

Auxiliary Variables in Mixture Modeling:
Using the BCH Method in Mplus to Estimate a
Distal Outcome Model and an Arbitrary
Secondary Model

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Mplus Web Notes: No. 21

Version 2

May 14, 2015

1 Introduction

In mixture modeling, indicator variables are used to identify an underlying latent categorical variable. In many practical applications we are interested in using the latent categorical variable for further analysis and exploring the relationship between that variable and other, auxiliary observed variables. If we use a direct approach where the auxiliary variables are included in the mixture model the latent class variable may have an undesirable shift in the sense that it is no longer measured simply by the original latent class indicator variables but now it is also measured by the auxiliary variables. The shift can be so substantial that the analysis can yield meaningless results because it is no longer based on the original latent class variable.

Different approaches have been proposed recently to remedy this problem and are discussed in detail in Asparouhov and Muthén (2014). Among these are the 3-step approach proposed by Vermunt (2010) and the approach of Lanza et al. (2013). Both of these approaches are implemented in Mplus and the details of that implementation are discussed in Asparouhov and Muthén (2014). It is pointed out in Asparouhov and Muthén (2014) that the 3-step approach does not resolve the problem of shifting classes completely. In some situations when the auxiliary variable is included in the final stage the latent class variable can shift substantially and invalidate the results. Mplus monitors the shift in classes with the 3-stage approach and if this shift is substantial results are not reported. This monitoring is conducted with the automatic Mplus commands DU3STEP and DE3STEP, however, if a manual 3-step approach is conducted the monitoring has to be done manually as well.

Further simulation studies conducted in Bakk and Vermunt (2014) confirm the finding that the 3-step approach fails in certain situations. Bakk and Vermunt (2014) also point out that the approach of Lanza (2013) for distal continuous outcomes, implemented in Mplus with the DCON command, can also fail due to assumptions underlying this method, primarily related to unequal variance across classes. The method yields poor results when the entropy is low and there is a substantial difference between the variances of the distal outcome across classes. If either one of these is not present then Lanza's method works well. With categorical distal outcome Lanza's method has no such drawbacks or any assumptions that can be violated.

A method proposed in Bray et al. (2014) appears to yield results similar to the method in Lanza et al. (2013) for continuous distal outcomes. This method also fails when the distal outcome has unequal variance across classes.

Bakk and Vermunt (2014) also consider in simulation studies the modified BCH method, BCH for short, described in Vermunt (2010) and also in Bakk et al. (2013). For the distal outcome model that evaluates the means across classes for a continuous auxiliary variable these simulations show that the BCH method substantially outperforms Lanza's method and the 3-step method. The BCH method avoids shifts in latent class in the final stage that the 3-step method is susceptible to. In its final stage the BCH method uses a weighted multiple group analysis, where the groups correspond to the latent classes, and thus the class shift is not possible because the classes are known. In addition, the BCH method performs well when the variance of the auxiliary variable differs substantially across classes, i.e., resolving the problems that Lanza's method is susceptible to.

The BCH method uses weights w_{ij} which reflect the measurement error of the

latent class variable. In the estimation of the auxiliary model, the i -th observation in class/group j is assigned a weight of w_{ij} and the auxiliary model is estimated as a multiple group model using these weights. The main drawback of the BCH method is that it is based on weighting the observations with weights that can take negative values. If the entropy is large and the latent class variable is measured without error then the weight w_{ij} is 1 if the i -th observation belongs to class j and zero otherwise. If the entropy is low, however, the weights w_{ij} can become negative and the estimates for the auxiliary model can become inadmissible. For example, it is possible that the variance of the distal outcome is estimated to a negative value or that the frequency table of a categorical auxiliary variable has a negative value. In such cases it would be difficult to utilize the BCH method beyond the basic distal outcome mean comparison model. Bakk and Vermunt (2014) show that the means of a continuous distal outcomes can be estimated correctly even when the sample group specific variances are negative. To obtain an admissible solution the estimated model holds equal the variances across group/class. In this simple model the mean and variance estimates are independent and thus the equal variance restriction has no effect on the mean estimates. However, if one is interested in evaluating the effect of the latent class variable on a more general auxiliary model it is not clear how to resolve the problems with inadmissible solutions due to negative weights.

