysis
- Monte Carlo simulation

Fully integrated in a general latent variable framework
What’s New in Mplus Version 7?

5 big new features:

1. Surprise
2. Factor analysis
   - Bi-factor EFA rotations, bi-factor ESEM, two-tier modeling
   - Bayesian EFA and CFA (BSEM), bi-factor BSEM
3. Analysis of several groups with approx. measurement invariance
   - using a Bayes approach (multiple-group BSEM)
   - using a two-level analysis with random intercepts and loadings
4. Analysis of individual differences SEM using measurement parameters that vary across subjects
5. Mixture analysis
   - Using a proper 3-step analyze-classify-analyze approach to investigate covariates and distal outcomes
   - Latent transition analysis with new output, covariates influencing transition probabilities, and probability parameterization
   - Exploratory LCA using Bayesian analysis

What’s New in Mplus Version 7, Continued

5 more big features:

1. 3-level SEM analysis, complex survey data handling, and multiple imputation
2. Cross-classified SEM analysis including random subjects and contexts (2 random modes)
3. IRT analysis with random items
4. Longitudinal analysis with approx. measurement invariance
   - using a Bayes approach (multiple-time point BSEM)
   - using cross-classified analysis of time and subjects
5. Analysis of changing membership over time

and 5 other new features:

1. Parallel analysis
2. LOOP plots (moderated mediation, cross-level interactions, etc)
3. Bayes plausible value factor score distribution plots for each subject
4. Two-tier algorithm
5. New convenience options: LOOP, DO, COV, DIFF, DO DIFF, MODEL=ALLFREE, auto-labeling, BY with random loadings, BITER = (minimum), TECH15, TECH16

- and if you don’t see what you had on your wish list, stay tuned for
  
  Version 7.1
  
  Version 7.2
  
  ...
Part 1:
  - Hardware and Timings
  - Recap of Bayesian Analysis in Mplus
  - Advances in Factor Analysis
  - Advances in Multiple-Group Analysis: Invariance Across Groups

Part 2:
  - Two-Level Random Loadings in IRT
  - Advances in Individual Differences Modeling: Invariance Across Subjects
  - Advances in Mixture Modeling
  - 3-Level Analysis, Complex Survey Data, and Multiple Imputation
  - Introductory Cross-Classified Analysis

Part 3:
  - Advanced Cross-Classified Analysis: Two Random Modes
  - Random Items in IRT
  - Advances in Longitudinal Analysis: Invariance Across Time
  - Advances in Longitudinal Analysis: Growth Modeling with Many Time Points

New option: PROCESSORS = a b; ! a = # processors, (b = # threads)
Can be used with: STARTS = c d; ! New default: c = 20, d = 4.

Multiple processors:
- Programming parallelized code, executable distributes the computing over different processors
- Parallelized code implemented for numerical integration (sample split into parts), missing data patterns, but not for Bayes
- Speed increases as a increases in PROCESSORS = a;

Multiple threads:
- Used with STARTS = , typically for TYPE = MIXTURE
- Different starting value sets are analyzed in different threads using different processors, possibly using more than one processor per thread
- Using as many threads as processors is fastest, but choosing fewer threads than processors is less memory demanding

Don’t settle for using an outdated computer. Use 64-bit instead of 32-bit, use several processors instead of only 1 or 2, use fast CPUs.

PC:
- Intel Core i7-3770K 3.5GHz/3.9GHz Turbo 8MB L3 Cache HD 4000 (Intel’s i7 processor released in April is also available for laptops)
- over-clocked to 4.2 to 4.5GHz
- 8 procs
- 32GB RAM
- 64-bit

Mac Pro:
- Intel Xeon 3.33GHz, 6-core
- 12 procs
- 24GB RAM
- 64-bit

Bengt’s PC as of June 2012: $1,500 Dell XPS 8500, i7-3770 with 8 processors, CPU of 3.40 GHz, 12 GB RAM, 64-bit.
Processors/Threads Available When?

- Multiple processors and multiple threads with random starts and multiple processors without random starts available when
  - TYPE=MIXTURE
  - Bayesian analysis with more than one chain if STVALUES=ML
  - Models that require numerical integration
- Multiple processors and multiple threads with random starts (w/o random starts, one processor is used) available when
  - TYPE=RANDOM
  - TYPE=TWOLEVEL and TYPE=THREELEVEL, continuous outcomes, ESTIMATOR= ML, MLR, and MLF without numerical integration
- Multiple processors but not multiple threads available when
  - Models with all continuous variables, missing data, and maximum likelihood estimation
  - Bayesian analysis with more than one chain
  - TYPE=TWOLEVEL, categorical outcomes, and ESTIMATOR= WLSMV

Timing Examples: 3 Computers

<table>
<thead>
<tr>
<th>Computer</th>
<th>Processor</th>
<th>RAM</th>
<th>Operating System</th>
</tr>
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<tbody>
<tr>
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<td>Intel Core 2 Duo CPU T8100</td>
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<td>Windows Vista 32-bit</td>
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<tr>
<td>(oldish laptop)</td>
<td>2.10 GHz</td>
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<tr>
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<tr>
<td>(2 dual-core desktop)</td>
<td>1333-MHz data rate</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.0 GHz</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dell XPS 8500</td>
<td>i7-3770</td>
<td>12 GB</td>
<td>Windows 7 64-bit</td>
</tr>
<tr>
<td>(new desktop June 2012)</td>
<td>8 processors</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>3.4GHz</td>
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</table>

Timing Examples

<table>
<thead>
<tr>
<th>Example 1</th>
<th>Example 2</th>
<th>Example 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor mixture model with 2 dimensions of integration, 2 classes, 18 binary outcomes and n=3314</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Factor mixture model with 2 dimensions of integration, 8 classes, 22 outcomes, and n=842</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Growth mixture model with 8 time points, count outcomes, 3 classes, starts = 32 8, 3 dimensions of integration, and n=1314</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Timing Examples: 3 Analyses

Example 1: Factor mixture model with 2 dimensions of integration, 2 classes, 18 binary outcomes and n=3314
Example 2: Factor mixture model with 2 dimensions of integration, 8 classes, 22 outcomes, and n=842
Example 3: Growth mixture model with 8 time points, count outcomes, 3 classes, starts = 32 8, 3 dimensions of integration, and n=1314
LOOP Option

LOOP is used in MODEL CONSTRAINT in conjunction with the PLOT option to create plots of one variable related to another, including a 95% confidence interval. An example:

**MODEL:** y ON x (p1);
**MODEL CONSTRAINT:**
PLOT(ypred);
**LOOP(age, 20, 50, 1);** ! 20 ≤ age ≤ 50 with steps of 1
ypred = p1*age;

- Plotting indirect effects with moderated mediation. Preacher, Rucker, Hayes (2007), MBR: Figure 3 - conditional indirect effect as a function of the moderator
- Plotting sensitivity graphs for causal effect mediation modeling. Imai et al. (2010), Psych Methods; Muthén (2011)

LOOP Example: Moderated Mediation of School Removal

**Model Equations:**
\[
\begin{align*}
\text{remove} &= \beta_0 + \beta_1 \text{agg5} + \beta_2 \text{tx} + \beta_3 \text{agg1} + \beta_4 \text{tx agg1} + \epsilon_1, \\
\text{agg5} &= \gamma_0 + \gamma_1 \text{tx} + \gamma_2 \text{agg1} + \gamma_3 \text{tx agg1} + \epsilon_2, \\
\text{agg1} &= \gamma_0 + (\gamma_1 + \gamma_3 \text{agg1}) \text{tx} + \gamma_2 \text{agg1} + \epsilon_2.
\end{align*}
\]

Indirect effect of tx on remove is \(\beta_1 (\gamma_1 + \gamma_3 \text{agg1})\), where agg1 moderates the effect of the treatment. Direct effect: \(\beta_2 + \beta_4 \text{agg1}\).

**Indirect Effect of Treatment as a Function of SD Units of the Moderator agg1**

**Indirect Effect:**

**Direct Effect:**

**PLOT Example Continued**

**inter = tx*agg1;**
**ANALYSIS:**
ESTIMATOR = BAYES;
PROCESSORS = 2; FBITER = 50000;
**MODEL:**
remove ON agg5 (beta1)
tx (beta2)
agg1 (beta3)
inter (beta4);
agg5 ON tx (gamma1)
agg1 (gamma2)
inter (gamma3);
**MODEL CONSTRAINT:**
PLOT(indirect direct);
! let moderate represent the range of the agg1 moderator
LOOP(moderate, -2, 2, 0.001);
indirect = beta1*(gamma1+gamma3*moderate);
direct = beta2+beta4*moderate;
**PLOT:**
TYPE = PLOT2;
DO Option

Example: Two groups, 9 factor loadings in each group, expressing the 9 group differences (note that the longer symbol – denotes a “dash”, namely a list, and the shorter symbol - denotes minus)

DO(1,9) diff# = lambda1# – lambda2#;
! Same as
! diff1 = lambda11 - lambda21;
! diff2 = lambda12 - lambda22:
! ... 
! diff9 = lambda19 - lambda29;

- Useful in MODEL CONSTRAINT to create NEW parameters
- Useful in MODEL PRIORS for Bayesian analysis
- Can also be used in DEFINE

BY with Random Loadings

ANALYSIS:
TYPE = TWOLEVEL RANDOM;
ESTIMATOR = BAYES;

MODEL:
% WITHIN %
s1-s10 | f BY y1-y10;
% BETWEEN %
[s1-s10];
s1-s10;

Easier than a series of statements like:
s | y ON f;

Bayesian Analysis: A Review of the Mplus Implementation

Mplus conceptualization:
- Mplus was envisioned 15 years ago as both a frequentist and a Bayesian program
- Bayesian analysis firmly established and its use growing in mainstream statistics
- Much less use of Bayes outside statistics
- Bayesian analysis not sufficiently accessible in other programs
- Bayes provides a broader platform for further Mplus development

Bayesian Analysis

Why do we have to learn about Bayes?
- More can be learned about parameter estimates and model fit
- Better small-sample performance, large-sample theory not needed
- Priors can better reflect substantive hypotheses
- Analyses can be made less computationally demanding
  - Frequentists can see Bayes with non-informative priors as a computing algorithm to get answers that would be the same as ML if ML could have been done
- New types of models can be analyzed
Overview of Bayesian Features In Mplus

- Single-level, multilevel, and mixture models
- Continuous and categorical outcomes (probit link)
- Default non-informative priors or user-specified informative priors (MODEL PRIORS)
- Multiple chains using parallel processing (CHAIN)
- Convergence assessment using Gelman-Rubin potential scale reduction factors (PSR \(\approx 1\))
- Posterior parameter distributions with means, medians, modes, and credibility intervals (POINT)
- Posterior parameter trace plots
- Autocorrelation plots
- Posterior predictive checking plots

Multiple Imputation (DATA IMPUTATION)

- Carried out using Bayesian estimation to create several data sets where missing values have been imputed
- The multiple imputation data sets can be used for subsequent model estimation using ML or WLSMV
- The imputed data sets can be saved for subsequent analysis or analysis can be carried out in the same run
- Imputation can be done based on an unrestricted H1 model using three different algorithms including sequential regressions
- Imputation can also be done based on an H0 model specified in the MODEL command

The set of variables used in the imputation of the data do not need to be the same as the set of variables used in the analysis

Single-level and multilevel data imputation are available

Multiple imputation data can be read using TYPE=IMPUTATION in the DATA command
Plausible Values (PLAUSIBLE)

- Plausible values are multiple imputations for missing values corresponding to a latent variable.
- Plausible values used in IRT contexts such as the ETS NAEP, The Nations Report Card (Mislevy et al., 1992).
- Available for both continuous and categorical latent variables (factors, random effects, latent classes).
- More informative than only an estimated factor score and its standard error or a class probability.
- Plausible values are more accurate than factor scores.
- Plausible values are given for each observation together with a summary over the imputed data sets for each observation and latent variable.
- Multiple imputation and plausible values examples are given in the Users Guide, Chapter 11.

Overview of Bayes News in Version 7

- Multiple-group and multiple time point analysis with approximate measurement invariance.
- 2-level analysis with random loadings.
- 3-level analysis with continuous (ML and Bayes) and categorical outcomes (Bayes only).
- Cross-classified analysis (Bayes only).
- EFA.
- Factor scores (plausible values).
- Kolmogorov-Smirnov convergence checking.

Bayesian Analysis Using Mplus: An Ongoing Project

Features that are not yet implemented include:

- ESEM.
- Logit link.
- Censored, count, and nominal variables.
- XWITH.
- Weights.
- C ON in mixtures.
- Mixture models with more than one categorical latent variable.
- Two-level mixtures.
- MODEL INDIRECT.
- MODEL CONSTRAINT except for NEW parameters.
- MODEL TEST.

News in Version 7: Bayesian EFA

- Bayesian estimation of exploratory factor analysis implemented in Mplus version 7 for models with continuous and categorical variables.
- Asymptotically the Bayes EFA is the same as the ML solution.
- Bayes EFA for categorical variable is a full information estimation method without using numerical integration and therefore feasible with any number of factors.
- New in Mplus Version 7: Improved performance of ML-EFA for categorical variables, in particular high-dimensional EFA models with Monte Carlo integration; improved unrotated starting values and standard errors.
The first step in the Bayesian estimation is the estimation of the unrotated model as a CFA model using the MCMC method. Obtain posterior distribution for the unrotated solution. To obtain the posterior distribution of the rotated parameters we simply rotate the generated unrotated parameters in every MCMC iteration, using oblique or orthogonal rotation. No priors. Priors could be specified currently only for the unrotated solution. If the unrotated estimation takes many iterations to converge, use THIN to reduce the number of rotations.

This MCMC estimation is complicated by identification issues that are similar to label switching in the Bayesian estimation of Mixture models. There are two types of identification issues in the Bayes EFA estimation.

The first type is identification issues related to the unrotated parameters: loading sign switching. Solution: constrain the sum of the loadings for each factor to be positive. Implemented in Mplus Version 7 for unrotated EFA and CFA. New in Mplus Version 7, leads to improved convergence in Bayesian SEM estimation:

$$\sum_{i=1}^{p} \lambda_{ij} > 0$$

The second type is identification issues related to the rotated parameters: loading sign switching and order of factor switching. Solution: Align the signs $s_j$ and factor order $\sigma$ to minimize MSE between the current estimates $\hat{\lambda}$ and the average estimate from the previous MCMC iterations $L$

$$\sum_{ij} (s_j \hat{\lambda}_{i(\sigma(j))} - L_{ij})^2$$

Minimize over all sign allocations $s_j$ and factor permutations $\sigma$.

Factor scores for the rotated solutions also available. Confidence intervals and posterior distribution plots. Using the optimal rotation in each MCMC iteration we rotate the unrotated factors to obtain the posterior distribution of the rotated factors. With continuous variables Bayes factor is computed to compare EFA with different number of factors. PPP value is computed with continuous or categorical variables.
Bayes Factors

- Bayes factors is an easy and quick way to compare models using BIC

$$BF = \frac{P(H_1)}{P(H_0)} = \frac{\text{Exp}(-0.5BIC_{H_1})}{\text{Exp}(-0.5BIC_{H_0})}$$

- Values of BF greater than 3 are considered evidence in support of H1
- New in Mplus Version 7: BIC is now included for all models with continuous items (single level and no mixtures)
- The above method can be used to easily compare nested and non-nested models

Bayes EFA: Simulation Study ($n = 500$)

Absolute bias, coverage and log-likelihood for EFA model with 7 factors and 35 ordered polytomous variables.

<table>
<thead>
<tr>
<th>Method</th>
<th>$\lambda_{11}$</th>
<th>$\lambda_{12}$</th>
<th>Log-Likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mplus Monte 500</td>
<td>.01(0.97)</td>
<td>.00(0.83)</td>
<td>-28580.3</td>
</tr>
<tr>
<td>Mplus Monte 5000</td>
<td>.01(0.96)</td>
<td>.00(0.87)</td>
<td>-28578.4</td>
</tr>
<tr>
<td>Mplus Bayes</td>
<td>.01(.90)</td>
<td>.00(.96)</td>
<td>-</td>
</tr>
<tr>
<td>Mplus WLSMV</td>
<td>.00(.94)</td>
<td>.00(.89)</td>
<td>-</td>
</tr>
<tr>
<td>IRTPRO MHRM</td>
<td>.00(.54)</td>
<td>.00(.65)</td>
<td>-28665.2</td>
</tr>
</tbody>
</table>

Bayes EFA: Example

Example is based on Mplus User’s Guide example 4.1 generated with 4 factors and 12 indicators.

```
DATA: FILE IS ex4.1.dat;
VARIABLE: NAMES ARE y1-y12;
ANALYSIS: TYPE = EFA 1 5; estimator=bayes;
```

We estimate EFA with 1, 2, 3, 4 or 5 factors.

Bayes EFA is the most accurate full information estimation method for high-dimensional EFA with categorical variables.
Bayes factor results: The posterior probability that the number of factors is 4 is: 99.59%. However, this is a power result - there is enough information in the data to support 4 factors and not enough to support 5 factors. Use BITER = (10000)

POSTERIOR PROBABILITIES FOR ALL MODELS:

1-FACTOR MODEL  0.0000
2-FACTOR MODEL  0.0000
3-FACTOR MODEL  0.0041
4-FACTOR MODEL  0.9959
5-FACTOR MODEL  0.0000

News in Version 7: Bayes Factor Scores Handling

- New improved language for factor scores with Bayesian estimation. The same language as for other estimators.
- SAVEDATA: FILE=fs.dat; SAVE=FS(300); FACTORS=factor names; This command specifies that 300 imputations will be used to estimate the factor scores and that plausible value distributions are available for plotting.
- Posterior mean, median, confidence intervals, standard error, all imputed values, distribution plot for each factor score for each latent variable for any model estimated with the Bayes estimator.
- Bayes factor score advantages: more accurate than ML factor scores in small sample size, Bayes factor score more accurate in secondary analysis such as for example computing correlations between factor.
Bayes Factor Scores Example

- Factor analysis with 3 indicators and 1 factor. Simulated data with N=45. True factor values are known. Bayes factor score estimates are more accurate. Bayes factor score SE are more accurate.
- ML factor scores are particularly unreliable when Var(Y) is near 0.

<table>
<thead>
<tr>
<th></th>
<th>ML</th>
<th>Bayes</th>
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</thead>
<tbody>
<tr>
<td>MSE</td>
<td>0.636</td>
<td>0.563</td>
</tr>
<tr>
<td>Coverage</td>
<td>20%</td>
<td>89%</td>
</tr>
<tr>
<td>Average SE</td>
<td>0.109</td>
<td>0.484</td>
</tr>
</tbody>
</table>

PSR Convergence Issues: Premature Stoppages

Due to Non-Identification

News in Version 7: Kolmogorov-Smirnov Convergence Test

- The Mplus default convergence criterion is the Potential Scale Reduction (PSR) criterion. The PSR is not sufficiently strict in certain cases, particularly when the model is not identified and an insufficient number of Bayes draws (iterations) has been used.
- A new more strict test of convergence is now implemented and reported as part of the Tech8 output.
- The test is based on computing the Kolmogorov-Smirnov (K-S) test for equal sample distribution.
- For each parameter the test uses 100 draws from each of the two MCMC chains and compares the two distributions.
Kolmogorov-Smirnov Convergence Test

- If convergence has been achieved the two distributions should be similar and the K-S test would not reject the hypothesis that the distributions are equal
- The test is based on the K-S statistic

\[ D = \sup |F_1(x) - F_2(x)| \]

where \( F_1(x) \) and \( F_2(x) \) are the sample distribution functions for chain 1 and 2
- Mplus computes \( D \) for each parameter and a p-value for the hypothesis that the two chains have the same distribution
- Large values of \( D \) and p-value < 0.05 indicate that convergence has not been achieved.
- Sometimes smaller values should be accepted because the mixing may be too slow to satisfy a strict criterion such as K-S. For more complex models p-values > 0.001 can be interpreted as confirmed convergence.