Two versions of the BCH method are implemented in Mplus. The first version is referred to as the automatic version. This procedure evaluates the mean of a continuous distal outcome variable across classes using the approach of Bakk and Vermunt (2014). In this version one simply specifies the measurement model for the latent class variable and specifies the auxiliary variable as such. The second

version is the manual version which allows us to estimate the effect of a latent class variable on an arbitrary auxiliary model. This version requires two separate runs. In the first run we estimate the latent class measurement model and save the BCH weights. In the second run we estimate the general auxiliary model conditional on the latent class variable using the BCH weights. Both BCH versions are illustrated in the next two sections.

2 The automatic BCH approach for estimating the mean of a distal continuous outcome across latent class

This approach is very similar to the DU3STEP and DE3STEP commands in Mplus. With the following input file we estimate a latent class model using the 8 binary indicator variables U_1, \dots, U_8 . We also independently estimate the mean of the auxiliary variable Y across the different classes with the BCH method.

Variable:

Names are U1-U8 Y;

Categorical = U1-U8;

Classes = C(4);

Auxiliary = Y(bch);

Data: file=a1.dat;

Analysis: Type = Mixture;

The model estimates for the latent class model are not affected by the auxiliary variable and the results for the auxiliary variable mean estimates can be located in the output file

3 Using Mplus to conduct the BCH method with an arbitrary secondary model

In many situations it would be of interest to estimate a more advanced secondary model with the BCH method. In the Mplus implementation the secondary model can be an arbitrary model with any number and types of variables. The model is essentially estimated as a multiple group model as if the latent class variable is observed. The BCH method uses group specific weights for each observation that are computed during the latent class model estimation. An outline of the procedure is as follows. First estimate a latent class model using only the latent class indicator variables and save the BCH weights. All variables that will be

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EQUALITY TESTS OF MEANS ACROSS CLASSES USING THE BCH PROCEDURE
WITH 3 DEGREE(S) OF FREEDOM FOR THE OVERALL TEST

Y

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	Mean	S.E.		Mean	S.E.
Class 1	-1.063	0.079	Class 2	-0.363	0.139
Class 3	1.416	0.096	Class 4	0.295	0.088
	Chi-Square	P-Value		Chi-Square	P-Value
Overall test	420.441	0.000	Class 1 vs. 2	15.023	0.000
Class 1 vs. 3	405.734	0.000	Class 1 vs. 4	99.557	0.000
Class 2 vs. 3	87.209	0.000	Class 2 vs. 4	17.808	0.000
Class 3 vs. 4	60.329	0.000			

used in the secondary model should be placed in the auxiliary variable command without any specification. That way the auxiliary variables will be saved in the same file as the BCH weights. This is step 1 of the estimation. In step 2 we simply specify the auxiliary model and we use the BCH weights as training data.

3.1 Regression auxiliary model

In the following example we estimate the auxiliary regression model of a dependent variable Y on a covariate X . We measure a 3-class latent variable using an LCA model with 10 binary items and then use that latent variable to estimate class specific regression Y on X . The example and the data are the same as the example presented on page 332 in Asparouhov and Muthén (2014). In the first step we use the following input file to estimate the LCA model and save the BCH weights

Variable:

Names=U1-U10 Y X;

Categorical = U1-U10;

Classes = C(3);

Usevar=U1-U10;

Auxiliary=Y X;

Data: file=manBCH.dat;

Analysis: Type = Mixture;

Savedata: File= manBCH2.dat; Save=bchweights;

Here the key command is **Save=bchweights;** which requests the BCH weight

for further analysis. In the second step the following input file can be used to estimate the class specific regression of Y on X .