Consider the following unidentified factor analysis model which has a free factor mean.

```
VARIABLE:
  NAMES = y1-y4;

DATA:
  FILE = kolm.dat;

ANALYSIS:
  ESTIMATOR = BAYES;

MODEL:
  f BY y1-y4*1;
  f@1;
  y1-y4*.5;
  [f*0];
```

After 1800 iterations the PSR criterion is satisfied. The K-S test however clearly rejects the convergence and also points out that the unidentified parameters are the indicator means as well as the factor mean.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>K-S Statistic</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter 4</td>
<td>0.3900</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

The Kolmogorov-Smirnov distribution test for parameter 4 has a P-value 0.0000, indicating discrepancy posterior distributions in the different MCMC chains. This may indicate non-convergence due to an insufficient number of MCMC iterations or it may indicate a non-identified model. Specify a larger number of MCMC iterations using the BITER option of the analysis command to investigate the problem.

Using the option BITER=50000(20000) we specify that the MCMC chain runs for a minimum of 20000 iterations and a maximum of 50000

With this option the example does not converge, i.e., the PSR criterion agrees with the K-S criterion

The PSR criterion tends to accidentally be satisfied if an insufficient number of Bayes draws (iterations) has been used. The K-S criterion should be checked especially when we are not clear if the model is identified.
A Factor Analysis Example: Holzinger-Swineford Data

Data are from the classic 1939 factor analysis study by Holzinger and Swineford (1939). Twenty-six tests intended to measure a general factor and five specific factors were administered to seventh and eighth grade students in two schools, the Grant-White school ($n = 145$) and the Pasteur school ($n = 156$). Students from the Grant-White school came from homes where the parents were mostly American-born, whereas students from the Pasteur school came largely from working-class parents of whom many were foreign-born and where their native language was used at home.

Source:
ML CFA versus BESEM CFA

- ML CFA uses a very strong prior with an exact zero loading
- BESEM uses a zero-mean, small-variance prior for the loading:

BSEM can be used to specify approximate zeros for
- Cross-loadings
- Residual correlations
- Direct effects from covariates
- Group and time differences in intercepts and loadings

BSEM CFA vs ML CFA: Holzinger-Swineford 19 Variables

<table>
<thead>
<tr>
<th>CFA Factor Loading Pattern:</th>
<th>Spatial</th>
<th>Verbal</th>
<th>Speed</th>
<th>Memory</th>
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<tbody>
<tr>
<td>visual</td>
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<td>0</td>
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<td>wordm</td>
<td>x</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>addition</td>
<td>x</td>
<td>0</td>
<td>0</td>
<td>x</td>
</tr>
<tr>
<td>code</td>
<td>x</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>counting</td>
<td>x</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>straight</td>
<td>x</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>wordr</td>
<td>x</td>
<td>0</td>
<td>0</td>
<td>x</td>
</tr>
<tr>
<td>numberr</td>
<td>x</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td>figure</td>
<td>x</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>object</td>
<td>x</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>numberf</td>
<td>x</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>figurew</td>
<td>x</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td>deduct</td>
<td>x</td>
<td>0</td>
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<td>numeric</td>
<td>x</td>
<td>0</td>
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<td>problems</td>
<td>x</td>
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<td>0</td>
<td>0</td>
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<td>series</td>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>arithmet</td>
<td>x</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
ML CFA Testing Results For Holzinger-Swineford Data For Grant-White (n = 145) And Pasteur (n = 156)

<table>
<thead>
<tr>
<th>Model</th>
<th>$\chi^2$</th>
<th>df</th>
<th>P-value</th>
<th>RMSEA</th>
<th>CFI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grant-White CFA</td>
<td>216</td>
<td>146</td>
<td>0.000</td>
<td>0.057</td>
<td>0.930</td>
</tr>
<tr>
<td>EFA</td>
<td>110</td>
<td>101</td>
<td>0.248</td>
<td>0.025</td>
<td>0.991</td>
</tr>
<tr>
<td>Pasteur CFA</td>
<td>261</td>
<td>146</td>
<td>0.000</td>
<td>0.071</td>
<td>0.882</td>
</tr>
<tr>
<td>EFA</td>
<td>128</td>
<td>101</td>
<td>0.036</td>
<td>0.041</td>
<td>0.972</td>
</tr>
</tbody>
</table>

EFA has 6 (Grant-White) and 9 (Pasteur) significant cross-loadings

BSEM CFA for Holzinger-Swineford

- CFA: Cross-loadings fixed at zero - the model is rejected
- A more realistic hypothesis: Small cross-loadings allowed
- Cross-loadings are not all identified in terms of ML
- Different alternative: Bayesian CFA with informative priors for cross-loadings: $\lambda \sim N(0, 0.01)$.
  
  This means that 95% of the prior is in the range -0.2 to 0.2

Input BSEM CFA 19 Items 4 Factors Crossloading Priors

```plaintext
VARIABLE:
  NAMES = id female grade age y agem school;
  ! grade = 7/8
  ! school = 0/1 for Grant-White/Pasteur
  visual cubes paper flags general paragrap sentence wordc wordm addition code counting straight wordr numberfr figurew deduct numeric problemr series arithmetic;
USEV = visual-figurew;
USEOBS = school eq 0;
DEFINE:
  STANDARDIZE visual-figurew;
ANALYSIS:
  ESTIMATOR = BAYES;
  PROCESSORS = 2;
  FBITER = 10000;
```
**Input BSEM CFA 19 Items 4 Factors Crossloading Priors (Continued)**

MODEL:
- spatial BY visual* cubes paper flags;
- verbal BY general* paragrap sentence wordc wordm;
- speed BY addition* code counting straight;
- memory BY wordr* numberr figurer object numberf figurew;
- spatial-memory@1;

  † cross-loadings:
  - spatial BY general-figurew*0 (a1-a15);
  - verbal BY visual-flags*0 (b1-b4);
  - verbal BY addition-figurew*0 (b5-b14);
  - speed BY visual-wordm*0 (c1-c9);
  - speed BY wordr-figurew*0 (c10-c15);
  - memory BY visual-straight*0 (d1-d13);

MODEL PRIORS:
- a1-d13 ∼ N(0,.01);

OUTPUT:
- TECH1 TECH8 STDY;
- TYPE = PLOT2;

---

**Bayesian Posterior Predictive Checking For The CFA Model For Grant-White**

CFA with small cross-loadings not rejected by Bayes PPC: 
\[ p = 0.361 \]

Conventional CFA model rejected by Bayes PPC: 
\[ p = 0.006 \]

---

Bayesian analysis

<table>
<thead>
<tr>
<th>Model</th>
<th>Z²</th>
<th>Df</th>
<th>P-value</th>
<th>RMSEA</th>
<th>CFI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grant-White</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CFA</td>
<td>216</td>
<td>146</td>
<td>0.000</td>
<td>0.057</td>
<td>0.930</td>
</tr>
<tr>
<td>EFA</td>
<td>110</td>
<td>101</td>
<td>0.248</td>
<td>0.025</td>
<td>0.991</td>
</tr>
</tbody>
</table>

Pasteur

<table>
<thead>
<tr>
<th>Model</th>
<th>Z²</th>
<th>Df</th>
<th>P-value</th>
<th>RMSEA</th>
<th>CFI</th>
</tr>
</thead>
<tbody>
<tr>
<td>CFA</td>
<td>261</td>
<td>146</td>
<td>0.000</td>
<td>0.071</td>
<td>0.882</td>
</tr>
<tr>
<td>EFA</td>
<td>128</td>
<td>101</td>
<td>0.036</td>
<td>0.041</td>
<td>0.972</td>
</tr>
</tbody>
</table>

**Grant-White Factor Loadings Using Informative Priors**

<table>
<thead>
<tr>
<th>Spatial</th>
<th>Verbal</th>
<th>Speed</th>
<th>Memory</th>
</tr>
</thead>
<tbody>
<tr>
<td>visual</td>
<td>0.64*</td>
<td>0.012</td>
<td>0.050</td>
</tr>
<tr>
<td>cubes</td>
<td>0.52*</td>
<td>-0.008</td>
<td>-0.010</td>
</tr>
<tr>
<td>paper</td>
<td>0.45*</td>
<td>0.040</td>
<td>0.041</td>
</tr>
<tr>
<td>flags</td>
<td>0.67*</td>
<td>0.046</td>
<td>-0.020</td>
</tr>
<tr>
<td>general</td>
<td>0.037</td>
<td>0.78*</td>
<td>0.049</td>
</tr>
<tr>
<td>paragrapp</td>
<td>-0.001</td>
<td>0.83*</td>
<td>-0.053</td>
</tr>
<tr>
<td>sentence</td>
<td>-0.045</td>
<td>0.88*</td>
<td>0.021</td>
</tr>
<tr>
<td>wordc</td>
<td>0.053</td>
<td>0.61*</td>
<td>0.096</td>
</tr>
<tr>
<td>wordm</td>
<td>-0.012</td>
<td>0.88*</td>
<td>-0.086</td>
</tr>
<tr>
<td>addition</td>
<td>-0.17*</td>
<td>0.030</td>
<td>0.79*</td>
</tr>
<tr>
<td>code</td>
<td>-0.002</td>
<td>0.054</td>
<td>0.56*</td>
</tr>
<tr>
<td>counting</td>
<td>0.013</td>
<td>-0.092</td>
<td>0.82*</td>
</tr>
<tr>
<td>straight</td>
<td>0.18*</td>
<td>0.043</td>
<td>0.63*</td>
</tr>
<tr>
<td>wordr</td>
<td>-0.040</td>
<td>0.044</td>
<td>-0.031</td>
</tr>
<tr>
<td>numberr</td>
<td>0.003</td>
<td>-0.004</td>
<td>-0.038</td>
</tr>
<tr>
<td>figurer</td>
<td>0.132</td>
<td>-0.024</td>
<td>-0.049</td>
</tr>
<tr>
<td>object</td>
<td>-0.139</td>
<td>0.014</td>
<td>0.029</td>
</tr>
<tr>
<td>numberf</td>
<td>0.099</td>
<td>-0.071</td>
<td>0.095</td>
</tr>
<tr>
<td>figurew</td>
<td>0.012</td>
<td>0.045</td>
<td>0.007</td>
</tr>
</tbody>
</table>