Variable:

Names = U1-U10 Y X W1-W3 MLC;

Usevar are Y X W1-W3;

Classes = C(3);

Training=W1-W3(bch);

Data: file=manBCH2.dat;

Analysis: Type = Mixture; Starts=0; Estimator=mlr;

Model:

%overall%

Y on X;

%C#1%

Y on X;

%C#2%

Y on X;

%C#3%

Y on X;

Note that the latent class indicator variables U1-U10 are not on the USEVAR list in this step. The key commands here are **Training=W1-W3(bch)**; which specifies the BCH weights to be used in this secondary analysis, **Starts=0**; because this is a multiple group analysis and random starting values are not

needed, and **Estimator=mlr**; because that estimator leads to better standard errors because the analysis utilizes weights, see Bakk and Vermunt (2014). The results of the auxiliary model estimation are found as usual in the output file of the second step run.

3.2 Regression auxiliary model combined with latent class regression

Distal outcomes are often studied in the presence of covariates so that the effect of the latent class variable on the distal is controlled for by those covariates. This is a variation on the modeling just discussed where the covariate X influences not only Y but also the latent class variable. Following is an illustration of the manual BCH estimation for such a model.

The auxiliary model we are interested in estimating with the BCH method is given by the following two equations

$$Y|C = \alpha_c + \beta_c X$$

$$P(C = c|X) = \frac{\text{Exp}(\gamma_{0c} + \gamma_{1c}X)}{\sum_c \text{Exp}(\gamma_{0c} + \gamma_{1c}X)}$$

We illustrate this BCH manual estimation with a four class model measured by 8 binary indicators U_i where

$$P(U_i = 1|C) = 1/(1 + \text{Exp}(s_{ci}\tau))$$

where $s_{1p} = -1$, $s_{4p} = 1$, $s_{2p} = 1$ for $p = 1, \dots, 4$, $s_{1p} = -1$ for $p = 5, \dots, 8$, $s_{3p} = -1$

for $p = 1, \dots, 4$ and $s_{3p} = 1$ for $p = 5, \dots, 8$. We set the value of τ to 1 to generate the data. We generate a single data set of size $N = 50000$ according to the above model. The first step model input is as follows.

Variable:

Names are U1-U8 y x;

Usevar=U1-U18;

Categorical = U1-U8;

Classes = C(4);

Auxiliary=Y X;

Data: file=1.dat;

Analysis: Type = Mixture; starts=0;

Savedata: File= 2.dat; Save=bchweights;

Model:

%Overall%

%c#1%

[U1\$1-U8\$1*-1.0] ;

%c#2%

[U1\$1-U4\$1*1.0 U5\$1-U8\$1*-1.0] ;

%c#3%

[U1\$1-U4\$1*-1.0 U5\$1-U8\$1*1.0] ;

%c#4%

[U1\$1-U8\$1*1.0] ;

Starting values are provided so that the class order does not reverse from the generated order. In real data analysis starting values are not needed. Instead, a large number of random starting value should be set using the **starts** command. The second step input is as follows

Variable:

Names = U1-U8 Y X W1-W4 MLC;

Usevar = Y X W1-W4;

Classes = c(4);

Training=W1-W4(bch);

Data: file=2.dat;

Analysis: Type = Mixture; starts=0;

Model:

%Overall%

C on X;

Y on X;

%c#1%

Y on X;

%c#2%

Y on X;

%c#3%

Y on X;

%c#4%

Y on X;

The results of this simulation are presented in Table 1. All estimates are close to the true parameter values and all but one of them are within the implied confidence limits. Thus we conclude that the manual BCH approach can be used for more complex auxiliary models. If we remove the variable Y from the above example we get an example where the auxiliary variable is a latent class predictor. Thus the BCH manual approach can be used as an alternative to the R3STEP auxiliary command which uses a 3-step estimation approach.

4 Simulation study with a continuous distal auxiliary outcome

In this section we extend the simulation studies presented in Section 6.1 of Asparouhov and Muthén (2014) to include the BCH method and the Lanza et al. (2013) method referred to as DCON. For completeness we describe the simulation and include the results already presented in that article.

We estimate a 2-class model with 5 binary indicator variables. The distribution for each binary indicator variable U is determined by the usual logit relationship

$$P(U = 1|C) = 1/(1 + \text{Exp}(\tau_c))$$

where C is the latent class variable which takes values 1 or 2 and the threshold value τ_c is the same for all 5 binary indicators. In addition we set $\tau_2 = -\tau_1$ for all five indicators. We choose three values for τ_1 to obtain different level of class separation/entropy. Using the value of $\tau_1 = 1.25$ we obtain an entropy of 0.7,

Table 1: Manual BCH estimation

Parameter	True Value	Estimated Value	SE
α_1	0	0.013	0.035
α_2	1	0.984	0.030
α_3	0	0.123	0.042
α_4	2	1.979	0.022
β_1	1	0.964	0.037
β_2	2	2.043	0.047
β_3	-1	-0.910	0.046
β_4	0	-0.005	0.027
γ_{11}	1	1.004	0.027
γ_{12}	0.5	0.542	0.029
γ_{13}	-0.3	-0.246	0.030

with value $\tau_1 = 1$ we obtain an entropy of 0.6, and with value $\tau_1 = 0.75$ we obtain an entropy of 0.5. The latent class variable is generated with proportions 43% and 57%. In addition to the above latent class model we also generate a normally distributed distal auxiliary variable with mean 0 in class one and mean 0.7 in class 2 and variance 1 in both classes. We apply the pseudo-class method, the 3-step method, Lanza’s method, the 1-step method, and the BCH method to estimate the mean of the auxiliary variable in the two classes.

Table 2 presents the results for the mean of the auxiliary variable in class 2. We generate 500 samples of size 500 and 2000 and analyze the data with the five methods. The results in Table 2 show that the BCH procedure and the 3-step procedure have almost identical performance in terms of bias, MSE and coverage. In this simulation the BCH method shows no bias and the coverage is near the nominal level with the exception of the case of low entropy of 0.5 and sample size of 500 where a small bias is observed which also leads to decrease of coverage.

Table 2: Distal outcome simulation study: Bias/Mean Squared Error/Coverage

N	Entropy	PC (E)	3-step (DU3STEP)	Lanza (DCON)	1-step	BCH
500	0.7	.10/.015/.76	.00/.007/.95	.00/.006/.92	.00/.006/.94	.00/.007/.94
500	0.6	.16/.029/.50	.01/.008/.94	.00/.007/.89	.00/.007/.94	.01/.008/.94
500	0.5	.22/.056/.24	.03/.017/.86	.00/.012/.80	.01/.012/.96	.03/.017/.86
2000	0.7	.10/.011/.23	.00/.002/.93	.00/.002/.89	.00/.002/.93	.00/.002/.93
2000	0.6	.15/.025/.03	.00/.002/.93	.00/.002/.87	.00/.002/.94	.00/.002/.94
2000	0.5	.22/.051/.00	.00/.004/.91	.00/.003/.80	.00/.003/.94	.00/.004/.91

Next we conduct a simulation study to compare the performance of the four different methods DU3STEP, DE3STEP, Lanza’s method and the BCH method in the situation when the distal variable variances are different across class. The two 3-step approaches DU3STEP and DE3STEP differ in the third step. The DU3STEP approach estimates different means and variances for the distal variable in the different classes while the DE3STEP approach estimates different means but equal variances. The second approach is more robust and more likely to converge but may suffer from the mis-specification that the variances are held equal in the different classes. We use the same simulation as above except that we generate a distal outcome in the second class with variance 20 instead of 1. The results for the mean in the second class are presented in Table 3.

It is clear from these results that the unequal variance 3-step approach (DU3STEP) is superior particularly when the class separation is poor (entropy level of 0.6 or less). The equal variance approach (DE3STEP) can lead to severely biased estimates when the class separation is poor and the variances are different across classes. Lanza’s method appears to have completely failed particularly when the class separation is poor. The BCH method appears to be slightly worse

Table 3: Distal outcome with unequal variance simulation study: Bias/Mean Squared Error/Coverage