**Number of significant cross-loadings:** 2 for Grant-White and 1 for Pasteur

---

*Significant loadings are denoted by asterisks.*

Bengt Muthén & Tihomir Asparouhov New Developments in Mplus Version 7 69/146

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Bengt Muthén & Tihomir Asparouhov New Developments in Mplus Version 7 71/146

Bengt Muthén & Tihomir Asparouhov New Developments in Mplus Version 7 72/146
### Effects Of Using Different Variances For The Informative Priors Of The Cross-Loadings For The Holzinger-Swineford Data: Grant-White

<table>
<thead>
<tr>
<th>Prior variance</th>
<th>95% cross-loading limit</th>
<th>PPP</th>
<th>Cross-loading (Posterior SD)</th>
<th>Factor corr. range</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.02</td>
<td>0.361</td>
<td>0.189 (.078)</td>
<td>0.443-0.557</td>
</tr>
<tr>
<td>0.02</td>
<td>0.28</td>
<td>0.441</td>
<td>0.248 (.096)</td>
<td>0.439-0.542</td>
</tr>
<tr>
<td>0.03</td>
<td>0.34</td>
<td>0.457</td>
<td>0.275 (.109)</td>
<td>0.432-0.530</td>
</tr>
<tr>
<td>0.04</td>
<td>0.39</td>
<td>0.455</td>
<td>0.292 (.120)</td>
<td>0.413-0.521</td>
</tr>
<tr>
<td>0.05</td>
<td>0.44</td>
<td>0.453</td>
<td>0.303 (.130)</td>
<td>0.404-0.513</td>
</tr>
<tr>
<td>0.06</td>
<td>0.48</td>
<td>0.447</td>
<td>0.309 (.139)</td>
<td>0.400-0.510</td>
</tr>
<tr>
<td>0.07</td>
<td>0.52</td>
<td>0.439</td>
<td>0.315 (.148)</td>
<td>0.395-0.508</td>
</tr>
<tr>
<td>0.08</td>
<td>0.55</td>
<td>0.439</td>
<td>0.319 (.156)</td>
<td>0.387-0.508</td>
</tr>
<tr>
<td>0.09</td>
<td>0.59</td>
<td>0.435</td>
<td>0.323 (.163)</td>
<td>0.378-0.506</td>
</tr>
<tr>
<td>1.00</td>
<td>0.62</td>
<td>0.427</td>
<td>0.327 (.171)</td>
<td>0.369-0.504</td>
</tr>
</tbody>
</table>

### Summary of Analyses of Holzinger-Swineford 19-Variable Data

- Conventional, frequentist, CFA model rejected
- Bayesian CFA with informative cross-loadings not rejected
- The Bayesian approach uses an intermediate hypothesis:
  - Less strict than conventional CFA
  - Stricter than EFA, where the hypothesis only concerns the number of factors
  - Cross-loadings shrunken towards zero; acceptable degree of shrinkage monitored by PPP
- Bayes modification indices obtained by estimated cross-loadings
- Factor correlations: EFA < BSEM < CFA

### Comparing BSEM And Target Rotation

- Target rotation: EFA rotation chosen to match zero target loadings using least-squares fitting
  - Similarities: Replaces mechanical rotation with judgement/hypotheses
  - Differences: For Target, specifying more than the necessary EFA restrictions does not affect fit and user-defined closeness to zero is replaced with least-squares fitting
- Results for Holzinger-Swineford data:
  - Results similar to EFA with 10 significant cross-loadings for Grant-White and 15 for Pasteur

### Comparing BSEM And ESEM

  - Similarities: Both ESEM and BSEM can be used for measurement models in SEM
  - Differences:
    - ESEM is EFA-oriented while BSEM is CFA-oriented
    - ESEM uses a mechanical rotation and the rotation is not based on information from other parts of the model
    - BSEM is applicable not only to measurement models
Bi-Factor Modeling

As popular today as in 1939.

Reise, Morizot, & Hays (2007). The role of the bifactor model in resolving dimensionality issues in health outcomes measures. Quality of Life Research, 16, 1931.
- Testlet modeling, e.g. for PISA test items
- Longitudinal modeling with across-time correlation for residuals of the same item

Bi-Factor Modeling Without Using CFA

New methods that do not use regular ML CFA:
- Bi-factor EFA (Jennrich & Bentler)
  - ROTATION = BI-GEOMIN
- Bi-factor ESEM
  - ROTATION = BI-GEOMIN (same as above)
  - Bi-factor ESEM with general CFA factor and ROTATION = GEOMIN for specific factors
- Bi-factor BSEM (no rotation)

Bi-Factor EFA: UG Ex4.7

TITLE: this is an example of a bi-factor exploratory factor analysis with continuous factor indicators
DATA: FILE = ex4.7.dat;
VARIABLE: NAMES = y1-y10;
ANALYSIS: TYPE = EFA 2 3;
  ROTATION = BI-GEOMIN;

The number of factors is the general factor plus the specific factors

Bi-Factor ESEM: UG Ex5.29

Same as Bi-Factor EFA

TITLE: this is an example of a bi-factor EFA
DATA: FILE = ex5.29.dat;
VARIABLE: NAMES = y1-y10;
ANALYSIS:  ROTATION = BI-GEOMIN;
MODEL: fg f1 f2 BY y1-y10 (*1);
OUTPUT: STDY;
Bi-Factor ESEM with CFA Factor and Regular Rotation: UG Ex5.30

TITLE: this is an example of bi-factor EFA with two items loading on only the general factor
DATA: FILE = ex5.30.dat;
VARIABLE: NAMES = y1-y10;
ANALYSIS: ROTATION = GEOMIN;
MODEL: fg BY y1-y10*;
f1-f2 BY y1-y8 (*1);
f1-f2 WITH f1-f2@0;
OUTPUT: STDY;

Bi-Factor BSEM (CFA-Like; No Rotation): UG Ex5.31

TITLE: this is an example of a Bayesian bi-factor CFA with two items loading on only the general factor and cross-loadings with zero-mean and small-variance priors
DATA: FILE = ex5.31.dat;
VARIABLE: NAMES = y1-y10;
ANALYSIS: ESTIMATOR = BAYES;
PROCESSORS = 2;
MODEL: fg BY y1-y10*;
f1 BY y1-y4 y5-y10 (f1xlam5-f1xlam10);
f2 BY y5-y8 y1-y4 y9-y10(f2xlam1-f2xlam6);
f1-f2 WITH f1-f2@0;
MODEL PRIORS: f1xlam5-f2xlam6~N(0,0.01);
PLOT: TYPE = PLOT2;

Holzinger-Swineford, 24 Variables: Input Excerpts for Bi-Factor EFA

USEVARIABLES = visual - arithmet;
USEOBSERVATIONS = school EQ 0;
ANALYSIS: TYPE = EFA 5 5;
ROTATION = BI-GEOMIN;
Bi-Factor EFA for Holzinger-Swineford’s 24-variable Grant-White data

<table>
<thead>
<tr>
<th></th>
<th>General</th>
<th>Spatial</th>
<th>Verbal</th>
<th>Speed</th>
<th>Memory</th>
</tr>
</thead>
<tbody>
<tr>
<td>visual</td>
<td>0.621*</td>
<td>0.384*</td>
<td>-0.065</td>
<td>0.072</td>
<td>0.002</td>
</tr>
<tr>
<td>cubes</td>
<td>0.433*</td>
<td>0.207</td>
<td>-0.103</td>
<td>-0.115</td>
<td>-0.118</td>
</tr>
<tr>
<td>paper</td>
<td>0.430*</td>
<td>0.343*</td>
<td>0.058</td>
<td>0.225</td>
<td>0.079</td>
</tr>
<tr>
<td>flags</td>
<td>0.583*</td>
<td>0.311*</td>
<td>-0.028</td>
<td>-0.077</td>
<td>-0.109</td>
</tr>
<tr>
<td>general</td>
<td>0.610*</td>
<td>-0.034</td>
<td>0.524*</td>
<td>0.001</td>
<td>-0.075</td>
</tr>
<tr>
<td>paragrap</td>
<td>0.554*</td>
<td>0.053</td>
<td>0.618*</td>
<td>0.012</td>
<td>0.102</td>
</tr>
<tr>
<td>sentence</td>
<td>0.572*</td>
<td>-0.037</td>
<td>0.622*</td>
<td>0.010</td>
<td>-0.064</td>
</tr>
<tr>
<td>wordc</td>
<td>0.619*</td>
<td>0.006</td>
<td>0.354*</td>
<td>0.038</td>
<td>-0.048</td>
</tr>
<tr>
<td>wordm</td>
<td>0.582*</td>
<td>-0.008</td>
<td>0.603*</td>
<td>-0.137</td>
<td>0.009</td>
</tr>
<tr>
<td>addition</td>
<td>0.508*</td>
<td>-0.528</td>
<td>-0.036</td>
<td>0.327</td>
<td>0.009</td>
</tr>
<tr>
<td>code</td>
<td>0.532*</td>
<td>-0.031</td>
<td>0.046</td>
<td>0.428*</td>
<td>0.310*</td>
</tr>
<tr>
<td>counting</td>
<td>0.568*</td>
<td>-0.229</td>
<td>-0.216*</td>
<td>0.302</td>
<td>-0.093</td>
</tr>
<tr>
<td>straight</td>
<td>0.643*</td>
<td>0.217</td>
<td>0.004</td>
<td>0.526*</td>
<td>-0.032</td>
</tr>
</tbody>
</table>

Bi-Factor EFA for Holzinger-Swineford, Continued

6 significant cross-loadings

Input for Bi-Factor ESEM with a General CFA Factor and Regular Rotation of the Specific Factors

USEVARIABLES = visual-arithmet;
USEOBSERVATIONS = school EQ 0;

ANALYSIS:

MODEL:

fg BY visual-arithmet*;
fg@1;
s1-s4 BY visual-figurew (*1);
fg WITH s1-s4@0;

OUTPUT: STDY;
Bi-Factor ESEM with a General CFA Factor and Regular Rotation of the Specific Factors

STDY Standardization

<table>
<thead>
<tr>
<th>General</th>
<th>Spatial</th>
<th>Verbal</th>
<th>Speed</th>
<th>Memory</th>
</tr>
</thead>
<tbody>
<tr>
<td>visual</td>
<td>0.574*</td>
<td>0.450*</td>
<td>-0.081</td>
<td>0.087</td>
</tr>
<tr>
<td>cubes</td>
<td>0.425*</td>
<td>0.277*</td>
<td>-0.140</td>
<td>-0.076</td>
</tr>
<tr>
<td>paper</td>
<td>0.382*</td>
<td>0.352*</td>
<td>0.067</td>
<td>0.151</td>
</tr>
<tr>
<td>flags</td>
<td>0.574*</td>
<td>0.356*</td>
<td>-0.070</td>
<td>-0.057</td>
</tr>
<tr>
<td>general</td>
<td>0.652*</td>
<td>-0.013</td>
<td>0.460*</td>
<td>0.034</td>
</tr>
<tr>
<td>paragrap</td>
<td>0.604*</td>
<td>0.038</td>
<td>0.580*</td>
<td>-0.031</td>
</tr>
<tr>
<td>sentence</td>
<td>0.614*</td>
<td>-0.018</td>
<td>0.574*</td>
<td>0.042</td>
</tr>
<tr>
<td>worde</td>
<td>0.628*</td>
<td>0.040</td>
<td>0.315*</td>
<td>0.106</td>
</tr>
<tr>
<td>wordm</td>
<td>0.657*</td>
<td>-0.042</td>
<td>0.539*</td>
<td>-0.126</td>
</tr>
<tr>
<td>addition</td>
<td>0.448*</td>
<td>-0.323*</td>
<td>-0.060</td>
<td>0.537*</td>
</tr>
<tr>
<td>code</td>
<td>0.468*</td>
<td>0.037</td>
<td>0.055</td>
<td>0.450*</td>
</tr>
<tr>
<td>counting</td>
<td>0.487*</td>
<td>-0.059</td>
<td>-0.252*</td>
<td>0.554*</td>
</tr>
<tr>
<td>straight</td>
<td>0.517*</td>
<td>0.394*</td>
<td>0.021</td>
<td>0.606*</td>
</tr>
</tbody>
</table>

6 significant cross-loadings

Input for Bi-Factor BSEM for Holzinger-Swineford

DATA: FILE = H-S Combined.txt;
VARIABLE: NAMES = id female grade agey agem school visual cubes paper flags general paragrap sentence worde wordm addition code counting straight wordr numberr figurer object numberf figurew deduct numeric problemr series numberf arithmet;
USEVARIABLES = visual-arithmet;
DEFINE: STANDARDIZE visual-arithmet;
ANALYSIS: ESTIMATOR = BAYES;
PROCESSORS = 2;
FBITER = 15000;
MODEL: g BY visual-arithmet*;
spatial BY visual-flags*;
general-arithmet*0 (s1-s20);

MODEL PRIORS:
s1-s20~N(0,0.01);
v1-v19~N(0,0.01);
sp1-sp20~N(0,0.01);
m1-m18~N(0,0.01);

OUTPUT: TECH1 TECH8 STDYX;
PLOT: TYPE = PLOT3;
Bi-Factor BSEM

<table>
<thead>
<tr>
<th>STDY Standardization</th>
<th>General</th>
<th>Spatial</th>
<th>Verbal</th>
<th>Speed</th>
<th>Memory</th>
</tr>
</thead>
<tbody>
<tr>
<td>visual</td>
<td>0.615*</td>
<td>0.387*</td>
<td>-0.014</td>
<td>0.003</td>
<td>0.003</td>
</tr>
<tr>
<td>cubes</td>
<td>0.434*</td>
<td>0.271</td>
<td>-0.041</td>
<td>-0.040</td>
<td>-0.040</td>
</tr>
<tr>
<td>paper</td>
<td>0.409*</td>
<td>0.359*</td>
<td>0.057</td>
<td>0.061</td>
<td>0.057</td>
</tr>
<tr>
<td>flags</td>
<td>0.583*</td>
<td>0.354*</td>
<td>0.017</td>
<td>-0.053</td>
<td>-0.033</td>
</tr>
<tr>
<td>general</td>
<td>0.603*</td>
<td>0.021</td>
<td>0.537*</td>
<td>0.040</td>
<td>-0.047</td>
</tr>
<tr>
<td>paragrap</td>
<td>0.577*</td>
<td>0.009</td>
<td>0.598*</td>
<td>-0.039</td>
<td>0.051</td>
</tr>
<tr>
<td>sentence</td>
<td>0.576*</td>
<td>-0.025</td>
<td>0.604*</td>
<td>0.009</td>
<td>-0.045</td>
</tr>
<tr>
<td>wordc</td>
<td>0.619*</td>
<td>0.032</td>
<td>0.361*</td>
<td>0.052</td>
<td>-0.016</td>
</tr>
<tr>
<td>wordm</td>
<td>0.618*</td>
<td>-0.022</td>
<td>0.593*</td>
<td>-0.102</td>
<td>0.000</td>
</tr>
<tr>
<td>addition</td>
<td>0.463*</td>
<td>-0.194*</td>
<td>0.009</td>
<td>0.641*</td>
<td>0.000</td>
</tr>
<tr>
<td>code</td>
<td>0.521*</td>
<td>0.001</td>
<td>0.047</td>
<td>0.398*</td>
<td>0.135</td>
</tr>
<tr>
<td>counting</td>
<td>0.505*</td>
<td>0.004</td>
<td>-0.101</td>
<td>0.542*</td>
<td>-0.042</td>
</tr>
<tr>
<td>straight</td>
<td>0.587*</td>
<td>0.191*</td>
<td>0.029</td>
<td>0.432*</td>
<td>-0.029</td>
</tr>
</tbody>
</table>

3 significant cross-loadings, all for the Spatial factor.

Summary of Bi-Factor Analysis for Holzinger-Swineford Data for the Grant-White School

- Bi-factor CFA does not fit
- EFA with bi-factor rotation: 6 significant cross-loadings
- Bi-factor ESEM with general CFA factor and regular rotation for the specific factors: 6 significant cross-loadings
- Bi-factor BSEM with no rotation: 3 significant cross-loadings

Muthén & Asparouhov (2012): Rejoinder: Mastering a New Method suggests that BSEM bi-factor analysis outperforms all other bi-factor analyses of the classic Holzinger-Swineford data

Bi-Factor BSEM, Continued

<table>
<thead>
<tr>
<th>STDY Standardization</th>
<th>General</th>
<th>Spatial</th>
<th>Verbal</th>
<th>Speed</th>
<th>Memory</th>
</tr>
</thead>
<tbody>
<tr>
<td>wordr</td>
<td>0.385*</td>
<td>-0.021</td>
<td>0.050</td>
<td>-0.004</td>
<td>0.440*</td>
</tr>
<tr>
<td>numbre</td>
<td>0.375*</td>
<td>0.013</td>
<td>-0.002</td>
<td>-0.021</td>
<td>0.386*</td>
</tr>
<tr>
<td>figurer</td>
<td>0.529*</td>
<td>0.102</td>
<td>-0.057</td>
<td>-0.051</td>
<td>0.348*</td>
</tr>
<tr>
<td>object</td>
<td>0.460*</td>
<td>-0.128</td>
<td>0.016</td>
<td>0.050</td>
<td>0.463*</td>
</tr>
<tr>
<td>number</td>
<td>0.560*</td>
<td>0.028</td>
<td>-0.104</td>
<td>0.083</td>
<td>0.267*</td>
</tr>
<tr>
<td>figurew</td>
<td>0.461*</td>
<td>-0.030</td>
<td>-0.003</td>
<td>0.002</td>
<td>0.190</td>
</tr>
<tr>
<td>deduct</td>
<td>0.662*</td>
<td>0.002</td>
<td>0.068</td>
<td>-0.105</td>
<td>0.029</td>
</tr>
<tr>
<td>numeric</td>
<td>0.672*</td>
<td>0.044</td>
<td>-0.067</td>
<td>0.099</td>
<td>-0.023</td>
</tr>
<tr>
<td>problem</td>
<td>0.665*</td>
<td>-0.002</td>
<td>0.054</td>
<td>-0.099</td>
<td>0.017</td>
</tr>
<tr>
<td>series</td>
<td>0.741*</td>
<td>0.048</td>
<td>0.040</td>
<td>-0.044</td>
<td>-0.032</td>
</tr>
<tr>
<td>arithmet</td>
<td>0.674*</td>
<td>-0.152*</td>
<td>0.066</td>
<td>0.159</td>
<td>0.022</td>
</tr>
</tbody>
</table>

Two-Tier Modeling

- New in Mplus Version 7: Two-tier algorithm. New algorithm that reduces the dimension of integration with ML estimation of SEM with categorical variables
- Multiple factors that load on different indicators and are uncorrelated are reduced to 1 dimensional integration
Two-Tier Simulation Study for a Bi-Factor Model

Table 1: Absolute bias, coverage and log-likelihood for the bifactor model.

<table>
<thead>
<tr>
<th>Method</th>
<th>$\lambda_{11}$</th>
<th>$\lambda_{12}$</th>
<th>$\tau_{11}$</th>
<th>Log-Likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mplus Two Tier</td>
<td>.02(.95)</td>
<td>.02(.97)</td>
<td>.02(.97)</td>
<td>-31664.1</td>
</tr>
<tr>
<td>Mplus Monte 500</td>
<td>.03(.92)</td>
<td>.07(.92)</td>
<td>.02(.94)</td>
<td>-31760.8</td>
</tr>
<tr>
<td>Mplus Monte 5000</td>
<td>.03(.94)</td>
<td>.01(.95)</td>
<td>.01(.96)</td>
<td>-31678.5</td>
</tr>
<tr>
<td>Mplus Bayes</td>
<td>.11(.89)</td>
<td>.07(.93)</td>
<td>.16(.89)</td>
<td>-16667.9</td>
</tr>
<tr>
<td>Mplus WLSMV</td>
<td>.06(.93)</td>
<td>.05(.95)</td>
<td>.13(.93)</td>
<td>-16767.9</td>
</tr>
<tr>
<td>IRT&amp;PRO Two Tier</td>
<td>.02(.98)</td>
<td>.00(.96)</td>
<td>.01(.98)</td>
<td>-31680.7</td>
</tr>
</tbody>
</table>

Table 2: Average standard error, ratio between average standard error and standard deviation for the bifactor model.