N	Entropy	DE3STEP	DU3STEP	Lanza(DCON)	BCH
500	0.7	.05/.147/.95	.00/.099/.94	.03/.129/.77	.00/.114/.93
500	0.6	.06/.174/.96	.00/.099/.95	.15/.397/.70	.00/.121/.94
500	0.5	.12/.822/.93	.01/.101/.95	1.20/5.755/.46	.04/.160/.94
2000	0.7	.05/.040/.92	.00/.027/.92	.03/.035/.76	.00/.029/.94
2000	0.6	.09/.056/.92	.00/.027/.93	.07/.056/.70	.00/.031/.93
2000	0.5	.11/.094/.95	.00/.029/.92	1.18/4.613/.44	.00/.041/.94

than the DU3STEP approach in terms of bias and MSE but the coverage remains good near the nominal level. Thus for the continuous distal variable estimation if the distal variable variances are unequal across class we can recommend only the DU3STEP and the BCH methods.

5 Simulation study with a non-normal distal auxiliary outcome

In Section 7.1 of Asparouhov and Muthén (2014) it was shown that when the distal outcome is not normally distributed the 3-step estimation can fail due to switching of the classes and the parameter estimates maybe severely biased. Further simulations illustrating this point were conducted in Bakk and Vermunt (2014). In this section we conduct a simulation study similar to the those in Bakk and Vermunt (2014).

We estimate and generate data according to a 4 class LCA model with 8 binary indicators. The class proportions are as follows: 0.375, 0.25, 0.1875 and 0.1875.

The measurement model is described as follows

$$P(U_p = 1|C) = 1/(1 + \text{Exp}(s_{cp}\tau))$$

where $s_{2p} = 1$, $s_{4p} = -1$, $s_{1p} = -1$ for $p = 1, \dots, 5$, $s_{1p} = 1$ for $p = 6, \dots, 8$, $s_{3p} = 1$ for $p = 1, \dots, 5$ and $s_{3p} = -1$ for $p = 6, \dots, 8$. We vary the value of τ to obtain different entropy value and class separation. If $\tau = 1.5$ the entropy is 0.7. If $\tau = 1.25$ the entropy is 0.6. If $\tau = 1$ the entropy is 0.5. The distal outcome in class 1 has the following bimodal distribution $0.5N(0, 0.1) + 0.5N(-2, 0.1)$, in class two it is also bimodal $0.75N(-2/3, 0.1) + 0.25N(2, 0.1)$, in class 3 it is the normal distribution $N(2, 0.1)$ and in class 4 it is the normal distribution $N(0.5, 0.1)$. We use three different sample sizes $N=2000$, 5000 and 10000 and generate and analyze 500 replications for each size. In this simulation we can expect that the DU3STEP, DE3STEP, 1-step and PC method to fail due to non-normality and we can expect Lanza's method to fail due to varying variances across class.

In Table 4 we present the results for the distal mean in class 2 for the most favorable case where Entropy=0.7 and $N = 10000$ for all of the estimation methods. No results are presented for the DE3STEP and DU3STEP because in almost all replications there was no convergence due to large differences between the step 1 class allocation and step 3 class allocation. Mplus will not report any results if substantial shift in the classes occur in step 3. The remaining methods fail dramatically as well with the exception of the BCH method. This simple simulation suggest that BCH may indeed be much more robust than any other method.

Next we evaluate the performance of the BCH method for different sample

Table 4: Non-normal distal outcome simulation study

Method	Bias	MSE	Coverage
DE3STEP	-	-	-
DU3STEP	-	-	-
Lanza	0.663	0.440	0.00
BCH	0.004	0.001	0.89
1-Step	0.643	0.414	0.00
PC	0.155	0.025	0.00

Table 5: Non-normal distal outcome simulation study for the BCH method

N	Entropy	Bias	MSE	Coverage	Std. Err/Std. Dev.
2000	0.7	0.00	0.007	0.89	0.82
5000	0.7	0.00	0.003	0.89	0.83
10000	0.7	0.00	0.001	0.89	0.81
2000	0.6	0.00	0.016	0.80	0.62
5000	0.6	0.00	0.005	0.82	0.66
10000	0.6	0.00	0.003	0.82	0.67
2000	0.5	0.05	0.057	0.58	0.42
5000	0.5	0.01	0.021	0.59	0.43
10000	0.5	0.00	0.010	0.67	0.43