<table>
<thead>
<tr>
<th>Method</th>
<th>$\lambda_{11}$</th>
<th>$\lambda_{12}$</th>
<th>$\tau_{11}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mplus Two Tier</td>
<td>0.12(0.97)</td>
<td>0.16(1.00)</td>
<td>0.19(1.10)</td>
</tr>
<tr>
<td>Mplus Monte 500</td>
<td>0.12(0.91)</td>
<td>0.15(0.99)</td>
<td>0.18(1.05)</td>
</tr>
<tr>
<td>Mplus Monte 5000</td>
<td>0.12(0.89)</td>
<td>0.15(0.94)</td>
<td>0.19(1.09)</td>
</tr>
<tr>
<td>Mplus Bayes</td>
<td>0.13(0.97)</td>
<td>0.16(1.01)</td>
<td>0.19(1.09)</td>
</tr>
<tr>
<td>Mplus WLSMV</td>
<td>0.13(0.97)</td>
<td>0.15(0.98)</td>
<td>0.18(1.08)</td>
</tr>
<tr>
<td>IRT&amp;PRO Two Tier</td>
<td>0.17(1.47)</td>
<td>0.16(1.01)</td>
<td>0.18(1.02)</td>
</tr>
</tbody>
</table>

Two-Tier Modeling: A Longitudinal Example

With categorical outcomes, ML estimation leads to numerical integration that is difficult with many dimensions/factors. The two-tier algorithm reduces the 6 dimensions of integration to 4. Bi-factor CFA has a similar structure that benefits from two-tier computing.

Two-Tier Modeling: Bi-Factor Model for PISA Math Items

The two-tier algorithm reduces the 6 dimensions of integration to 2.

Cai, Yang, & Hansen (2011) Generalized full-information item bifactor analysis. Psychological Methods, 16, 221-248
Two-Tier Estimation

- The two-tier integration method can be estimated in Mplus prior to Version 7 as a two-level multiple group model where the general factors are between level factors, while the specific factors are within level factors and the multiple groups represent the different blocks of variables that are correlated beyond the general factors.
- In Mplus Version 7 this is no longer necessary and the two-tier estimation can be used when the model is set up as a regular single level model.
- The program will automatically determine if the model is a bifactor-like model that allows for more optimal two-tier integration and will set up the two-level multiple group model automatically. On the surface you can not tell if the model is estimated through a special two-level setup.

Fixed versus Random Groups

- **Fixed mode:**
  - Inference to only the groups in the sample
  - Small to medium number of groups
- **Random mode:**
  - Inference to a population of groups from which the current set of groups is a random sample
  - Medium to large number of groups

Advances in Multiple-Group Analysis: Invariance Across Groups

- An old dilemma
- Two new solutions

Two Solutions

- New solution no. 1, suitable for a small to medium number of groups
  - A new BSEM approach where group is a fixed mode:
    - Multiple-group BSEM
    - Approximate invariance allowed
- New solution no. 2, suitable for a medium to large number of groups
  - A new Bayes approach where group is a random mode
  - No limit on the number of groups
COV Option

When choosing Bayesian priors, parameters are often specified as independent, but can also be allowed to covary. This is accomplished with the COVARIANCE option in MODEL PRIORS. An example:

MODEL:
y ON x1(a)
x2(b);
MODEL PRIORS:
a ~ N(10, 4);
b ~ N(6, 1);
COV(a,b)=0.5;

which says that the prior bivariate distribution of a and b has a covariance of 0.5, which translates to a correlation of 0.25:

\[ \frac{0.5}{\sqrt{4} \sqrt{1}} = 0.25 \]

COV Option Continued

The COVARIANCE option can also be used to specify small differences between parameters. Note that

\[ V(a - b) = V(a) + V(b) - 2 \text{cov}(a,b), \] (4)

so that if \( V(a) = V(b) = 1000 \), using \( \text{cov}(a,b) = 999.995 \) gives \( V(a - b) = 0.01 \). With a normal distribution, this means that the difference has a 95% chance of being between \(-0.196\) and \(+0.196\), that is, in a small range around the zero mean.

Example: Two parameters a and b for which we want to apply zero-mean, small-variance priors

\[ a - b \sim N(0, 1000); \] ! non-informative priors
\[ \text{COV}(a, b)=999.995; \]

DIFF Option

Used with MODEL PRIORS in Bayesian analysis to simplify specifying differences between parameters.

Example: The difference between the parameters a and b

DIFF(a, b) ~ N(0, 0.01);

This is the same as the two statements

\[ a - b \sim N(0,1000); \] ! non-informative prior
\[ \text{COV}(a, b)=999.995; \]
DO DIFF Option

DO DIFF is used to express parameter differences between large sets of parameters and groups/timepoints.

Example: Group differences for 4 parameters in 3 groups. Let \( \text{lam}_{jk} \) denote a factor loading for group/timepoint \( j \) and variable \( k \):

\[
\text{DO}(1,4) \ \text{DIFF}(\text{lam1}_{#} - \text{lam3}_{#}) \sim N(0,0.01);
\]

! for variable 1 this results in approximate invariance across the 3 groups:

\[
\text{lam1}_{1} \approx \text{lam2}_{1} \\
\text{lam1}_{1} \approx \text{lam3}_{1} \\
\text{lam2}_{1} \approx \text{lam3}_{1}
\]

! etc. for the lam parameters for variables 2-4

Auto Labeling Option for Multiple Groups

Used in conjunction with DO DIFF for multiple-group Bayes, which is carried out using TYPE=MIXTURE and KNOWNCLASS. For instance, with a factor measured by 4 variables in 3 groups:

**ANALYSIS:** TYPE = MIXTURE; ! 3 classes
ESTIMATOR = BAYES;
PROCESSORS = 2;
MODEL = ALLFREE;

**MODEL:**

\%

f BY y1-y4* (lam#_{1} - lam#_{4});

! the above gives labels for all 3 groups (group is #)

**MODEL PRIORS:**

\%

DO(1,4) DIFF(lam1_{#} - lam3_{#})\sim N(0,0.01);

Auto labeling saves an enormous amount of typing.

New Solution No. 1: Group is Fixed Mode. UG Ex5.33

**TITLE:** this is an example of a Bayesian multiple group model with approximate measurement invariance

**DATA:** FILE = ex5.33.dat;

**VARIABLE:** NAMES = u y1-y6 group;

**USEVARIABLES =** y1-y6 group;

**CLASSES =** c(10);

**KNOWNCLASS =** c(group = 1-10);

**ANALYSIS:** TYPE = MIXTURE;

**ESTIMATOR =** BAYES;

**PROCESSORS =** 2;

**MODEL =** ALLFREE;

**MODEL:**

\%

f1 BY y1-y3* (lam#_{1}-lam#_{3});

f2 BY y4-y6* (lam#_{4}-lam#_{6});

[y1-y6] (nu#_{1}-nu#_{6});

\%

f1-f2@0;

**MODEL PRIORS:**

\%

DO(1,6) DIFF(lam1_{#} - lam10_{#})\sim N(0,0.01);

**OUTPUT:** TECH1 TECH8;

**PLOT:** TYPE = PLOT2;

New Solution No. 2: Group is Random Mode

Two-level Factor Analysis with Random Loadings

Consider a single factor \( \eta \). For factor indicator \( r \) \((r = 1, 2, \ldots, p)\) for individual \( i \) in group (cluster) \( j \),

\[
y_{rij} = \nu_{ij} + \lambda_{ij} \eta_{ij} + \epsilon_{ij}.
\]

\[
\eta_{ij} = \eta_{j} + \zeta_{ij}, (this \ may \ be \ viewed \ as \ \eta_{B_{j}} + \eta_{W_{ij}})
\]

\[
\nu_{ij} = \nu_{i} + \delta_{\nu_{i}},
\]

\[
\lambda_{ij} = \lambda_{r} + \delta_{\lambda_{ij}},
\]

where \( \nu_{i} \) is the mean of the \( r^{th} \) intercept and \( \lambda_{r} \) is the mean of the \( r^{th} \) factor loading. Because the factor loadings are free, the factor metric is set by fixing \( V(\zeta_{ij}) = 1 \) (the between-level variance \( V(\eta_{j}) \) is free).

Note that the same loading is multiplying both the between- and within-level parts of the factor \( \eta \).
Two-Level Factor Analysis with Random Loadings: 3 Model Versions

\[ y_{rij} = \nu_{rij} + \lambda_{rij} \eta_{ij} + \epsilon_{ij}, \quad (9) \]
\[ \eta_{ij} = \eta_i + \zeta_{ij}, \quad (this \ may \ be \ viewed \ as \ \eta_{Bj} + \eta_{Wij}) \quad (10) \]
\[ \nu_{ij} = \nu_i + \delta_{ij}, \quad (11) \]
\[ \lambda_{ij} = \lambda_i + \delta_{ij}, \quad (12) \]

A first alternative to this model is that \( V(\eta_i) = 0 \) so that the factor with random loadings has only within-level variation. Instead, there can be a separate between-level factor with non-random loadings, measured by the random intercepts of the \( y \) indicators as in regular two-level factor analysis, \( y_{ij} = \lambda_{Bj} \eta_{Bj} + \zeta_{ij} \), where \( y_{ij} \) is the between part of \( y_{rij} \). A second alternative is that the \( \lambda_{Bj} \) loadings are equal to the means of the random loadings \( \lambda_i \).

Part 1: Random factor loadings (decomposition of the factor into within- and between-level parts)

```
TITLE: this is an example of a two-level MIMIC model with continuous factor indicators, random factor loadings, two covariates on within, and one covariate on between with equal loadings across levels
DATA: FILE = ex9.19.dat;
VARIABLE: NAMES = y1-y4 x1 x2 w clus;
WITHIN = x1 x2;
BETWEEN = w;
CLUSTER = clus;
ANALYSIS: TYPE = TWOLEVEL RANDOM;
ESTIMATOR = BAYES;
PROCESSORS = 2;
BITER = (1000);
MODEL: %WITHIN%
  s1-s4 | f BY y1-y4;
f@1;
f ON x1 x2;
%BETWEEN%
  f ON w;
  f;
PLOT: TYPE = PLOT2;
OUTPUT: TECH1 TECH8;
```

Part 2: Random factor loadings and a separate between-level factor

```
MODEL: %WITHIN%
  s1-s4 | f BY y1-y4;
f@1;
f ON x1 x2;
%BETWEEN%
  fb BY y1-y4;
  fb ON w;

f@0; is the between-level default
```

Part 3: Random factor loadings and a separate between-level factor with loadings equal to the mean of the random loadings

```
MODEL: %WITHIN%
  s1-s4 | f BY y1-y4;
f@1;
f ON x1 x2;
%BETWEEN%
  fb BY y1-y4* (lam1-lam4);
  fb ON w;
  [s1-s4*1] (lam1-lam4);
```
Monte Carlo Simulations for Groups as Random Mode: Two-Level Random Loadings Modeling

- The effect of treating random loadings as fixed parameters
  - Continuous variables
  - Categorical variables
- Small number of clusters/groups

The Effect of Treating Random Loadings as Fixed Parameters with Continuous Variables

<table>
<thead>
<tr>
<th>parameter</th>
<th>Bayes</th>
<th>ML with fixed loadings</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_1$</td>
<td>0.00(0.97)</td>
<td>0.20(0.23)</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>0.01(0.95)</td>
<td>0.14(0.66)</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>0.01(0.96)</td>
<td>0.00(0.80)</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>0.02(0.89)</td>
<td>0.00(0.93)</td>
</tr>
</tbody>
</table>

Ignoring the random loadings leads to biased mean and variance parameters and poor coverage. The loading is unbiased but has poor coverage.

The Effect of Treating Random Loadings as Fixed Parameters in Categorical Variables

<table>
<thead>
<tr>
<th>parameter</th>
<th>Bayes</th>
<th>WLSMV with fixed loadings</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_1$</td>
<td>0.05(0.96)</td>
<td>0.17(0.63)</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>0.03(0.92)</td>
<td>0.13(0.39)</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>0.05(0.91)</td>
<td>0.11(0.70)</td>
</tr>
</tbody>
</table>

Ignoring the random loadings leads to biased mean, loading and variance parameters and poor coverage.

Random Loadings with Small Number of Clusters/Groups

- Many applications have small number of clusters/groups. How many variables and random effects can we use?
- Independent random effects model - works well even with 50 variables (100 random effects) and 10 clusters
- Weakly informative priors are needed to eliminate biases for cluster level variance parameters
- Correlated random effects model (1-factor model) - works only when "number of clusters > number of random effects". More than 10 clusters are needed with 5 variables or more.
- What happens if you ignore the correlation: standard error underestimation, decreased accuracy in cluster specific estimates
- Using BSEM with 1-factor model for the random effects and tiny priors $N(1, \sigma)$ for the loadings resolves the problem.

- Survey of 67 hospitals, $n = 7168$ employee respondents, approximately 100/hospital
- 6 dimensions of an overall "quality improvement implementation" based on the Malcom Baldrige National Quality Award criteria
- Focus on 10 items measuring a leadership dimension

Hospital Data: Old and New Factor Analysis Alternatives

- **Hospital as Fixed Mode:**
  - Old approach: Conventional multiple-group factor analysis
  - New approach: BSEM multiple-group factor analysis

- **Hospital as Random Mode:**
  - Old approach: Conventional two-level factor analysis
  - New approach: Bayes random loadings two-level factor analysis (random factor variances also possible)

Hospital as Fixed Mode:

**Conventional Multiple-Group Factor Analysis**

Regular ML analysis:

```
USEVARIABLES = lead21-lead30! info31-info37!
straqp38-straqp44 hru45-hru52 qm53-qm58 hosp;
MISSING = ALL(-999); !CLUSTER = hosp;
GROUPING = hosp (101 102 104 105 201 301-306
308 310-314 316-320 322 401-403 405-409 412-416
501-503 505-512 602-609 612-613 701 801 901-908);
ANALYSIS: ESTIMATOR = ML;
PROCESSORS = 8;
MODEL:
  lead BY lead21-lead30;
PLOT: TYPE = PLOT2;
OUTPUT: TECH1 TECH8 MODINDICES(ALL);
```

Maximum-likelihood analysis with $\chi^2$ test of model fit and modification indices.

Holding measurement parameters equal across groups/hospitals results in poor fit with many moderate-sized modification indices and none that sticks out as much larger than the others.

Conventional multiple-group factor analysis "fails".
New Solution No. 1: Group as Fixed Mode using Multiple-Group BSEM

BSEM Input Excerpts for Hospital Data

USEVARIABLES = lead21-lead30 hosp;
MISSING = ALL (-999);
CLASSES = c(67);
KNOWNCLASS = c(hosp=101 102 104 105 201 301-306 308 310-314 316-320 322 401-403 405-409 412-416 501-503 505-512 602-609 612-613 701 801 901-908);

ANALYSIS:
TYPE = MIXTURE;
ESTIMATOR = BAYES;
BITERATIONS = (2000); ! min number of Bayes iterations
PROCESSORS = 2;
MODEL = ALLFREE;
! changes the mixture default of across-class equality of 
! nu, Lambda, Psi, and Theta parameters when using BY

BSEM Input Excerpts for Hospital Data, Continued

Use the auto-labeling feature (# is the class number) in MODEL and specify approximate measurement equality for intercepts and factor loadings across classes/groups in MODEL PRIORS:

MODEL:

MODEL: %OVERALL%
lead BY lead21-lead30* (lam#1-lam#10);
[lead21-lead30] (nu#1-nu#10);
%c#67%
lead@1; ! sets the metric of the factor

MODEL PRIORS:
DO(1,10) DIFF(lam1#-lam67#)∼N(0,0.05);
DO(1,10) DIFF(nu1#-nu67#)∼N(0,0.05);

PLOT:
TYPE = PLOT2;
OUTPUT: TECH1 TECH8;

BSEM Analysis of Hospital Data

BSEM Estimates for Hospital Data

67-group BSEM requires 2142 parameters (!)

Computing time: 3:07:35
BSEM Estimates for Hospital 66

<table>
<thead>
<tr>
<th>LEAD</th>
<th>BY</th>
<th>LEAD1</th>
<th>0.983</th>
<th>0.086</th>
<th>0.000</th>
<th>0.818</th>
<th>1.154</th>
<th>*</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEAD2</td>
<td>1.077</td>
<td>0.101</td>
<td>0.000</td>
<td>0.877</td>
<td>1.269</td>
<td>*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LEAD3</td>
<td>0.927</td>
<td>0.092</td>
<td>0.000</td>
<td>0.749</td>
<td>1.113</td>
<td>*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LEAD4</td>
<td>0.994</td>
<td>0.084</td>
<td>0.000</td>
<td>0.826</td>
<td>1.156</td>
<td>*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LEAD5</td>
<td>1.091</td>
<td>0.081</td>
<td>0.000</td>
<td>0.933</td>
<td>1.248</td>
<td>*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LEAD6</td>
<td>1.080</td>
<td>0.089</td>
<td>0.000</td>
<td>0.912</td>
<td>1.258</td>
<td>*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LEAD7</td>
<td>1.047</td>
<td>0.086</td>
<td>0.000</td>
<td>0.874</td>
<td>1.212</td>
<td>*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LEAD8</td>
<td>1.076</td>
<td>0.088</td>
<td>0.000</td>
<td>0.908</td>
<td>1.248</td>
<td>*</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Means

| LEAD  | 0.383  | 0.099  | 0.000  | -0.567 | -0.177 | *  |       |   |

Intercepts

| LEAD1 | 3.701  | 0.087  | 0.000  | 3.528  | 3.863  | *  |       |   |
| LEAD2 | 3.460  | 0.096  | 0.000  | 3.274  | 3.642  | *  |       |   |
| LEAD3 | 3.524  | 0.090  | 0.000  | 3.344  | 3.694  | *  |       |   |
| LEAD4 | 3.702  | 0.083  | 0.000  | 3.533  | 3.859  | *  |       |   |
| LEAD5 | 3.739  | 0.087  | 0.000  | 3.571  | 3.909  | *  |       |   |
| LEAD6 | 3.762  | 0.085  | 0.000  | 3.578  | 3.922  | *  |       |   |
| LEAD7 | 3.598  | 0.092  | 0.000  | 3.407  | 3.770  | *  |       |   |
| LEAD8 | 3.431  | 0.099  | 0.000  | 3.238  | 3.635  | *  |       |   |
| LEAD9 | 3.423  | 0.085  | 0.000  | 3.263  | 3.593  | *  |       |   |
| LEAD10| 3.613  | 0.090  | 0.000  | 3.430  | 3.778  | *  |       |   |

Variances

| LEAD  | 0.677  | 0.118  | 0.000  | 0.491  | 0.943  | *  |       |   |

BSEM Measurement Intercept Differences Across Hospitals: Intercept for Item 3 for all 67 Hospitals

<table>
<thead>
<tr>
<th>Average</th>
<th>Std. Dev.</th>
<th>Deviations from the Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>NU1_3</td>
<td>3.401</td>
<td>0.042</td>
</tr>
<tr>
<td>NU2_3</td>
<td>-0.140</td>
<td>-0.048</td>
</tr>
<tr>
<td>NU3_3</td>
<td>0.187</td>
<td>-0.176</td>
</tr>
<tr>
<td>NU4_3</td>
<td>-0.054</td>
<td>-0.219</td>
</tr>
<tr>
<td>NU5_3</td>
<td>0.021</td>
<td></td>
</tr>
</tbody>
</table>