sizes and entropy levels. The results are presented in Table 5. The estimates are unbiased in all cases with small bias being visible for smaller sample sizes and entropy levels. On the other hand the coverage drops substantially particularly when the entropy is low. Also the ratio of the standard errors to the standard deviation, which should be near 1 for large sample sizes is consistently smaller and it does not improve with increasing the sample size. For example in the last row of Table 5 we see that even when the sample size is 10000 and entropy is 0.5 the ratio is 0.43, i.e., the standard errors are underestimated by 57% and should be nearly twice to what the method currently computes. This has been noted also in Bakk and Vermunt (2014) and has been suggested there that the underestimation occurs due to unaccounted variability of the posterior probabilities that are used as weights in step 3. The BCH method heavily depends on these posterior probabilities and one can expect that this effect is substantial. When the class separation is large the underestimation disappears which also reflects the diminished variability in the posterior probabilities. At this point no reasonable method is available to resolve this shortcoming although bootstrapping would resolve this problem and it can be run in Mplus as external montecarlo where the bootstrap samples are obtained separately.

6 Summary

Many methods have been proposed in recent years for mixture modeling with auxiliary variables. To clarify the choice of method, Table 6 and 7 list the Mplus options, give their intended use, and give recommendations on which method should be used for which purpose.

Table 6: Alternative auxiliary settings for mixture modeling

DU3STEP	
Useage:	Continuous distal outcomes
Description; reference:	Classification-error corrected; Vermunt (2010) and Asparouhov-Muthén (2014)
Pros and cons:	Susceptible to class changes. Mplus will not report results if the class formation changes. Manual version also available for an arbitrary auxiliary model, including controlling for covariates. Estimates unequal distal variances across classes.
Recommendation:	Preferred method for continuous distal outcomes Use when Mplus reports results, i.e., there are no class formation changes, otherwise use BCH.

BCH	
Useage:	Continuous distal outcomes
Description; reference:	Measurement-error weighted; Bakk and Vermunt (2014)
Pros and cons:	Avoids class changes. Avoids the DCON shortcomings with class-varying variances for distals. Manual version also available for an arbitrary auxiliary model, including controlling for covariates. Possible SE underestimation with low entropy.
Recommendation:	Preferred method for continuous distal outcomes

DCAT	
Useage:	Categorical distal outcomes
Description; reference:	Distal treated as covariate; Lanza et al. (2013)
Pros and cons:	Avoids class changes
Recommendation:	Preferred method for categorical distal outcomes

R3STEP	
Useage:	Covariates
Description; reference:	Classification-error corrected; Vermunt (2010)
Pros and cons:	Works well
Recommendation:	Recommended method with covariates

Table 7: Alternative auxiliary settings for mixture modeling, continued

DE3STEP	
Usage:	Continuous distal outcomes. Equal distal variances across classes
Description; reference:	Classification-error corrected; Vermunt (2010) and Asparouhov-Muthén (2014)
Pros and cons:	Susceptible to class changes and class-varying variances. Mplus will not report results if the class formation changes.
Recommendation:	Inferior to BCH and DU3STEP. Use only when DU3STEP does not converge.

DCON	
Usage:	Continuous distal outcomes
Description; reference:	Distal treated as covariate; Lanza et al. (2013) and Asparouhov-Muthén (2014)
Pros and cons:	Avoids class changes. Sensitive to class-varying variances for distals when entropy is low
Recommendation:	Inferior to BCH and DU3STEP when DU3STEP does not change the class formation. Use only when entropy is higher than 0.6. If variance appears to be varying across class more than a factor of 2 do not use this method. This check can be done using most likely class assignment - it is not done automatically by Mplus. Use only for methods research purposes

E	
Usage:	Continuous distal outcomes
Description; reference:	Pseudo-class (PC) method; Wang et al. (2005)
Pros and cons:	Gives biased results
Recommendation:	Superseded by BCH and DU3STEP. Use only for methods research purposes

R	
Usage:	Covariates
Description; reference:	Pseudo-class (PC) method; Wang et al. (2005)
Pros and cons:	Gives biased results
Recommendation:	Superseded by R3STEP. Use only for methods research purposes

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