Displaying Non-Invariant Items for BSEM: Hospitals With Significant Differences Compared to the Mean (Prior V = 0.01 + Prior V = 0.05)

<table>
<thead>
<tr>
<th>Item</th>
<th>Loadings</th>
<th>Intercepts</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>48 + 67</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>24, 33, 45 + 47</td>
<td>33, 37, 40, 45, 47 + 24, 39, 50, 52, 57</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>45, 51, 64 + 3, 6</td>
</tr>
<tr>
<td>4</td>
<td>-</td>
<td>7, 34</td>
</tr>
<tr>
<td>5</td>
<td>- + 67</td>
<td>15 + 3, 34, 41, 53</td>
</tr>
<tr>
<td>6</td>
<td>- + 25, 67</td>
<td>52, 60 - 52</td>
</tr>
<tr>
<td>7</td>
<td>67 + 47</td>
<td>52 + 5, 11, 26</td>
</tr>
<tr>
<td>8</td>
<td>45 + 36</td>
<td>17, 60 + 1, 7, 26, 45, 53, 55, 59</td>
</tr>
<tr>
<td>9</td>
<td>- + 67</td>
<td>52 + 51</td>
</tr>
<tr>
<td>10</td>
<td>-</td>
<td>67</td>
</tr>
</tbody>
</table>

Group as Random Mode: Conventional Two-Level Factor Analysis

- Recall random effects ANOVA (individual i in cluster j):
  $$y_{ij} = \nu + \eta_j + \epsilon_{ij} = y_{Bj} + y_{Wj}$$  \hfill (13)

- Two-level factor analysis ($r = 1, 2, \ldots, p$ items; 1 factor on each level):
  $$y_{rij} = \nu_r + \lambda_r \eta_j + \epsilon_{rij} + \lambda W_{ij}$$ $$\eta_{ij} = \eta_{Bj} + \eta_{Wij}$$  \hfill (14)

- Alternative expression often used in 2-level IRT:
  $$y_{rij} = \nu_r + \lambda_r \eta_j + \epsilon_{rij}$$ $$\eta_{ij} = \eta_{Bj} + \eta_{Wij}$$  \hfill (15)

so that $\lambda$ is the same for between and within.
Input Excerpts for Hospital as Random Mode: Conventional Two-Level Factor Analysis

USEVARIABLES = lead21-lead30;
MISSING = ALL (-999);
CLUSTER = hosp;

ANALYSIS:
TYPE = TWOLEVEL;
ESTIMATOR = ML;
PROCESSORS = 8;

MODEL:
%WITHIN%
leadw BY lead21-lead30* (lam1-lam10);
leadw@1;

%BETWEEN%
leadb BY lead21-lead30* (lam1-lam10);
leadb;

OUTPUT: TECH1 TECH8 MODINDICES(ALL);

Results for Hospital as Random Mode: Conventional Two-Level Factor Analysis

Equality of within- and between-level factor loadings cannot be rejected by $\chi^2$ difference testing

10% of the total variance in the leadership factor is due to between-hospital variation

No information about measurement invariance across hospitals

New Solution No. 2: Hospital as Random Mode using Two-Level Analysis with Random Loadings

In the interest of time, this Hospital data analysis is not reported.

Instead, an application for binary items in PISA data for 40 countries is shown in the next part.

Multiple-Group BSEM: Cross-Cultural Comparisons


- Data from the ISSP 2003 National Identity Module
- 34 countries, n=45,546
- 5 measurements of nationalism and patriotism
- Expected 2-factor structure

Many other country comparisons, e.g. TIMMS
Two-factor CFA with measurement invariance across all 34 countries: 
$\chi^2 (334) = 9669, p = 0 (!)$

Group-specific misfit evenly spread over the countries

Modification indices show a multitude of similarly large values

Alternatives:

- EFA for each group
- ESEM for all groups
- BSEM for all groups

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Australia ($n = 2146$): EFA $\chi^2 (1) = 0.402$

<table>
<thead>
<tr>
<th>GEOMIN ROTATED LOADINGS (* significant at 5% level)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>V21 1.521*</td>
</tr>
<tr>
<td>V22 0.569*</td>
</tr>
<tr>
<td>V26 0.001</td>
</tr>
<tr>
<td>V29 -0.041</td>
</tr>
<tr>
<td>V35 0.223*</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>GEOMIN FACTOR CORRELATIONS (* significant at 5% level)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>1 1.000</td>
</tr>
<tr>
<td>2 0.268*</td>
</tr>
</tbody>
</table>

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Germany ($n = 1281$): EF A

<table>
<thead>
<tr>
<th>GEOMIN ROTATED LOADINGS (* significant at 5% level)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>V21 1.159*</td>
</tr>
<tr>
<td>V22 0.448*</td>
</tr>
<tr>
<td>V26 0.002</td>
</tr>
<tr>
<td>V29 -0.078*</td>
</tr>
<tr>
<td>V35 0.022</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>GEOMIN FACTOR CORRELATIONS (* significant at 5% level)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>1 1.000</td>
</tr>
<tr>
<td>2 0.110</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ESTIMATED RESIDUAL VARIANCES</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>V21 -0.343</td>
</tr>
<tr>
<td>V22 0.759</td>
</tr>
<tr>
<td>V26 0.577</td>
</tr>
<tr>
<td>V29 0.567</td>
</tr>
<tr>
<td>V35 0.630</td>
</tr>
</tbody>
</table>

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USA ($n = 1210$): No convergence for EFA

**ESEM:**

**ANALYSIS:**

**MODEL:**

f1-f2 BY v21-v35 (*1);
v21-v35 (var1-var5);

**MODEL CONSTRAINT:**

DO(1,5) var#>0;
Nationalism and Patriotism Data: ESEM Solution for USA (STDY), $\chi^2(1) = 4.763$

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>S.E.</th>
<th>Est./S.E.</th>
<th>Two-Tailed P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1 BY</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V21</td>
<td>1.000</td>
<td>0.000</td>
<td>15267.805</td>
<td>0.000</td>
</tr>
<tr>
<td>V22</td>
<td>0.413</td>
<td>0.027</td>
<td>15.480</td>
<td>0.000</td>
</tr>
<tr>
<td>V26</td>
<td>-0.020</td>
<td>0.035</td>
<td>-0.553</td>
<td>0.580</td>
</tr>
<tr>
<td>V29</td>
<td>0.002</td>
<td>0.008</td>
<td>0.303</td>
<td>0.762</td>
</tr>
<tr>
<td>V35</td>
<td>0.087</td>
<td>0.037</td>
<td>2.353</td>
<td>0.019</td>
</tr>
<tr>
<td>F2 BY</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V21</td>
<td>-0.001</td>
<td>0.000</td>
<td>-4.245</td>
<td>0.000</td>
</tr>
<tr>
<td>V22</td>
<td>0.224</td>
<td>0.035</td>
<td>6.466</td>
<td>0.000</td>
</tr>
<tr>
<td>V26</td>
<td>0.641</td>
<td>0.042</td>
<td>15.418</td>
<td>0.000</td>
</tr>
<tr>
<td>V29</td>
<td>0.503</td>
<td>0.035</td>
<td>14.249</td>
<td>0.000</td>
</tr>
<tr>
<td>V35</td>
<td>0.551</td>
<td>0.040</td>
<td>13.939</td>
<td>0.000</td>
</tr>
<tr>
<td>F2 WITH F1</td>
<td>0.270</td>
<td>0.049</td>
<td>5.489</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Residual Variances

|                |          |       |           |                    |
| V21            | 0.000    | 0.000 | 24.485    | 0.000              |
| V22            | 0.729    | 0.024 | 30.424    | 0.000              |
| V26            | 0.595    | 0.049 | 12.075    | 0.000              |
| V29            | 0.747    | 0.035 | 21.467    | 0.000              |
| V35            | 0.663    | 0.040 | 16.703    | 0.000              |

No convergence due to large negative residual variances for many countries

Constraining residual variances to be non-negative gives convergence but the measurement invariance model has $\chi^2(496) = 13893$

Nationalism and Patriotism Data: Multiple-Group ESEM

Multiple-group BSEM allows cross-loadings similar to ESEM, but also allows approximate measurement invariance

With a small number of variables, setting the metric by fixing factor variances at 1 may lead to poor mixing in Bayesian analysis. This occurs in these data.

An alternative is to fix the factor loading at 1 in one group and let the other groups’ factor loading be approximately 1 by small-variance priors

Nationalism and Patriotism Data: Multiple-Group BSEM

CLASSES = c(34);  
KNOWNCLASS = c(cntry = 1 2 4 6-8 10-22 24-28 30-33 36 37 40-43);  
ANALYSIS: TYPE = MIXTURE;  
ESTIMATOR = BAYES;  
PROCESSORS = 2;  
MODEL = ALLFREE;  
MODEL:  
%OVERALL%  
nat BY v21* (lam#_1)  
v22 (lam#_2);  
pat BY v26* (lam#_3)  
v29-v35* (lam#_4-lam#_5);  
[v21-v35] (nu#_1-nu#_5);  
pat BY v21-v22* (xlam#_1-xlam#_2);  
nat BY v26-v35* (xlam#_3-xlam#_5);  

Bengt Muthén & Tihomir Asparouhov  New Developments in Mplus Version 7 144/146
%c#1%
nat BY v21@1;
pat BY v26@1;
[nat-pat@0];
%c#34%
[nat-pat];

MODEL PRIORS:
DO(2,2) DIFF(lam1_-lam34_-)~N(0,0.01);
DO(4,5) DIFF(lam1_-lam34_-)~N(0,0.01);
DO(1,5) DIFF(nu1_-nu34_-)~N(0,0.01);
DO(2,34) lam#_1~N(1,0.01);
DO(2,34) lam#_3~N(1,0.01);
DO(1,5) xlam1_-xlam34_-~N(0,0.01);

34-Country Factor Mean Estimates: Patriotic Factor.
Measurement Difference Prior Variance 0.001 Versus 0.01